A NEW WAVE ENERGY CONVERSION METHOD BASED ON INERTIAL PENDULUM

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Abstract. As one of the renewable energies, the absorption and utilization of wave energy is always an important research field domestically and abroad. But research on autonomous energy obtaining for mobile system or system without power for itself is seldom. In this paper, the working mechanisms of an innovative sea wave energy converter are described, together with the analysis of wave force and hydrodynamic parameters. The proposed structure, based on inertial pendulum, is studied. The kinematics and dynamics models are established based on which the simulations have been implemented. Influential factors on energy capture width ratio are discussed, and the relation surface of load radius, inertial pendulum mass ratio to load and capture width ratio is gained. It is concluded finally that the proposed system can absorb sufficient wave energy. And if the structure is selected reasonably, maximal capture width ratio can be gained. It is proved that the proposed model is effective.

Key Words. wave energy, inertial pendulum, hydrodynamic, capture width ratio and modeling.

1. Introduction

Recently, the exploration of ocean mineral resources and the detection of ocean environment have accelerated the development of submersible technology. And there have been many researches on power supply for moving load underwater [1]. But there is little research on some other moving load, such as passive/vehicle system etc. As well known, there are about 70% areas of earth covered by ocean, and there is much huge wave energy in ocean. So, research on the utilization mechanism of wave energy facing autonomous load will have a wide range of potential applications.

At present, one of the researches that have been developed is to make use of wave energy conversed in power supply of the submarine system, such as a docking system developed by the AUV Laboratory at Massachusetts Institute of Technology (MIT), which can support the Autonomous Underwater Vehicle (AUV) to exchange power and data, and to transfer energy. In 1997, trials of this system on sea were done, and the results demonstrated that the maximum energy transfer rate of this system was estimated to be 80% [2]. In 2001, A. Maridan et al completed sea trials of their docking system for AUV, this trial mainly aimed at the complex task of docking an AUV, and the conversion efficiency was not mentioned [3].

Many efforts have been made to capture wave energy [4-12], all of which can be divided into three classes. The first are floats or pitching devices that utilize the bobbing or pitching action of a floating object, and the object can be mounted to a floating raft or to a device fixed on the ocean floor. The second are oscillating
water columns (OWC) devices that utilize the wave-driven rise and fall of water in a cylindrical shaft. The rising and falling water column drives air into and out of the top of the shaft, powering an air-driven turbine. The third are wave surge or focusing devices that rely on a shore-mounted structure to channel and concentrate the waves, driving them into an elevated reservoir. The above methods are mostly developed for exchanging wave energy into electricity, but not for passive/vehicle submarine system, so there are many restrictions.

Aiming at solving problem of the autonomous energy absorption of artificial load underwater, in this paper, a method of wave energy conversion is presented, and the energy absorption mechanism, based on inertial pendulum, is adopted. Meanwhile, the relative kinematics and dynamics analysis are completed, and the key hydrodynamic coefficients for simulation are gained.

2. Analysis of conversion mechanism of inertial pendulum and force of system

2.1. Wave energy absorption mechanism of inertial pendulum load. The pendulum without friction, in general mean, will reciprocate under the effect of gravity. If an outside excitation acts on the pendulum and makes it move, an inertial pendulum system is formed. And if the reciprocation of wave is used as outside excitation to drive the load with inertial pendulum to move, the inertial pendulum will move too. The movement of inertial pendulum is converted into usable mechanical energy, and the wave energy can be absorbed and utilized.

Based on this idea, a mechanical design of inertial pendulum used in absorbing wave energy is proposed, the diagram of this system is shown in Fig.1.

Suppose that an inertial pendulum with a mass \( m \) and multi-DOF is installed in a load with a mass \( M \), and the whole system is ensured to float near the water surface. Besides gravity and buoyancy, the forces on the system are mainly wave forces. Under the action of wave force, the load will move with wave, and the velocity and acceleration of load are produced, and then the hydrodynamic forces are produced too.

![Fig.1 The diagram of inertial pendulum system](image)

2.2. Analysis of load forces. According to the analysis above, the load with inertial pendulum will be affected by gravity, buoyancy, wave forces and hydrodynamic forces, etc. In order to make the analysis and evaluation easy, the load is designed as an ellipse with two symmetry-planes.

The coordinate is defined as Fig.2, in which the coordinate \( E-XeYeZe \) is ground coordinate system, and the coordinate \( O-xyz \) is kinetic coordinate system of load. The velocity coordinate of load is defined as Fig.3, in which \( u,v,w \) are the three velocity components of kinetic coordinate system’s origin, and \( p,q,r \) are the three angular velocity components.
With the two coordinates systems defined above, suppose that
· The gravity of the system is equal to the buoyancy of system, and the system is near the water surface.
· The wave is regular.

2.2.1. Mathematical model of wave force (toque). The wave with a certain period and less viscosity is a natural phenomenon of the ocean.

With Froude-Krylov’s suppose that the pressure distribution of wave will not be affected by the existence of load, pressure distribution of a load underwater is [14]

\[ \Delta p(x, z, t) = -\rho a e^{-kz} \cos(kx \cos \gamma - ky \sin \gamma - \omega e t) \]

where, \( \rho \) is the consistency of liquid; \( g \) is the gravity acceleration; \( a \) is the height of wave; \( k \) is the wave number; \( \omega \) is the frequency of wave; \( \gamma \) is the angle of wave direction or the encounter angle, and \( \omega_e \) is the encounter frequency. Then the forces and toques acting on load can be indicated by

\[
\begin{align*}
\vec{F}_{\text{wave}} &= - \oint \Delta p \vec{n} dS \\
\vec{M}_{\text{wave}} &= - \oint \Delta p (\vec{n} \times \vec{r}) dS
\end{align*}
\]

where, \( S \) is the wet-surface area of load; \( \vec{n} \) is the unit external normal vector of \( S \), and \( \vec{r} \) is the position vector of a random point on the load surface.

Based on the GAUSS theorem, the integral of formula (2) can be transformed from along surface to along the volume. The wave force vector \( \vec{F} \) and wave toque vector \( \vec{M} \) can be projected to kinetic coordinate. Due to only the load’s movement with wave being considered, i.e., the angle of wave direction \( \gamma \) in formula (1) is equal to zero, the forces (and toques) acting on load are longitudinal force, vertical force, and torque around the
vertical surface. Thus, formula (2) is simplified as

\[
\begin{align*}
F_{x\text{wave}} &= -\iiint_V \frac{\partial p}{\partial x} dV \\
F_{y\text{wave}} &= -\iiint_V \frac{\partial p}{\partial y} dV \\
M_{x\text{wave}} &= -\iiint_V (x \frac{\partial p}{\partial y} - y \frac{\partial p}{\partial x}) dV
\end{align*}
\]

2.2.2. Hydrodynamic forces. There are mainly two kinds of hydrodynamic forces. They are viscous forces and the inertia forces acting on the load underwater.

1)Viscous forces

Suppose that the coefficients of vertical force are only related with the parameters of movement on the vertical section, and the longitudinal force is only related with movement on the horizontal section \([15]\), the viscous forces can be described as follows

\[
\begin{align*}
F_{xV} &= \frac{1}{2} C_x \rho V_T^2 S \\
F_{yV} &= \frac{1}{2} C_y \rho V_T^2 S \\
F_{zV} &= \frac{1}{2} C_z \rho V_T^2 S \\
M_{xV} &= \frac{1}{2} m_x \rho V_T^2 SL \\
M_{yV} &= \frac{1}{2} m_y \rho V_T^2 SL \\
M_{zV} &= \frac{1}{2} m_z \rho V_T^2 SL
\end{align*}
\]

where, \(F_{xV} \sim M_{zV}\) stand for viscous forces and toques, respectively; \(C_x, C_y, C_z, m_x, m_y, m_z\) are the coefficients of longitudinal force, vertical and side force, and coefficients of rolling torque, yaw torque and pitching torque, respectively, which will be gained by test or estimation; \(V_T\) is the relative velocity between load coordinate system and ground coordinate system; \(L\) is load length, and \(S\) is the maximum section area of load.

2) Inertial forces

The relations between inertial force and acceleration or angular acceleration of load are linear. The number of all of the 6-DOF inertial forces of system is 36, as

\[
g_i = -\sum_{j=1}^{6} \lambda_{ij} \dot{v}_j
\]

where, \(g_i (i = 1, 2, \cdots, 6)\) denote the 6-DOF inertial forces, and \(\dot{v}_j (j = 1, 2, \cdots, 6)\) denote the acceleration \(\dot{u}, \dot{v}, \dot{w}\), and the angular acceleration \(\dot{\phi}, \dot{\theta}, \dot{\psi}\) of load, respectively.

2.2.3. Gravity torque. The wave torque acting on load will bring the bias of mass center. Therefore, the gravity torque is produced.

Define that the angle between the Ox axes of load coordinate and horizontal plane is the angle \(\theta\), then the translation matrix from ground coordinate system to load coordinate system can be gained as follow

\[
C^E_O = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Based on the analyses above, the gravity torque is

\[
R_G \times C^E_O G = G(-z_G \cos \theta, z_G \sin \theta, x_G \cos \theta - y_G \sin \theta)
\]

where, \(R_G = x_G i + y_G j + z_G k\) is the coordinate vector of load mass center in the load coordinate system, and \(i, j, k\) are the unit coordinate vectors of load coordinate system axes.

3. Modeling of kinematics and dynamics

3.1. Kinematics analysis. The transition matrix from load to ground is got from formula(6)

\[
C^G_O = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
Suppose that the center coordinate of load mass in the ground coordinate system is 
\((X_e, Y_e, Z_e)\), the velocity is \((\dot{X}_e, \dot{Y}_e, \dot{Z}_e)\), then the kinematics equation is

\[
\begin{align*}
\dot{r} &= \dot{\theta} \\
\dot{X}_e &= u \cos \theta - v \sin \theta \\
\dot{Y}_e &= u \sin \theta + v \cos \theta \\
V_T &= \sqrt{u^2 + v^2}
\end{align*}
\]

### 3.2. Dynamic modeling

The momentum theorem is used to model dynamic equations.

#### 3.2.1. Momentum equations

The momentum equation is dedicated as

\[
m \frac{dV_G}{dt} = F
\]

where, \(m\) is load mass; \(F\) is resultant force on load underwater, \(V_G = V_T + V_R\) is the absolute velocity of mass center, and \(V_R\) is the relative velocity of mass center.

On the assumption that the gravity force is equal to buoy force, and from wave forces having been gained, we know that there is only 3-DOF of load. Thus, it is obtained in the velocity coordinate system of load

\[
w = \dot{w} = 0, \ p = \dot{p} = 0, \ q = \dot{q} = 0
\]

Then, the dynamic equations of the whole system under water are

\[
\begin{align*}
m(\dot{u} - y_G \dot{r} - vr - x_G r^2) &= F_x \\
m(\dot{v} + x_G \dot{r} + ur - y_G r^2) &= F_y
\end{align*}
\]

#### 3.2.2. Momentum moment equations

Suppose that the momentum moment of load relative to mass center is \(L\), and then by the theorem of momentum moment, we can know that

\[
dL \frac{dt}{dt} = M
\]

where, \(L = J_{xG} \pi + J_{zG} qj + J_{yG} rk\); \(J_{xG}, J_{yG}, J_{zG}\) are rotational inertias of three axes in translational load coordinate system, respectively, and \(M\) is the resultant torque vector.

From formula (11), only the rotation about the z axes is considered, then the rotational inertia of load circling in z axes of load coordinate system is

\[
J_z = J_{zG} + m(x_G^2 + y_G^2)
\]

The little quantities \(x_G^2, y_G^2, z_G^2\) can be ignored. The formula (11) and (14) are put into formula (13), then

\[
J_z \dot{r} + m x_G \dot{v} + m x_G u r = M_z
\]

where, \(M_z\) is the resultant torque in z axes.

The kinematics and dynamics equations of load on the effect of wave forces and hydrodynamic forces can be gained by formulas (3), (9), (12), and (15)
4. Simulation experiments

To validate the feasibility of inertial pendulum system and equations established above, simulation experiments of the simple model described in Fig.1 are implemented under. Suppose the period of wave is 5s, the height of wave is 4m, and the density of water is 1000kg/m³. In order to analyze and simulate the movement of system, the shape of load is designed as a spheroid, the radius of load is 0.22m, the total mass of system is 44kg, and the mass of pendulum is 20kg.

4.1. Hydrodynamic coefficients. The center of buoyancy is selected in the origin of the load coordinate system. The inertial hydrodynamic coefficients can be gained [15]

\[ \lambda_{11} = 22.3011, \lambda_{22} = 22.3011, \lambda_{26} = \lambda_{66} = 0 \]

Because the viscous hydrodynamic coefficients are related to the incidence angle, an exact expression can not be elicited. Based on the expression of viscous hydrodynamic coefficients and incidence angle, the three viscous hydrodynamic coefficients in formula (6) \( C_x, C_y, m_z \) can be described as Fig.4.

![Fig.4 The relationship of viscous force coefficients and the variable time](image)

4.2. Simulation results. Under the conditions of above wave system models, as well as the kinetics and dynamics models are established, the simulation of system movement, under the water, is implemented. The simulation results of forces, velocity, and energy obtained are shown as Fig.5 and Fig.6.

![Image of simulation results](image)
In Fig. 5 (a), Force X is the horizontal wave force of load, Force Y is the vertical force of load, and Torque Z is the wave torque circling the EZe axes in the anticlockwise. And in Fig. 5 (b), Velocity X, Velocity Y and Angular Velocity Z are the velocities of horizontal direction, vertical direction, and the angular velocity circling the EZe axes, respectively.

The energy obtained by inertial pendulum and whole system are shown in Fig. 6 (a) and Fig. 6 (b), respectively. It can be seen that the whole energy obtained by inertial pendulum,
in five seconds, is 1056.1Nm, and obtained by system is 3785.6Nm. The average energy obtained by inertial pendulum is \( \bar{E}_P = 211.22 \text{Nm} \), and by whole system is \( \bar{E}_S = 757.12 \text{Nm} \).

Based on the potential flow theory what can be known is that the mean energy in the unit horizontal area can be indicated as \([16]\)

\[
E = \frac{1}{8} \rho g H^2
\]

Then, the whole energy in the section areas of proposed system is \( \bar{E} = 2982.3 \text{Nm} \). The capture width ratio of inertial pendulum and system, are

\[
\begin{align*}
\varepsilon_f &= \frac{\bar{E}_P}{\bar{E}} = 7.08\% \\
\varepsilon &= \frac{\bar{E}_S}{\bar{E}} = 25.4\%
\end{align*}
\]

If the emphasis is put on capture width ratio of inertial pendulum, it is known that there is 7.08% wave energy being absorbed by inertial pendulum, which is higher than the oscillating buoy device put forward by Y. L. Su \([13]\), of which the capture width ratio is 1.429%. The conversion efficiency is high enough for a simple system. If whole energy obtained is converted into electric energy, which is suffice to meet control and other electric expenditure of a sea artificial system. If multiple bodies are combined as literature \([2]\) stated, the conversion efficiency will be improved extremely. It should be noted that wave energy possess random kinetic energy and potential energy, if an movable system want to absorb these energies, a device fit to this kind of external excitation must be adopted. Therefore, the inertial pendulum method is a good scheme.

Besides the simulation test done above, a large number of tests about diverse conditions have been done. From the conclusions obtained, when wave conditions are invariable, load radius and mass ratio of inertial pendulum to load are influential factors to capture width ratio. The relation surface of radius of load, mass ratio of inertial pendulum and capture width ratio is shown in Fig.7.

![Fig.7 The relation surface of load radius, inertial pendulum mass ratio to load and capture width ratio](image)

The simulation results indicate that the kinematics and dynamics equations established are correct, which can reflect the external movement of system. The movement of whole system and internal movement included can be gained by simulation. Therefore, it is concluded that if the structures of load and pendulum are selected properly, the system can obtain adequate power.

5. Conclusions

The aim of the research is to establish a means of energy of production for some submarine system. A pendulum based scheme of wave energy absorption is presented. The system forces are analyzed, and the kinematics and dynamics models are established. The simulation results for system movement and the hydrodynamic parameters are obtained.
The analysis on energy obtained by pendulum and whole system are made. The simulation results have proved that the scheme is feasible. The capture width ratio of inertial pendulum system is high enough for absorbing the wave energy, which is able to be utilized in some system without power for itself.

References


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