Dynamic Modeling and Analysis of Shield TBM Cutterhead Driving System

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1 Introduction

Presently many civil projects of building subways are launched within China and around the world because crowded traffic conditions can be improved and relieved by constructing subways in the metropolitan areas. For most of these projects, the constraints such as construction site, city environment, road traffic, and other factors make it almost impossible to use the traditional digging methods for excavation. Instead the shield tunnel boring machine (TBM) is one type of large-scale underground equipment that can excavate the subways under these constraints. The basic advantages of shield TBM are high safety, rapid excavation speed, and low manpower. Meanwhile shield TBM has the merits of fewer disturbances to residents and less damage to the environment compared with other excavating methods. Thus, the shield TBM has become the primary choice for excavating the subways or tunnels. The shield TBM cutterhead is a key component because it cuts the rocks and soil through its rotations (the cutterhead has a separate thrust system). The cutterhead undertakes the task of excavating rocks and soil, so the cutterhead’s driving system plays an important role in a shield TBM. At present, many publications [7–16] generally introduce the structure and composition of the shield TBM cutterhead and its driving system, overall shield TBM composition, excavating performances and field operation experiences, and the shield tunneling methods. A dynamic model of cutterhead driving system has not been established. In this paper, we describe the driving system of the shield TBM cutterhead that is driven by multiple induction motors. A dynamic model of shield TBM cutterhead driving system is established to help better understand and analyze the cutterhead driving system in theory, to achieve multimotor synchronous control, and to acquire better dynamic and steady-state control properties of the cutterhead. The dynamic model of the driving system is simulated and its performances are studied. The dynamic model offers better understanding of the basic input-output characters of the system. Also, some potential problems of the driving system are discussed, which provides theoretical guidance for an actual project design and solid theoretical foundations for achieving the cutterhead’s multiple motors synchronous control. This paper is arranged as follows: Sec. 2 introduces the cutterhead and its driven modes. The generalized nonlinear time-varying (G-NLTV) dynamic model of shield TBM cutterhead driving system is formulated in Sec. 3. In Sec. 4, the dynamic model is simulated and the results are analyzed; meanwhile, the issues contained in the driving system are discussed. Section 5 reviews the study contents and briefly concludes the results.

2 Shield TBM Cutterhead and Its Driven Modes

The shield TBM cutterhead can be in the form of panel or spoke, of course hybrid form (panel and spoke), as shown in Figs. 1(a) and 1(b). The cutterhead section views are displayed in Figs. 1(c) and 1(d). The cutterhead utilizes the motor-driven mode and its inner mechanical transmission structure is shown in Fig. 2(a). Multiple active pinions mesh a central large passive gear, which is driven by an induction motor. Each active pinion is driven by an induction motor. The cutterhead profile drawing is shown in Fig. 2(b). Figure 3 shows the comprehensive driving system of the cutterhead. The multiple gears driving structure is employed to synthesize the electrical magnetic torque (EMT) of the driving motors. The cutterhead has the same central shaft as the large gear. Once the large gear rotates, the cutterhead is driven immediately. Hence, the large gear and cutterhead have an equal rotary angular speed. Figures 4(a)–4(c) shows the induction motor, reduction gear box (reducer), and coupling that are used in the cutterhead driving system. The reducer is a torque converter that cuts down the induction motor’s speed, so the reducer’s output torque is amplified.

The shield TBM has two driving modes: one is the hydraulic-driven mode and the other is the motor-driven mode. The cutterhead rotation speed is low due to the complex geological condi-
tions and large excavation areas, which requires the cutterhead driving system to have high-power and large output torque. The cutterhead cuts rocks and soil; thus, the cutterhead driving system is one of the largest energy-consuming segments of the shield TBM. To ensure cutting ability and effectiveness of the cutterhead, currently, most cutterheads apply the hydraulic-driven mode. The shield TBM with large digging diameter often uses hydraulic-driven mode. However, with the development of variable frequency technology of motors, motor-driven mode is gradually being applied in the shield TBM cutterhead, which is especially important to the shield TBM with small or medium excavation diameter. The hydraulic-driven mode can adapt to soft and hard rock conditions. The hydraulic-driven mode has better interoperability, good synchronous performance, and mature technology, and is thus more practical, but its drawbacks are larger size, higher noise, lower efficiency, and requires lots of auxiliary equipments. Motor-driven mode also has some shortcomings compared with the hydraulic-driven mode. The motor-driven cutterhead has smaller output torque and is not widely used. The critical drawback is the lack of mature technology and susceptibility to breakdown. In addition, its control system requires a good working environment. The underground operation environment is usual very harsh, which will affect the cutterhead control system. However, the motor-driven mode has many advantages such as higher efficiency, less power consumption, easier maintenance, and lower noise. The rotary position of the motor-driven cutterhead is easily located and remains at the given position. In addition, the rotary direction of motor-driven cutterhead is easily changed by altering the electrical phase sequence of motors.

3 Generalized Nonlinear Time-Varying Dynamic Model of the Cutterhead Driving System

3.1 Preliminary Preparation. Because of the manufacturing and installing process variations, mesh requirement differences, and a variety of other uncertainties, gear backlash is inevitable. At the same time, the transmission error naturally exists in the driving process, so the gear transmission is considered as a nonlinear process. The gear backlash can be shown, as in Figs. 5(a) and 5(b). As the mesh point diversifies, the practical backlash is varying as well and such a variation in the gear backlash is not considered in this paper. The gear backlash is simplified and shown in Fig. 5(b). The nonlinear dynamic model is shown in Fig. 6 when the gear transmission error and backlash are considered.

Based on the gear mesh dynamic in [1–6], the relative position function, elastic mesh force, and torque are obtained, respectively, where $e(t)$ is transmission error and $2\Delta_b$ is the total gear backlash.

$$p_2 = \begin{cases} \theta_{pl} \cdot r_i - \theta_m \cdot r_m - e(t) = \Delta_b \\ 0 \\ \theta_{pl} \cdot r_i - \theta_m \cdot r_m - e(t) + \Delta_b \end{cases}, \quad \begin{align*} \theta_{pl} \cdot r_i - \theta_m \cdot r_m - e(t) & \geq \Delta_b \\
- \Delta_b & < \theta_{pl} \cdot r_i - \theta_m \cdot r_m - e(t) < \Delta_b \\
\theta_{pl} \cdot r_i - \theta_m \cdot r_m - e(t) & \leq - \Delta_b \end{align*}$$
To simplify the dynamic model, the mesh damping $c_j$ is ignored. Hence, the elastic mesh force and torque of the pinion $i$ can be expressed as

$$F_i = f(p_i) = k_i p_i + c_i (\dot{\theta}_m - \dot{\theta}_m - \dot{r}_m - \dot{e}(t))$$  \hspace{1cm} (2)

$$M_{ij} = F_i \cdot r_i = k_i p_i + c_i (\dot{\theta}_m - \dot{r}_m - \dot{r}_m - \dot{e}(t))$$  \hspace{1cm} (3)

Fig. 2  (a) Cutterhead inner mechanical transmission structure; (b) the cutterhead profile
The system physical parameters are defined as

\[ \Delta = \Delta_i/r_i, \quad k_i^1 = k_i r_i, \quad k_i = k_i^1 r_i = k_i r_i^2, \quad i_m = r_m/r_i = r_m/r \]

where \( \Delta \) is the relative backlash and is a dimensionless parameter, \( k_i^1 \) and \( k_i \) are the elastic mesh force and torque coefficients, respectively, \( i_m \) is the gear transmission ratio. The mesh stiffness \( k_i(t) \) and transmission error \( e(t) \) are time-varying parameters that can be expressed as the Fourier series

\[ k_i(t) = k_0 + \sum_{j=1}^{\infty} k_j \cos(2 j \pi f z t + \phi_{k,j}) \]

\[ e(t) = e_0 + \sum_{j=1}^{\infty} e_j \cos(2 j \pi f z t + \phi_{e,j}) \]

where \( f_z \) is mesh frequency, and \( \phi_{k,j} \) and \( \phi_{e,j} \) are the phase of the mesh stiffness and transmission error, respectively. Two nonlinear time-varying parameters \( \varphi_i \) and \( \beta_i \) are brought to simplify the elastic mesh force and torque function. Defining the nonlinear parameters \( \varphi_i \) and \( \beta_i \)

\[ \varphi_i = \begin{cases} 1 |\theta_{\varphi_i} - \theta_{\varphi_m} + i_m - e(t)/r_i| \geq \Delta \\ 0 |\theta_{\varphi_i} - \theta_{\varphi_m} + i_m - e(t)/r_i| < \Delta \end{cases} \]
yield magnetic torque balance equations of the induction motors. According to the torque balance equations, the electrical lose the generality, we can assume that the cutterhead is driven by linear dynamic model of the shield TBM cutterhead and not to large, so the coupling is not ignored. To establish a general non-larger than the coupling mass significantly. The coupling itself is inertia can be ignored. In fact, the motor rotor mass cannot be equations are obtained:

\[ T_{e1} = J_1 \ddot{\theta}_1 + b_1 \dot{\theta}_1 + T_{e11} \]  

\[ T_{e2} = J_2 \ddot{\theta}_2 + b_2 \dot{\theta}_2 + T_{e22} \]

Likewise, the corresponding torque balance equations of the nth induction motor are

\[ T_{e1} = J_{1n} \dot{\theta}_1 + b_{1n} \dot{\theta}_1 + M_{11} \]

\[ T_{e2} = J_{2n} \dot{\theta}_2 + b_{2n} \dot{\theta}_2 + M_{22} \]

\[ T_{en} = J_{en} \dot{\theta}_n + b_{en} \dot{\theta}_n + M_{en} \]

where \( T_{en} \) is output torque of motor \( i \), \( M_{1i} \) is input torque of reducer \( i \), \( M_{2i} \) is the output torque of reducer \( i \) \((i=1,2,\ldots,n)\). Substituting Eqs. (11)–(15) into Eqs. (10)–(14), respectively, then the following equations are acquired after collation:

\[ T_{e1} = (J'_{d1} + J_{c1}) \dot{\theta}_1 + (b'_{d1} + b_{c1}) \dot{\theta}_1 + M_{11} \]

\[ T_{e2} = (J'_{d2} + J_{c2}) \dot{\theta}_2 + (b'_{d2} + b_{c2}) \dot{\theta}_2 + M_{22} \]

\[ T_{en} = (J'_{d1} + J_{c1}) \dot{\theta}_n + (b'_{d1} + b_{c1}) \dot{\theta}_n + M_{en} \]

We define the system parameters \( J_d = J'_{d1} + J_{c1} \) and \( J_{c1} = b'_{d1} + b_{c1} \) \((i=1,2,\ldots,n)\). Then Eqs. (16)–(18) can be rewritten in the following generalized form:

\[ T_{e1} = J_{d1} \ddot{\theta}_1 + b_{d1} \dot{\theta}_1 + M_{11} \]

\[ T_{e2} = J_{d2} \ddot{\theta}_2 + b_{d2} \dot{\theta}_2 + M_{22} \]

\[ T_{en} = J_{dn} \ddot{\theta}_n + b_{dn} \dot{\theta}_n + M_{en} \]

The input-output relationships of the ith reducer can be described by the equation

\[ \theta_i = q \theta_{pi}, \quad M_i = q M_{1i} \]

The torque balance equation of the ith pinion can be written as

\[ T_{e1} = J_{d1} \ddot{\theta}_1 + b_{d1} \dot{\theta}_1 + M_{11} \]

\[ T_{e2} = J_{d2} \ddot{\theta}_2 + b_{d2} \dot{\theta}_2 + M_{22} \]

\[ T_{en} = J_{dn} \ddot{\theta}_n + b_{dn} \dot{\theta}_n + M_{en} \]
The torque balance equations of large gear are obtained as
\[ TL_i = J_i \ddot{\theta}_i + b_i \dot{\theta}_i + M_{e,i} \quad (i = 1, 2, \ldots, n) \] (21)

The elastic mesh torque of the \( i \)th pinion and large gear has been presented in Sec. 3.1
\[ M_{ei} = k_{ei} \varphi_i \times (\theta_{pi} - \theta_m + i_m + \beta_i) \quad (i = 1, 2 \ldots n) \] (22)

The torque balance equations of large gear are obtained as
\[ M_i = J_i \ddot{\theta}_i + b_i \dot{\theta}_i + M_{e,i} \quad (i = 1,\ldots, n) \] (23)
\[ M_i = \sum_{i=1}^{n} M_{ei} \quad \text{for} \quad i = 1,\ldots, n \] (24)

where \( T_L \) is total load torque. The load torque has a large influence on the cutterhead driving system and can directly affect the shield TBM operation. During the operation, if the load torque is larger than the maximum design driving torque of the cutterhead, the cutterhead components could be damaged. For instance, the driving motor could be damaged due to the overload, and sometimes the cutter tools could fall off due to too large load torque, and bearings and other support apparatus could also be damaged. Many factors such as the cutterhead’s design parameters and structure, as well as geology conditions directly affect the load torque. The design parameters and structure of a cutterhead include the cutting depth, cutterhead opening ratio, and excavation diameter. The geology conditions contain the properties of soil, rocks, and boulders. As the complex and uncertain variation in geological conditions, the actual load torque will usually hop and change with time. References [17–21] studied the estimation and computation of the load torque for shield TBM. The load torque consists of the following six parts that are presented in Ref. [19]:

\[ TL = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = \sum_{j=1}^{6} T_j \] (25)

where \( T_1 \) is the resistant torque of soil and rocks, \( T_2 \) is the friction torque between the front of the cutterhead and soil and rocks, \( T_3 \) is the friction torque between the back of the cutterhead and soil and rocks, \( T_4 \) is the friction torque between the bulkhead and soil and rocks, \( T_5 \) is the stirring torque when the cutterhead stirs the soil and rocks, and \( T_6 \) is the friction torque of the cutterhead bearings and sealed chamber. Detailed derivations and descriptions of the each load torque component and its parameters can be found in Ref. [19]. Hence the computation formulae of the load torque are directly presented here

\[ T_1 = D^2 v \cos \alpha / 8 N_c \]
\[ T_2 = \pi \eta H_0 \mu D^3 / 12 \]
\[ T_3 = \pi \eta H_0 \mu (D^3 - D_1^3) / 12 \]
\[ T_4 = \pi \eta H_0 \mu B (D_1 + D_2^3) / 12 \]
\[ T_5 = \pi \eta H_0 \mu D_1 L_2 R_3 \]
\[ T_6 = (1.414 G_0 + K_\mu H) \mu D_1^2 / 2 \]

Equations (19) and (25) constitute the NLTV dynamic model, so, the general time-varying nonlinear dynamic model of the cutterhead driving system is obtained

\[ T_{ei} = J_i \ddot{\theta}_i + b_i \dot{\theta}_i + M'_{ei}, \quad \theta_i = q \theta_{pi}, \quad M'_{ei} = q M_{ei} \]
\[ M_{ei} = k_{ei} \varphi_i \times (\theta_{pi} - \theta_m + i_m + \beta_i) \]
\[ M'_{ei} = J_i \ddot{\theta}_{pi} + b_i \dot{\theta}_{pi} + M_{ei} \quad (i = 1, 2, \ldots, n) \]
The NLTV equation (Eq. 27) can be transformed into a state-space dynamic model, where \( \mathbf{x} \) is a state vector, \( \mathbf{y} \) is an output vector, and \( \mathbf{u} \) is a control vector

\[
\begin{align*}
\mathbf{x} &= \begin{pmatrix} \theta_{i1} & \theta_{i2} & \cdots & \theta_{in} & \dot{\theta}_{i1} & \dot{\theta}_{i2} & \cdots & \dot{\theta}_{in} & \omega_{i1} & \omega_{i2} & \cdots & \omega_{im} \end{pmatrix}^T \\
\mathbf{y} &= \begin{pmatrix} \omega_{i1} & \omega_{i2} & \cdots & \omega_{im} \end{pmatrix}^T \\
\mathbf{u} &= \begin{pmatrix} T_{i1} & T_{i2} & \cdots & T_{in} & T_L \end{pmatrix}^T
\end{align*}
\]

The nonlinear time-varying multiple input and multiple output (MIMO) state-space dynamic model of the driving system is obtained

\[
\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}
\]

\[
\mathbf{y} = \mathbf{C}\mathbf{x}
\]
load torque reduces the system nonlinearity to some degree. With the load torque increasing, the maximum oscillation peak amplitude (MOPA) and oscillation duration time (ODT) of the response speed will increase, and the amplitude of the cutterhead response speed will decrease as well. The MOPA of the cutterhead speed reaches 260.8 rpm and the ODT of the speed is about 2.05 s, as the load torque is 750 kN m. The driving system’s ability to resist load disturbances is studied, which is shown in Fig. 11(a). As the load torque experiences a tremendous step change, the speed immediately appears the oscillation behaviors that will sustain a few seconds. So, it is necessary to intensify the ability of the driving system to resist large change in the load torque, and this is an important issue that needs consideration in achieving the cutterhead’s multiple motors synchronous control.

The relationships of some system characteristics with the parameters are examined, as shown in Figs. 11(b) and 14. System characteristics such as steady-state speed ripple (SSR), and ODT and MOPA of the speed are affected by large gear inertia, load torque, gear transmission error, and relative backlash. The effects of large gear inertia and load torque on the system characteristics are shown in Figs. 11(c) and 11(d). Large gear inertia and load torque have an impact on the SSR, and ODT and MOPA of the speed. The ODT of the speed is extended when the large gear inertia is increased, but the increasing amount is small. As the load torque increases, the ODT of the speed will be extended, too. The MOPA of the speed and SSR will be heightened with the load torque increasing, but the MOPA of the speed and SSR are obviously reduced when large gear inertia increases. Therefore, oscillation speed could be restrained and steady speed ripple could be reduced by increasing the large gear inertia. In the case of a load torque of 750 kN m, the speed’s MOPA declines from 260.8 rpm to 89.8 rpm by increasing the inertia from 47.3 to 400, whereas the ODT of the speed increases by less than 3 s. Therefore, a preliminary method to restrain oscillation and reduce speed ripple is proposed by increasing large gear inertia.

The effects of gear transmission error and relative gap on the driving system are demonstrated in Figs. 12–14. The relationships of the ODT and MOPA of the speed with the gear transmission error and relative gap are studied in different load torque conditions. As the transmission error increases, the MOPA of the speed will heighten in the unloaded case, but the MOPA of the speed will decrease when the relative backlash increases. In the loaded case, the MOPA of the speed will increase with the relative backlash and transmission error enlarging. The ODT of the speed has complex relationships with the gear transmission error and relative gap, which is shown in Figs. 12(b), 13, and 14(b). The ODT of the speed may be extended or reduced, as the gear transmission error and relative gap increase. The study finds that the transmission error and relative gap have great influence on the ODT and MOPA of the speed. When the gear gap remains as a fixed value, the greater transmission error will bring about a larger MOPA of the speed. The MOPA of the speed is mostly affected by the transmission error, whereas the ODT of the speed is mainly affected by the transmission error and relative gap.

The effects of multiple motors’ steady-state torque asynchrony on the driving system are shown in Figs. 15 and 16. The corresponding response speeds in both unloaded and loaded cases reflect that the steady-state torque of multiple motors is unsynchronized despite of possessing identical transient characteristics, which will have an effect on the amplitude of steady-state speed and speed ripple. The amplitude of steady-state speed in the motor’s torque inconsistency case fluctuates larger than in the case of
torque synchronization, which is manifested in the unloaded and loaded cases. However, the inconsistency of multiple motors’ torque does not cause large speed oscillation phenomenon in the unloaded case.

The impacts of the transient characteristics difference in the multimotor torque synchronization on the driving system are displayed in Figs. 17 and 18. The response speed in the unloaded case is shown in Fig. 17, whereas the response speed in the loaded case is shown in Fig. 18. The cutterhead response speed demonstrates that the multiple motors have identical steady-state torque (ordinary sense of torque synchronization); however, its torque transient characteristics has some differences, which still causes the driving system to appear as larger steady-state speed ripple and speed fluctuation in both the unloaded and loaded cases. The maximum amplitude of the speed ripple is larger than 1.5 rpm, which is shown in Figs. 17 and 18. The driving system shows that larger steady-state speed fluctuation is due to larger differences in the multimotor torque’s dynamic characteristics, despite the multiple motors having identical steady-state torque. Therefore, the
transient characteristics difference in the multiple motors’ torque synchronization has a negative impact on the driving system.

The conclusion is that the multiple motor torque synchronization is an important issue that should be resolved. Studying the cutterhead driving system’s dynamic model finds that multiple motor torque synchronization should include at least two requirements. First, each motor should have consistently or approximately the same steady-state torque. Second, the transient performance differences in the multiple motor torque should be controlled within a certain range. In other words, each motor has the identical or similar dynamic characteristics of the torque. However, at present, most of the synchronous control systems merely require a consistency of the steady-state character of synchronous controlled variables, whereas the requirements of transient characteristics of the synchronously controlled variables are very little requested or not considered at all. Specifically, for the shield TBM cutterhead driving system, not only the steady-state torque of multiple motors is required to be consistent, but also the dynamic characteristics of the multiple motor torque cannot be ignored and must be controlled to achieve some consistency for better performance of the shield TBM.

5 Conclusion

A generalized NLTV dynamic model is proposed and studied in this paper for the shield TBM cutterhead driving system. A nonlinear time-varying MIMO state-space dynamic model is also presented. To study the driving system, the dynamic model is simulated, and the effects of system parameters on the dynamic response of the driving system are investigated. The simulation results reveal the following:

- First, the load torque has a large impact on the driving system. With the load torque increasing, the MOPA of the speed will raise and the steady amplitude of the response speed will decrease at the same time. The SSR and ODT of the speed will also heighten with the load torque increasing. The driving system is sensitive to larger variations in the load torque, which is a key issue that should be considered.
- Second, the ODT of the cutterhead speed will be slightly extended by increasing the large gear inertia. However, increasing the large gear inertia reaps the benefits from restraining speed oscillation, reducing the MOPA of the speed and SSR. Thus, a simple method to restrain oscillation and to reduce the speed ripple is proposed by heightening the large gear inertia.
- Third, the gear transmission error and backlash will affect the driving system character. In the unloaded case, the MOPA of the speed will increase with the gear transmission error increasing, but the MOPA of the speed will decrease with the relative backlash increasing. In the loaded case, the MOPA of the speed will increase, as the relative backlash and transmission error enlarge. The ODT of the speed has complex relationships with the gear transmission error and relative gap. As the gear relative gap and transmission error
rise, the ODT of the speed may increase, but it may also decrease in some cases. The gear transmission error mainly tends to affect the MOPA of the speed, whereas the ODT of the speed is affected by the gear backlash and transmission error.

- Fourth, the corresponding response speeds in both the unloaded and loaded cases reflect that the asynchrony of the steady-state torque of multiple motors will affect the amplitude of the steady-state speed and speed ripple, even if motors have identical transient characteristics. The multiple motors have the identical steady-state torque, however, if the motors’ transient characteristics have some differences, which still causes the driving system to have large steady-state speed ripple and speed fluctuation in both unloaded case and loaded case. In addition, multimotor torque synchronization also involves a load balance problem, which is not discussed in this paper. Thus, multimotor torque synchronization is a critical issue that should be solved.

Through studying, some issues that should be considered in the shield TBM cutterhead driving system are summed up as follows: (1) speed oscillation in the startup phase and during excavation process; (2) the multiple motors torque synchronization issue; (3) driving system is sensitive to the large variations in the load torque that may cause speed oscillation in the operation; (4) how to control the cutterhead to rotate smoothly and steadily under the complex geological conditions and uncertain excavation load.

The dynamic model proposed here tries to describe the driving system, and thus, it can be expected to predict the cutterhead speed and driving system dynamic response. The generalized NLTV dynamic model of shield TBM cutterhead driving system provides a solid theoretical foundation for analyzing, designing, and achieving the cutterhead’s multiple motor synchronous control.

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Nomenclature

- \( J_{di} \) = inertia of induction motor rotor \( i \)
- \( \theta_{pi} \) = angular displacement of the pinion \( i \)
- \( b_{di} \) = viscous damping of induction motor rotor \( i \)
- \( q \) = speed reduction ratio of reducer
- \( J_{ci} \) = inertia of the pinion \( i \)
- \( \theta_{i} \) = angular displacement of the motor rotor \( i \)
- \( b_{zi} \) = viscous damping of the motor coupling \( i \)
- \( \theta_{m} \) = angular displacement of the large gear
- \( p_{ci} \) = inertia of the pinion \( i \)
- \( \omega_{i} \) = rotary angular speed of induction motor \( i \)
- \( b_{pCi} \) = viscous damping of the pinion \( i \)
- \( \omega_{pi} \) = rotary angular speed of pinion \( i \)
- \( J_{m} \) = inertia of the large gear
- \( T_{ei} \) = electrical magnet torque of induction motor \( i \)
- \( b_{m} \) = viscous damping of the large gear
- \( \omega_{m} \) = angular speed of large gear
- \( J_{di} \) = rotor \( i \) inertia after equivalent coupling \( i \) inertia
- \( n \) = number of driving motor
Appendix A: Matrices A, B, and C

\[
\begin{align*}
A_{11} &= O_{(n+1) \times (n+1)}, \quad A_{12} = l_{(n+1) \times (n+1)}, \quad B_{11} = O_{(n+1) \times (2n+1)}, \quad C_{11} = O_{(n+1) \times (n+1)}, \quad C_{12} = l_{(n+1) \times (n+1)} \\
A_{21} &= \begin{pmatrix} A_{21}^{11} & A_{21}^{12} \\ A_{21}^{13} & A_{21}^{14} \end{pmatrix}_{(n+1) \times (n+1)}, \quad B_{21} = \begin{pmatrix} B_{21}^{11} & B_{21}^{13} \\ B_{21}^{12} & B_{21}^{14} \end{pmatrix}_{(n+1) \times (2n+1)}, \quad A_{21}^{22} = -\frac{1}{J_m} \sum_{i=1}^{n} k_i \varphi_i \omega_i^2 \\
A_{21}^{11} &= B_{21}^{13} = \text{diag}(-k_1 \varphi_1 J_1 - k_2 \varphi_2 J_2 \cdots - k_m \varphi_m J_m), \quad B_{21}^{12} = -1/J_m \\
A_{21}^{12} &= \begin{pmatrix} k_1 \varphi_1 \omega_1 J_1 & k_2 \varphi_2 \omega_2 J_2 & \cdots & k_m \varphi_m \omega_m J_m \end{pmatrix}_{1 \times n}, \quad A_{21}^{21} = B_{21}^{13} = \begin{pmatrix} k_1 \varphi_1 \omega_1 J_1 & k_2 \varphi_2 \omega_2 J_2 & \cdots & k_m \varphi_m \omega_m J_m \end{pmatrix}_{1 \times n} \\
B_{21}^{11} &= \text{diag}(q_1 J_1, q_2 J_2, \cdots, q_n J_n), \quad B_{21}^{13} = (B_{21}^{13})^T = O_{n \times 1}
\end{align*}
\]
A_{22} = \text{diag}(-b_1/J_1, -b_2/J_2, \cdots, -b_i/J_i, \cdots, -b_n/J_n, -b_m/J_m)_{(n+1) \times (n+1)}

where \( I \) is a unit matrix and \( O \) is a zero matrix. The parameters are defined as

\[ J_{ci} = J^p_{ci} + J^s_{ci}, \quad b_{ci} = b^p_{ci} + b^s_{ci} \quad (i = 1, 2, \ldots, n) \]

Appendix B: Simulation Parameters

\[ n = 10 \]

\[ q = 165.3 \]

\[ r_m = 0.4 \text{ m} \]

\[ J_m = 47.30 \text{ kg m}^2 \]

\[ b_m = 0.90 \text{ kg m}^2/\text{rad s} \]
\[ r_i = r = 0.1 \text{ m} \]
\[ \Delta = \Delta/\Delta = 0.0001 \]
\[ e_i(t) = e(t) = 0.0001 \text{ m} \]
\[ J_{di} = 2.11 \text{ kg m}^2 \]
\[ b_{di} = 0.25 \text{ kg m}^2/\text{rad s} \]
\[ J_{ei} = 0.1767 \text{ kg m}^2 \]
\[ b_{ei} = 0.105 \text{ kg m}^2/\text{rad s} \]
where \( i = 1, 2, \ldots, n = 10 \).

References


