Angular acceleration estimation and feedback control: An experimental investigation

J.D. Han a, Y.Q. He a,b, W.L. Xu c,*

a Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China
b Graduate School of the Chinese Academy of Sciences, Beijing 100039, China
c Institute of Technology and Engineering, Massey University, Auckland, New Zealand

Received 18 January 2006; accepted 21 May 2007

Abstract

Angular acceleration estimation and its application in acceleration feedback control are investigated experimentally in the paper. In combination of Newton Predictor (NP) with Kalman Filter (KF), a new predictive estimator for angular acceleration, called Newton Predictor Enhanced Kalman Filter (NPEKF) is proposed. This estimator provides a wide bandwidth and a small phase lag of the estimated acceleration while attenuating noises. Based on the estimated acceleration an acceleration feedback control (AFC) is presented for multiple degree-of-freedom (DOF) mechatronic system. The design of AFC is specified in terms of its stability and ability in suppressing dynamic disturbances. Experiments are conducted on a 2-DOF direct-drive manipulator. The frequency responses of the acceleration estimated by NP, KF and NPEKF are compared with those of the measured acceleration via linear accelerometer. The performance of AFC using the estimated acceleration is assessed against that using the measured acceleration. This study has shown that the proposed NPEKF estimator is able to supply the AFC with reliable required acceleration.

© 2007 Elsevier Ltd. All rights reserved.

Keywords: Acceleration estimation; Newton Predictor; Kalman Filter; Acceleration feedback control

1. Introduction

Dynamic cross-coupling among motion axes is one of the most critical factors that degrade the tracking performance of the independent joint control of multi-DOF machines, such as robotic manipulators and CNC machines. Model-based compensation can not effectively remove the coupling effects as an accurate model is hardy obtainable. In view of this, Spong et al. proposed acceleration as the input to the system where the actuator produces directly commanded acceleration instead of torque [1]. In this way the dynamic couplings no longer impair the tracking performance of each individual axis and consequently need not to be taken into account. Following this pioneering idea, considerable work has been done, showing that acceleration feedback control is effective and robust in suppressing torque disturbances [2–8]. However, there still are a few practical issues in the acceleration measurement to be resolved.

A first problem with the above scheme is the mounting of accelerometers on a machine, especially for the measurement of angular acceleration [9]. A further problem is the couplings among the measurements of angular accelerations of different joints. To decouple the measurement couplings requires accurate kinematic parameters so that the acceleration feedback control is prevented from real applications. Consequently, estimating the acceleration from velocity or angle measurements has been proposed [11–15]. An obvious advantage of it is the fact that most of multi-DOF systems are instrumented with encoders and/or tachometers and no further hardware is thus required for acceleration estimation.
Although acceleration is the differentiation of velocity or double-differentiation of position, such a differential operator is practically infeasible for acceleration estimation due to severe noises in computation. The use of low-pass filters introduces additional phase delay which further limits the performance of the closed loop [10]. Other methods include observers or estimators [11–15], but paying attention to both noise rejection and acceleration prediction is a real challenge to the estimator design. To reduce the phase lag of the estimation, the Recursive Linear Smoothed Newton (RLSN) method was proposed, where the acceleration is estimated by Newton Predictor and enhanced by some form of recursive low-pass filter [13–15]. The RLSN becomes weak when estimating the acceleration not in polynomial expression. In recognition of the Kalman Filter (KF)’s ability of both filtering and prediction, Belanger et al. put forward a KF-based acceleration estimation approach, where angular acceleration is generated by passing white noise through a linear and stable all-integrator transfer function [16]. However, the performance of the above acceleration estimators is yet to be substantiated by accelerometer.

This paper presents the proposed acceleration estimator NPEKF and its application in acceleration feedback control with experimental validations. The rest of this paper is organized as follows. Section 2 briefly introduces the KF and RLSN for acceleration estimation and presents the proposed NPKF estimator. Section 3 specifies the design of the acceleration feedback control. Experimental procedure, results and discussion are given in Section 4, followed by the conclusion in Section 5.

2. Angular acceleration estimation

2.1. The NP estimator

Let the acceleration be expressed in a polynomial form

\[ a(t) = \lambda_0 + \lambda_1 t + \lambda_2 t^2 + \cdots + \lambda_{M-1} t^{M-1} + \lambda_M t^M \]  

(1)

where \( a(t) \) is the acceleration at time \( t \), \( M \) the assumed order of \( a(t) \), and \( \lambda_i, i \in [0, M] \) the polynomial coefficients. The acceleration signal can be predicted by the \( n \)-step-leading Newton Predictor [13],

\[ H^m_{\lambda}(z) = \frac{a_{\text{est}}}{a_{\text{raw}}} = \sum_{k=0}^{M} (1 - z^{-k})^k \]  

(2)

where \( a_{\text{raw}}(t) \) is the raw acceleration after the differentiation of velocity or double-differentiation of position and \( a_{\text{est}}(t) \) is the estimated acceleration.

2.2. The KF estimator

The acceleration estimation can be expressed in a state-space model

\[
\begin{align*}
x_k &= A x_{k-1} + \omega_k - 1 \\
z_k &= H x_k + v_k \\
y_k &= C x_k
\end{align*}
\]  

(3)

where \( A \) is the system matrix, \( H \) the measurement matrix, and \( C \) the output matrix. \( x_k = [\theta_k, \dot{\theta}_k, \alpha_k]^T \) the state vector, \( \theta_k \) the position signal, \( \dot{\theta}_k \) the velocity signal, \( \alpha_k \) the acceleration signal, \( y_k \) the estimated acceleration, \( \omega_k \) and \( v_k \) the process and measurement noises, respectively, which are assumed to be independent of each other. \( Q \) the covariance matrix of the process noise, and \( r \) the covariance of the measurement noise.

Let a priori and a post-priori estimation errors be

\[
\begin{align*}
\varepsilon_k &= x_k - \hat{x}_k \\
\bar{\varepsilon}_k &= x_k - \hat{x}_k
\end{align*}
\]  

(5)

The KF-based acceleration estimation can be expressed as,

\[
\begin{align*}
\hat{x}_{k+1} &= A \hat{x}_k + B v_k \\
P_{k+1} &= A P_k A^T + Q \\
\hat{x}_k &= \hat{x}_{k+1} + P_k H^T (H P_k H^T + r)^{-1} (z_k - H \hat{x}_k) \\
P_k &= (I - P_k H^T (H P_k H^T + r)^{-1} H) P_k
\end{align*}
\]  

(6)

where

\[
\begin{align*}
P_{k+1} &= E[\varepsilon_k \varepsilon_k^T] \\
P_k &= E[\varepsilon_k \varepsilon_k^T]
\end{align*}
\]  

(7)

2.3. The proposed NPEKF

In the proposed NPKF the angular acceleration is first estimated by Kalman Filter and the estimated acceleration with noise being attenuated is then passed through the Newton Predictor to further reduce the phase lag caused by Kalman Filter. There are two remarks when applying the NPKF estimator:

1. The historical states used by the NPKF have to be limited since the NP makes use of a polynomial model Eq. (1) within a small time window and a longer history would thus result in significant error in the model.
2. The order of the NP Predictor needs to be neither low nor high as the low order would make any inaccurate signal lead to an intolerant level of noise and the high order model would make real-time implementation challenging. Moreover, the computational burden
increases as the order increases, and a high order predictor will ‘introduce’ extra noise because more noisy historical values are required. The 1-step-2nd-order Newton Predictor below was chosen by experiments.

Let \( M = 2, n = 1 \) in Eq. (2), yielding the Newton Predictor,

\[
H_2^1(z) = \sum_{k=0}^{2} (1 - z^{-1})^k = 3 - 3z^{-1} + z^{-2}
\]

in which only three past states are required to predict the acceleration, and the time window is small enough to satisfy the 2nd-order assumption. Fig. 1 shows the NPEKF scheme for the acceleration estimation.

3. Acceleration feedback control

3.1. The AFC for single joint

The differential equation governing the dynamics of a single joint of multi-axis machine is described as,

\[
\tau_a - \tau_a = J \ddot{z}
\]

where \( \tau_a \) is the torque generated by the actuator, \( J \) the total moment of inertia of both the rotor and the payload, \( \ddot{z} \) the rotor acceleration, and \( \tau_a \) is the total disturbing torque acted on the rotor axis.

The AFC can be designed as

\[
\tau_a = k_a (a_{cmd} - \dot{\theta})
\]

where \( k_a \) is the gain of the acceleration closed loop, and \( a_{cmd} \) the acceleration command.

Substituting Eq. (10) into Eq. (9) yields:

\[
\ddot{\theta} = \frac{k_a}{J + k_a} a_{cmd} - \frac{1}{J + k_a} \tau_a = \frac{1}{J + k_a} a_{cmd} - \frac{1}{J + k_a} \tau_a
\]

where \( J \) is the constant part of \( J \) and \( \Delta J \) the time-varying (uncertain) part. When the acceleration feedback gain \( k_a \) is chosen as,

\[
k_a \gg \max(\Delta J, \tau_a)
\]

Eq. (11) becomes

\[
\ddot{\theta} \approx \frac{k_a}{k_a + J} a_{cmd}
\]

which implies that by a large gain the AFC can suppress both the disturbing torque and the uncertain disturbance in payload.

3.2. The AFC for multi-axis machines

The dynamics of a \( n \)-DOF robot-type machine can be expressed as,

\[
M(q) \ddot{q} + h(q, \dot{q}) = u
\]

where \( h(q, \dot{q}) = C(q, \dot{q}) \ddot{q} + B \dot{q} + g(q) \), \( q \in \mathbb{R}^n \) the vector of generalized coordinates, \( M(q) \in \mathbb{R}^{n \times n} \) the symmetric, positive definite inertia matrix, \( C(q, \dot{q}) \ddot{q} \in \mathbb{R}^n \) the vector of the centripetal and Coriolis torque, \( g(q) \in \mathbb{R}^n \) the torque due to gravity, \( B \dot{q} \in \mathbb{R}^n \) the torque due to viscous friction and \( u \in \mathbb{R}^n \) the generated torque.

Acceleration feedback control, which is used as an inner loop for robust enhancement, can be designed as:

\[
u = K_a (v - \dot{q})
\]

where \( v \) is the outer loop control, \( K_a = \text{diag}(k_{a1}, k_{a2}, \ldots, k_{an}) \) the AFC gain and \( k_{ai} > 0, i = 1, 2, \ldots, n \).

Substituting Eq. (15) into Eq. (14) yields

\[
\ddot{M}(q) \ddot{q} + \ddot{h}(q, \dot{q}) = v
\]

where \( \ddot{M} = I + K_a^{-1}M, \ddot{h}(q, \dot{q}) = K_a^{-1}h(q, \dot{q}) \). The outer loop control of \( v \) can be designed as a normal PD control, namely,

\[
v = \dot{q}_d - K_p e_1 - K_v e_2 + \Delta \ddot{v}
\]

where \( e_1 = q - q_d, e_2 = \dot{q} - \dot{q}_d, \dot{q}_d \) is the desired trajectory, \( K_p \) and \( K_v \) the constant gains of PD control, for example,

\[
K_p = \text{diag}(\omega_1^2, \omega_2^2, \ldots, \omega_n^2),
K_v = \text{diag}(2\zeta_1 \omega_1, 2\zeta_2 \omega_2, \ldots, 2\zeta_v \omega_n)
\]

and \( \Delta \ddot{v} \) is the compensation for globally asymptotical stability.

Substituting Eq. (17) into Eq. (16) yields

\[
\ddot{e} = A e + B (\Delta \ddot{v} + \ddot{\eta})
\]

and

\[
\ddot{\eta} = \bar{E} \Delta \ddot{v} + \bar{E} (\ddot{q}_d - Ke) - \bar{M}^{-1} \ddot{h}
\]

where

\[
\bar{E} = \bar{M}^{-1} - I, \quad K = [K_p, K_v], \quad e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},
\]

\[
A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}
\]

In order to estimate the worse case bound on \( \ddot{\eta} \), the following three assumptions, Eqs. (20)–(22) are made,
\[ \sup_{t > 0} \| \tilde{q}_d \| \leq Q_d \]  
\[ M \leq \| M(q) \| \leq \overline{M} \]  
\[ |h| \leq \phi(e, t) \] for a known function \( \phi \)  

According to Eq. (20),
\[ \| \tilde{E} \| = \| M^{-1} - I \| = \| M(M + K_a)^{-1} \| \leq \beta < 1 \] and
\[ \frac{\| K_a \|}{\| K_a \| + \| M \|} \leq \| M^{-1}(q) \| = \| (I + K_a^{-1}M)^{-1} \| \leq \frac{\| K_a \|}{\| K_a \| + \| M \|} \]  

By designing
\[ \Delta \tilde{v} = \begin{cases} -\tilde{\rho}(e, t) \frac{B^T P e}{\| B^T P e \|} & \text{if} \quad \| B^T P e \| \neq 0 \\ 0 & \text{if} \quad \| B^T P e \| = 0 \end{cases} \]  

where
\[ \tilde{\rho}(e, t) = \frac{1}{1 - \beta} \left[ \beta Q_d + \| K \| \cdot \| e \| + \frac{\| K_a \|}{\| K_a \| + \| M \|} \cdot \phi(e, t) \right] \]

and \( P \) is the unique positive definite symmetric solution to the Lyapunov equation
\[ A^T P + PA + Q = 0, \]

there exists the following theorem.

**Theorem.** Given the system of Eq. (14) and the control of Eqs. (15), (17) and (25), the closed loop system of Eq. (18) is globally asymptotically stable.

**Proof.** See Appendix. \( \square \)

By introducing AFC as an inner loop, the following conclusions can be drawn:

1. If \( \| K_a \| \gg \| M \| \), then \( \beta \approx 0 \) (see Eq. (23)), and for both sides of Eq. (24),
\[ \frac{\| K_a \|}{\| K_a \| + \| M \|} \approx 1, \quad \frac{\| K_a \|}{\| K_a \| + \| M \|} \approx 1, \]
i.e., \( \| M^{-1}(q) \| \approx 1 \)

2. If \( \| K_a \| \gg \phi(e, t) \), then \( \phi(e, t) \rightarrow 0 \)

3. If \( \| K_a \| \gg \max \{ \| M \|, \phi(e, t) \} \), then \( \tilde{\rho}(e, t) \approx \| K \| \cdot \| e \| \) (see Eq. (26)), and \( \Delta \tilde{v} \approx -\| K \| \cdot \| e \| \frac{B^T P e}{\| B^T P e \|} \) which has been reduced to an additional proportional control. This means that a high-gain AFC as an inner loop can guarantee the normal PD control tracking a desired trajectory in globally asymptotically stable way.

### 3.3. The design of AFC for real implementation

Unlike Eq. (9) that represents the dynamics of ideal-rigid body, the open loop transfer function of real system from the generated torque \( (\tau_a) \) to the acceleration \( (\ddot{\theta}) \) can be derived as [4,6],
\[ G_{ao}(s) = \frac{\ddot{\theta}(s)}{\tau_a(s)} = \frac{1}{J} \frac{G_s(s) \prod_{j=1}^{N} \left(1 - s^2/\omega_j^2\right)}{1 + 2\zeta_j \omega_j + s^2/\omega_j^2} \]  

where \( \omega_j \) and \( \zeta_j \) are, respectively, the natural frequency and the damping coefficient of the \( j \)th resonant mode, and \( G_s(s) \) the transfer function of either the accelerometer or the estimator. In general, \( G_s(s) \) is only a proportional gain within its bandwidth, i.e.
\[ G_s(s) = \begin{cases} k_s, & \omega_s \in [0, \omega_s] \\ 0, & \text{other} \end{cases} \]  

where \( k_s \) is a positive constant and \( \omega_s \) the sensitive bandwidth of the accelerometer/estimator. Substituting Eq. (28) into (15) yields
\[ G_{ao}(s) = \frac{a_{out}(s)}{\tau_a(s)} = \frac{k_s}{J} \prod_{j=1}^{N} \left(1 - s^2/\omega_j^2\right) \]  

in which \( a_{out}(s) \) is the measured/estimated acceleration.

The AFC can be designed in terms of the following three requirements:

1. The absolute closed loop stability while avoiding the resonance in Eq. (29). That is,
\[ G_{ao}(s) = \frac{\ddot{\theta}(s)}{\tau_a(s)} = \frac{G_s(s)G_{ao}(s)}{1 + G_s(s)G_{ao}(s)} \]

2. The AFC closed loop bandwidth. It needs to be large enough such that the AFC has little impact on its outer loop, i.e., the phase and the magnitude decay- 
hing within the outer loop bandwidth must be little,
\[ \angle G_{ao}(s) = \angle \frac{G_s(s)G_{ao}(s)}{1 + G_s(s)G_{ao}(s)} \bigg|_{s=jo\omega} \leq \xi \]  

3. The 20\log ||G_{ao}(s)|| = 20\log \left| \frac{G_{ao}(s)}{a_{out}(s)} \right| \bigg|_{s=jo\omega} \leq \sigma \]  

where \( \omega_s \) is the bandwidth of the outer control loop 
and, \( \omega_s \) the bandwidth of the disturbance \( \tau_a \), \( \xi \) and \( \sigma \) pre-designed constants.
(3) The control gain in $G_a(s)$. It should be as large as possible as long as the criterion (1) and (2) are not destroyed, because the function of AFC is dependent on its gain, the larger the better.

4. Experimental study

4.1. The setup

The experiment was to compare the NP-, KF-, and NPEKF-based angular acceleration estimations and to evaluate the performance of the AFC based on estimated and measured accelerations. The experiment was conducted with the first joint of a 2-DOF direct-drive manipulator. As shown in Fig. 2, the manipulator consists of two joints, each of which is actuated by a DC torque motor independently, and instrumented with an encoder of 80,000 counts per revolution. A linear accelerometer with the bandwidth [0, 200 Hz] and sensitivity of $10^{-5} g$ is mounted on the joint, and the angular acceleration of the joint is calculated simply by

$$\dot{\theta} = \frac{a_{\text{mea}}}{l}$$  \hspace{1cm} (33)

where $a_{\text{mea}}$ is the measured linear acceleration, and $l$ is the length between the accelerometer and the joint axis.

4.2. Experiments for acceleration estimation

4.2.1. Experimental scheme

The experimental scheme for acceleration estimation is presented in Fig. 3. HP 3562A Dynamic Signal Analyzer generates a swept sine signal as the set-point torque of the DC-motor for joint 1, and gets the same signal back to its channel-1. The channel-2 of HP3562A takes in the estimated acceleration or measured acceleration, depending on what experiment is performed. The frequency response between Ch1 and Ch2 can be measured by HP3562A. The NP, KF and NPEKF estimators are implemented on an Industrial PC (IPC) and the angular position measured by the encoder is sampled by the IPC at 10 ms circle and fed into the three estimators.

A total of four experiments were carried out. Referring to Fig. 3, they are,

1. $G_{ao}(s) = a_{\text{mea}}(s)/\tau_d(s)$, the frequency response of the accelerometer measurement with respect to the set-point torque of the DC-motor. This is done by keeping $K_1$ closed and $K_2$ open.
2. $G_{ao}(s) = \hat{a}_{\text{NP}}(s)/\tau_d(s)$, by keeping $K_1$ open and $K_2$ closed and turning the RLSN on.
3. $G_{ao}(s) = \hat{a}_{\text{KF}}(s)/\tau_d(s)$, by keeping $K_1$ open and $K_2$ closed and turning the KF on.
4. $G_{ao}(s) = \hat{a}_{\text{NPEKF}}(s)/\tau_d(s)$, by keeping $K_1$ open and $K_2$ closed and turning the NPKF on.

4.2.2. Results and discussion

Fig. 4 shows the frequency responses of the measured and estimated accelerations over the torque and Table 1 lists some major comparisons of the phase and the magnitude at frequency points of 10, 15 and 20 Hz. It can be seen clearly that NPEKF significantly improves the estimation and is superior to both KF- and NP-based estimations. At 10 Hz, the phase lag of the proposed NPEKF estimator has only $5^\circ$ larger than the measured one, and is only one-third of that of KF estimator, less than one-eighth of that of RLSN estimator. This demonstrates not only good approximation of NPEKF to the measurement of accelerometer, but also improvement of NPEKF over other two estimators. The acceleration estimated by NPEKF matches the measured acceleration in a frequency range less than 10 Hz.

4.3. Experiments for the AFC

4.3.1. Experimental scheme

The block diagram of the AFC is shown in Fig. 5. Two switches $K_1$ and $K_2$ are used to conduct the following three experiments,
There is no AFC, by opening both $K_1$ and $K_2$.

(2) The AFC uses the NPEKF-estimated acceleration, by keeping $K_1$ closed and $K_2$ open.

(3) The AFC uses the measured acceleration, by keeping $K_1$ open and $K_2$ closed.

In this section, all of the position, velocity and acceleration signals are needed, but only the position and velocity signals are actually measured, while the acceleration signal is the estimated one by the NPEKF proposed in this paper.

### 4.3.2. Experimental results on open loop model of the AFC

$G_{ao}(s) = a_{mea}(s)/\tau_d(s)$ and $\tilde{G}_{ao}(s) = \tilde{a}_{NPEKF}(s)/\tau_d(s)$ are the open loop transfer functions of the measured and estimated accelerations, respectively. By opening $K_1$ and the $K_2$ in Fig. 5, the frequency response of $G_{ao}(s)$ is actually

---

**Table 1**

<table>
<thead>
<tr>
<th>Source</th>
<th>Phase (°)</th>
<th>Magnitude (dB)</th>
<th>Phase (°)</th>
<th>Magnitude (dB)</th>
<th>Phase (°)</th>
<th>Magnitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RLSN estimator</td>
<td>-125</td>
<td>-12.5</td>
<td>-140</td>
<td>-17</td>
<td>-173</td>
<td>-23</td>
</tr>
<tr>
<td>KF estimator</td>
<td>-45</td>
<td>-2</td>
<td>-80</td>
<td>-3</td>
<td>-115</td>
<td>-8.8</td>
</tr>
<tr>
<td>NPEKF estimator</td>
<td>-15</td>
<td>1.5</td>
<td>-45</td>
<td>2.5</td>
<td>-84</td>
<td>5</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>-10</td>
<td>0</td>
<td>-22</td>
<td>0</td>
<td>-30</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Magnitude figures are compared to the measured one.*
measured by HP3562A Dynamic Signal Analyzer and presented as the curve ① in Fig. 6. The synthesized transfer function is given by Eq. (34) and shown in curve ② in Fig. 6. The result clearly shows that the real measurement can be approximated by the following equation:

\[
G_{ao}(s) = \frac{a_{mea}(s)}{a_d(s)} = \frac{100}{s + 100}
\]  

(34)

The frequency response of \(\hat{G}_{ao}(s)\) can be obtained by adding another lag-component to \(G_{ao}(s)\):

\[
\hat{G}_{ao}(s) = \frac{\hat{a}_{NPKF}(s)}{a_d(s)} = G_{ao}(s) \frac{100}{s + 100}
\]  

(35)

This is because of the fact that the bandwidth of NPEKF estimator is about 15 Hz (see Fig. 4d). By choosing \(\omega_v = 10 \text{ Hz}, \omega_i = 1 \text{ Hz}, \xi = -10^\circ, \sigma = -30 \text{ dB}\), the control law for AFC is designed according to Eqs. (31) and (32),

\[
G_a(s) = \frac{2000}{(s + 10)(s + 200)} \times K
\]  

(36)

where \(K\) is the control gain.

### 4.3.3. Experimental results on AFC closed loop

With \(K_1\) being closed and \(K_2\) open in Fig. 5, the closed loop frequency response of the AFC based on the estimated acceleration, i.e., \(G_{ac}(s) = \hat{a}_{NPKF}(s)/a_d(s)\) can be experimented. Curve ① shown in Fig. 7 is \(G_{ac}(s)\) with \(K = 50\) in Eq. (36). When \(K_2\) is closed and \(K_1\) is open, the frequency response of \(G_{ac}(s) = a_{mea}(s)/a_d(s)\) can be obtained. Curve ② in Fig. 7 is the one with \(K = 100\). It can be seen from the results that the AFC based on the estimated acceleration is stable and its frequency response is close to that based on real measurement within its bandwidth of about 30 Hz.

### 4.3.4. Influence of the AFC on velocity closed loop

The influence of the AFC on the velocity control was studied by experimentally constructing the frequency response of the velocity closed loop with or without AFC.
as its inner control loop. In Fig. 8, curve ① presents the frequency response of the velocity closed loop without the AFC, curve ② the one with the AFC based on the estimated acceleration, and curve ③ the one with the AFC based on the measured acceleration. The velocity control law \( G_v(s) \) in Fig. 5 was kept unchanged during the experiments. Their comparisons are made at different frequency points and the major results are given in Table 2. It can be seen clearly that the stability of the velocity closed loop does not change significantly after the introduction of the AFC.

4.3.5. The performance of AFC on disturbance suppression

Fig. 9 shows the experimental results regarding the AFC’s ability in suppressing disturbance, where curve ① is for the frequency response of \( v_{\text{out}}(s)/\tau_d(s) \) without AFC, ② with the AFC using estimated acceleration, and ③ with the AFC using accelerometer measurement. Major results at various frequency points are given in Table 3. It can be seen that both AFCs using the estimated acceleration and the accelerometer measurement are able to suppress a disturbance within frequency range of \([0, 10 \text{ Hz}]\), but they are worse off as the disturbance frequency increases. The AFC using the accelerometer measurement is around six times stronger than that using the estimation acceleration at the frequency of 10 Hz, and such a difference becomes smaller as the disturbance frequency decreases. This is because the bandwidth of the NPEKF estimator is smaller than that of the accelerometer.

5. Conclusion

An ordinary Kalman Filter was enhanced for angular acceleration estimation by means of Newton Predictor, where the phase lag of the KF estimator is reduced due to the predicting ability of the NP. The acceleration feedback control was introduced to enhance the independent PD control of multi-DOF mechatronic system. Experiments were carried out to compare the NP-, KF- and NPEKF-based acceleration estimation. The results show that the phase lag in the acceleration estimated by NPEKF is only 1/8 that estimated by NP and 1/3 that by KF at 10 Hz sine torque input. The NPEKF is satisfactory when the frequency of the input torque is low. The results also conclude that the acceleration estimated by NPEKF is con-

---

Table 2

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Without AFC</th>
<th>With AFC using accelerometer</th>
<th>With AFC using estimated acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Hz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnitude</td>
<td>0</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Phase (°)</td>
<td>−2.1</td>
<td>−6</td>
<td>−1</td>
</tr>
<tr>
<td>10 Hz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Magnitude</td>
<td>0</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td>Phase (°)</td>
<td>−3.6</td>
<td>−9.3</td>
<td>−3</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Without AFC</th>
<th>With AFC using estimated acceleration</th>
<th>With AFC using accelerometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistance (dB)</td>
<td>5</td>
<td>−26.3</td>
<td>−31</td>
</tr>
<tr>
<td>Magnitude</td>
<td>1/36</td>
<td>1/5</td>
<td>1/15</td>
</tr>
<tr>
<td>10 Hz</td>
<td>−1.7</td>
<td>−9.2</td>
<td>−15.8</td>
</tr>
<tr>
<td>Resistance (dB)</td>
<td>1/14.1</td>
<td>1/6.6</td>
<td>1/17.6</td>
</tr>
<tr>
<td>Magnitude</td>
<td>−11</td>
<td>−1.7</td>
<td>1</td>
</tr>
<tr>
<td>20 Hz</td>
<td>−11</td>
<td>−1.7</td>
<td>1</td>
</tr>
<tr>
<td>Resistance (dB)</td>
<td>1/5.6</td>
<td>1/15.9</td>
<td></td>
</tr>
<tr>
<td>Magnitude</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Magnitude figures are compared to AFC using the measured acceleration.
consistent with the one measured by accelerometer within [0, 10 Hz].

Experiments were also conducted to compare the AFCs using the estimated acceleration and the measured acceleration. The results show that, in terms of disturbance suppressing capability the AFC with the estimated acceleration exhibits five times less than the AFC with the measured acceleration at 1 Hz and, the control system with the AFC using the estimated acceleration is still 3.5 times stronger than that without AFC at the disturbance of frequency of 10 Hz, and 26 times stronger at the disturbance of frequency of 1 Hz.

Appendix. Proof of Theorem 1

Let \( V(\epsilon) = \epsilon^T P \epsilon \) to be the Lyapunov candidate, then

\[
\dot{V}(\epsilon) = \epsilon^T P \dot{\epsilon} + \epsilon^T \dot{P} \epsilon
\]

Substituting Eq. (18) into Eq. (A1) yields

\[
\dot{V}(\epsilon) = \epsilon^T (A^T P + PA) \epsilon + 2 \epsilon^T PB (\Delta \ddot{v} + \eta)
\]

\[
= -e^T Q e + 2 e^T PB (\Delta \ddot{v} + \eta)
\]

(A2)

Set \( w = B^T P \epsilon \), then

\[
\dot{V}(\epsilon) = -e^T Q e + 2 w^T (\Delta \ddot{v} + \eta)
\]

(A3)

If \( w = 0 \) then

\[
\dot{V}(\epsilon) = -e^T Q e < 0
\]

(A4)

If \( w \neq 0 \), substituting \( \Delta \ddot{v} = -\frac{\epsilon}{||\epsilon||} \) into Eq. (A3),

\[
\dot{V}(\epsilon) = -e^T Q e + 2 w^T \left(-\frac{w}{||w||} + \eta \right)
\]

\[
= -e^T Q e + 2 \left(-\frac{\rho w w}{||w||} + w^T \eta \right)
\]

\[
\leq -e^T Q e + 2 ||w||(-\rho + ||\eta||) < 0.
\]

(A5)

References


