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Backstepping Based Global Exponential Stabilization of a Tracked Mobile Robot with Slipping Perturbation

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Abstract

While the nonholonomic robots with no-slipping constraints are studied extensively nowadays, the slipping effect is inevitable in many practical applications and should be considered necessarily to achieve autonomous navigation and control purposes especially in outdoor environments. In this paper the robust point stabilization problem of a tracked mobile robot is discussed in the presence of track slipping, which can be treated as model perturbation that violates the pure nonholonomic constraints. The kinematic model of the tracked vehicle is created, in which the slipping is assumed to be a time-varying parameter under certain assumptions of track-soil interaction. By transforming the original system to the special chained form of nonholonomic system, the integrator backstepping procedure with a state-scaling technique is used to construct the controller to stabilize the system at the kinematic level. The global exponential stability of the final system can be guaranteed by Lyapunov theory. Simulation results with different initial states and slipping parameters demonstrate the fast convergence, robustness and insensitivity to the initial state of the proposed method.

Keywords: tracked mobile robot, nonholonomic system, stabilization, backstepping, Lyapunov function

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1 Introduction

The autonomous control and navigation tasks of mobile robots in outdoor environments have received considerable attentions in recent years, especially in the researches of planetary or polar explorations, urban reconnaissance, rescue mission scenarios, and so on[1–3]. Compared to the applications of structured indoor environments, outdoor tasks mentioned above seem to be much more challenging since the interactive effectiveness between robots and dynamic environments is time-varying and unpredictable. In view of that tracked robots are thought to be better suited for outdoor applications than wheeled robots due to the larger contact area of tracks that provides them with better stability and traction at various ground conditions[4]. However, the special mechanism of the tracked vehicles makes the slipping effect become a crucial factor for the health and activity of the mobile robot. Moreover, even the turning itself is produced by slipping between tracks and ground. In order to achieve accurate and reliable navigation and control purposes, the slipping effect must be considered seriously to introduce more stability and robustness.

Traditionally the control problem of mobile robots can be divided into two kinds of questions: point stabilization and tracking reference trajectory. The point stabilization problem of mobile robots has been investigated as the stabilization control of nonholonomic systems, or in other words, mechanical systems with nonholonomic constraints, by many researchers. It is well known that according to Brockett's necessary conditions for stability[5], the nonholonomic systems with restricted mobility cannot be stabilized to a desired point by smooth, or even continuous static-state feedback, although it is controllable. To deal with this problem, some researchers have proposed controllers that utilize discontinuous control laws, piecewise continuous control laws, smooth time-varying control laws, or hybrid controllers to achieve point stabilization[6–8]. Among the above researches a widely used method is to transform

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the original system, either locally or globally, to a chained form via coordinates and state-feedback transformation, as explained and illustrated by Murray and Sastry[9]. Several important results have been made to guarantee the exponential stabilization of nonholonomic systems in chained form at kinematic level[10–12].

Compared to the stabilization problem for driftless nonholonomic control systems, which is well understood nowadays, relatively fewer attentions[13] have been attracted to the nonholonomic system with model uncertainties or perturbations that violates the pure nonholonomic constraints such as slipping effect. For a class of chained form systems with constant parameters and drift terms, a robust nonlinear state law is provided with a discontinuous control law using backstepping techniques by Jiang[14,15]. Some other results such as various structures[16], sliding model[17] and adaptive controller[18] can also be found in the literatures. However, all of the above results are constrained to some special structures of system models and seem more complicated to practical applications, and therefore, more or less unsuited to the control with time-varying slipping perturbation.

In this paper, the point stabilization of a tracked mobile robot with slipping is considered. Based on certain assumptions of track-soil interaction, the side slipping effect is parameterized as model perturbation and the kinematic model of the tracked robot is constructed along with the time varying slipping parameter. Under the consideration of slipping the pure nonholonomic constraints with no-slip assumption are violated. The original system is transformed to a chained system, and a new control scheme combined with state-scaling technique and backstepping method is then derived to design the stabilizing controller that achieves global exponential stability in the presence of slipping perturbation.

2 Preliminary and problem statement

In this section a general kinematic model with slipping of tracked vehicles is developed. The slipping is described by a slipping parameter. Certain assumptions are made to simplify the mathematical model: (1) Only the plane motion is considered; (2) The angular velocities of the driven wheel are relatively small. Fig. 1 shows the platform of the vehicle undergoing general planar motion.

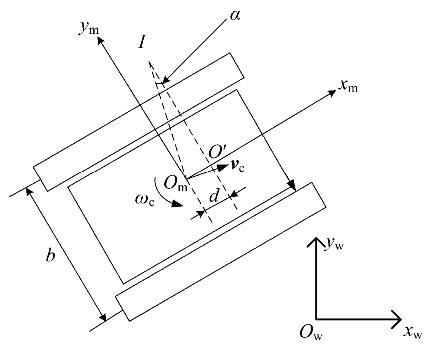

Fig. 1 Platform of the vehicle undergoing general planar motion.

In order to describe the motion of the tracked vehicle, a fixed reference frame $F_1(x_w, y_w)$ and a moving frame $F_2(x_m, y_m)$ attached to the vehicle body with origin at the mass center O_m of the vehicle are defined. The rotation transformation matrix between F_1 and F_2 can be defined as

$$ {}^{\mathrm{m}}_{\mathrm{w}}\boldsymbol{R} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix}, \tag{1} $$

where ψ denotes the heading angle of the robot.

The motion of the vehicle is composed of the translation velocity $\boldsymbol{v}_c=[v_x\ v_y]^T$ and the rotational velocity $\omega_c = d\psi/dt$, where v_x and v_y stand for the projections of $\boldsymbol{v}_c$ on the axes of frame F_2. v_y is termed as the side-slipping velocity. Point I is the instantaneous centre of rotation. Because of the slipping, it often shifts ahead O_m by an amount d. An angle α exists between the line connecting I to O_m and the perpendicular of I to x axis of frame F_2.

The slipping parameter can be described by the amount d, or equivalently defined as $\sigma = \tan\alpha$. In this paper, the mount d is chosen to stand for sideslip. We can easily get the geometrical relationship between d and σ as

$$ \sigma = \frac{d}{|\boldsymbol{v}_c|} = \frac{d}{\sqrt{v_x^2 + v_y^2}}. \tag{2} $$

In frame F_2, a kinematic suitable model can be written as

$$ \begin{cases} \dot{x}_m = v_x = \dfrac{r\omega_L + r\omega_R}{2} \\ \dot{y}_m = v_y = -\dfrac{-r\omega_L + r\omega_R}{b} d, \\ \dot{\psi}_m = \omega_c = \dfrac{-r\omega_L + r\omega_R}{b} \end{cases} \tag{3} $$

where b is the distance between two tracks; r is the radius of the wheels which drive the tracks; ω_L and ω_R denote the angular velocities of the left and right wheels, respectively.

Upon the transformation from F_2 to F_1, the kinematic model is modeled as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos\psi & d\sin\psi \\ \sin\psi & -d\cos\psi \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}, \tag{4}$$

or simply re-writing Eq. (4) as $\dot{\boldsymbol{q}} = \mathrm{S}(\boldsymbol{q})\boldsymbol{\eta}$, where $\boldsymbol{q} = (x, y, \psi)^{\mathrm{T}}$ denotes the pose of the tracked vehicle in the inertial Cartesian frame F_1; (x, y) is the position of the mass center of the robot; v is the forward velocity while ω is the angular velocity of the robot. Here the auxiliary input $\boldsymbol{\eta}$ is defined as

$$\boldsymbol{\eta} = \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_x \\ \omega_c \end{pmatrix} = \begin{pmatrix} \dfrac{r\omega_L + r\omega_R}{2} \\ \dfrac{-r\omega_L + r\omega_R}{b} \end{pmatrix}, \tag{5}$$

and $\boldsymbol{u} = (\omega_L, \omega_R)^{\mathrm{T}}$ is regarded as the real control input which can be used to control $\boldsymbol{\eta}$ according to the relationship $\boldsymbol{\eta} = \boldsymbol{T}\boldsymbol{u}$ with the transformation matrix $\boldsymbol{T}$ as

$$\boldsymbol{T} = r\begin{pmatrix} 1/2 & 1/2 \\ -1/b & 1/b \end{pmatrix}. \tag{6}$$

The relationship $\boldsymbol{u} = \boldsymbol{T}^{-1}\boldsymbol{\eta}$ can be derived from Eq. (6) as

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \boldsymbol{T}^{-1}\begin{pmatrix} v \\ \omega \end{pmatrix} = \frac{1}{r}\begin{pmatrix} 1 & -b/2 \\ 1 & b/2 \end{pmatrix}\begin{pmatrix} v \\ \omega \end{pmatrix}. \tag{7}$$

Since the transformation matrix $\boldsymbol{T}$ is always nonsingular, Eq. (4) will be considered as the kinematic model with input $\boldsymbol{\eta}$ for simplification in the rest of the paper.

Note that the lateral velocity $v_y = -d\omega$, which determines velocity of slipping, is not integrable and hence the nonholonomic constraint can be obtained as

$$\begin{pmatrix} \sin\psi & -\cos\psi & d \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \boldsymbol{A}\dot{\boldsymbol{q}} = 0, \tag{8}$$

which shows that the hypothesis of "pure rolling and non slipping condition" for the vehicle is not satisfied.

As mentioned by some researchers[19–21], the slipping parameter d mainly depends on two kinds of factors: (i) the soil condition of the ground, such as dry sand, sandy loam, clayey soil, dry clay, and so on; (ii) the velocity of the vehicle that affects the centrifugal forces. If we assume that the tracked vehicle is moving on the flat ground with soil of homogeneous texture, and behaves as a rigid-perfectly plastic material, that is, no recoverable elastic deformation occurs and the soil is rigid under increasing load until a stress condition at which failure occurs is reached. Moreover, the vehicle is moving at a relatively slow speed. Based on these hypotheses the following assumption that gives a relatively unsophisticated slipping model can be introduced according to the above mentioned references as

$$d = k\omega^2, \tag{9}$$

where k is an unknown coefficient related to the ground condition for (i). Here the slipping parameter is approximately proportional to the second power of the angular velocity, which is also consistent to (ii). In this paper, this relatively simple model of the slipping will be used to stabilize the tracked robot, and our purpose is to give some preliminary results for the control of tracked robot with slipping. It should be noted that the slipping parameter can be estimated on-line by some nonlinear estimators such as extended Kalman filter[22].

The stabilization control problem of the tracked robot with slipping studied in this paper is stated as follows:

Control Objective: Given the kinematic model Eq. (4) of the tracked robot with slipping, the initial point $\boldsymbol{q}_0$ and the reference target point $\boldsymbol{q}_d$, the goal is to design a control law $\boldsymbol{\eta}(t)$ such that

$$\lim_{t\to\infty}\|\boldsymbol{q}(t) - \boldsymbol{q}_d\| = 0, \tag{10}$$

where the signal $\|\boldsymbol{x}\|$ stands for the second-order norm of the vector $\boldsymbol{x}$. Note that the target point $\boldsymbol{q}_d$ is a constant vector, for simplicity we can set it to origin, that is, $\boldsymbol{q}_d = \boldsymbol{0}$. The control objective can then be transformed to

$$\lim_{t\to\infty}\|\boldsymbol{q}(t)\| = 0. \tag{11}$$

Furthermore, if there exits a control law $\boldsymbol{\eta}(t)$ such that

$$\|\boldsymbol{q}(t)\| \le \delta(\|\boldsymbol{q}_0\|)e^{-\varepsilon t}, \forall t \ge 0, \tag{12}$$

where $\varepsilon > 0$ and $\delta = \delta(\|\boldsymbol{q}_0\|) > 0$ is only dependent on the initial conditions, the original system can be said to be globally exponentially stabilized to the origin by the proposed control law $\boldsymbol{\eta}(t)$. Note that the function δ is said to belong to class K because it is strictly increasing and $\delta(0) = 0$. The system is therefore K-exponentially stable. Here the definition of K-exponential stability is not the same as the normal exponential stability, in which $\delta(\|\boldsymbol{q}_0\|)$ is set to $\delta(\|\boldsymbol{q}_0\|) = \beta\|\boldsymbol{q}_0\|$ with β being a positive constant. The exponential stability with form Eq. (12) is often called as K-exponential stability corresponds to a weaker form of stability than the usual concept of exponential stability[11]. However, these two kinds of exponential stability are equal with respect to the rate of convergence. Therefore, the notion "exponential stabilization" is used in this paper with discrimination.

3 Controller design using backstepping

In this section a globally stabilizing controller with exponential convergence rates is constructed to solve our stabilization problem by a so-called integrator backstepping technique.

Firstly, the original system of Eq. (4) is transformed into the chained form by the following nonsingular coordinate transformation (which is a global diffeomorphism) $\boldsymbol{x} = (x_0, x_1, x_2)^{\mathrm{T}} = \boldsymbol{\Phi}(\boldsymbol{q})$ as

$$\begin{cases} x_0 = \psi \\ x_1 = x\sin\psi - y\cos\psi \\ x_2 = x\cos\psi + y\sin\psi \end{cases}, \quad (13)$$

and the invertible input transformation $\boldsymbol{u} = (u_0, u_1)^{\mathrm{T}} = \boldsymbol{\Omega}(\boldsymbol{q})\boldsymbol{\eta}$ as

$$\begin{cases} u_0 = \omega \\ u_1 = v - x_2\omega \end{cases}. \quad (14)$$

The chained form of the original kinematic model by Eq. (4) can be deduced as

$$\begin{cases} \dot{x}_0 = u_0 \\ \dot{x}_1 = (x_2 + d)u_0 = (x_2 + ku_0^2)u_0 \\ \dot{x}_2 = u_1 \end{cases}. \quad (15)$$

The triangular structure of the above chained form system Eq. (15) suggests that we can design the control inputs u_0 and u_1 in two separate stages by dividing the system into x_0 sub-system, x_1 and x_2 sub-systems. For the x_0 sub-system a global exponential stabilization law can be designed easily as

$$u_0 = -\lambda_0 x_0, \quad (16)$$

where λ_0 is a positive design parameter to be determined. A Lyapunov function candidate for the x_0 sub-system is selected as

$$V_0 = \frac{1}{2}x_0^2, \quad (17)$$

so that the time derivative of V_0 satisfies

$$\dot{V}_0 = -\lambda_0 x_0^2, \quad (18)$$

which guarantees the global exponential stability of the x_0 sub-system. Moreover, with control input of Eq. (16) the corresponding solution of the x_0 sub-system can be obtained directly as

$$x_0(t) = x_0(0)e^{-\lambda_0 t}. \quad (19)$$

For the x_1 and x_2 sub-systems, it should be noted from Eq. (19) that the x_0 sub-system stabilizes to zero, the control input $u_0(t)$ also converges to zero as t goes to infinity. It will cause serious troubles in controlling the x_1 and x_2 sub-systems, because the sub-systems are uncontrollable at the limit, as stated in some previous researches[14]. Thus a state-scaling strategy is introduced here by the follow discontinuous transformation of the x_1 and x_2 sub-systems as

$$\begin{cases} z_1 = x_1 / x_0 \\ z_2 = x_2 \end{cases}, \quad (20)$$

where $x_0(t)$ can not be zero if the initial point $x_0(0)$ is not zero according to Eq. (19). When $x_0(0) = 0$, that is, the singular case happens, we can choose u_0 to be a non-zero value u^* for some time $t_1 > 0$ until the value $x_0(t_1)$ is no longer zero, then at time t_1 the controller u_0 will switch into Eq. (16) to avoid the singularity of the transformation of Eq. (20).

In the rest of the section we will focus on designing the control input u_1 provided $x_0(0)$ is not zero. The x_1 and x_2 sub-systems are then transformed to the following z system as

$$\begin{cases} \dot{z}_1 = -\lambda_0(z_2 + d - z_1) \\ \dot{z}_2 = u_1 \end{cases}. \quad (21)$$

The following target is to design the control input u to stabilize the sub-system Eq. (21). Observed from the special structure of the sub-system, the common backstepping method can be applied in two separate steps by divided the system to z_1 and z_2 sub-systems further.

For z_1 sub-system, denote $\xi_1 = z_1$ and introduce the Lyapunov function candidate as

$$V_1 = \frac{1}{2}\xi_1^2, \tag{22}$$

where z_2 is treated as the virtual control input. Then the time derivative of V_1 can be computed as

$$\dot{V}_1 = \xi_1\dot{\xi}_1 = -\lambda_0\xi_1\left(z_2 + d - z_1\right). \tag{23}$$

The virtual control input z_2 can be chosen as

$$z_2 = \alpha_1\left(x_0, z_1\right) = \left(\lambda_1 + 1\right)z_1 - d, \tag{24}$$

where λ_1 is a new positive design parameter to be determined; α_1 is a smooth function satisfying

$$\alpha_1(x_0, 0) = 0, \forall x_0 \in \mathbb{R}. \tag{25}$$

Consequently Eq. (23) implies

$$\dot{V}_1 = -\lambda_0\lambda_1\xi_1^2 \le 0, \tag{26}$$

thus the z_1 sub-system is then globally asymptotically stabilized by the virtual control input Eq. (24). In view of that a new variable ξ_2 can be defined as $\xi_2 = z_2 - \alpha_1$. For z_2 sub-system, the following Lyapunov function candidate can be introduced as

$$V_2 = V_1 + \xi_2^2 / 2, \tag{27}$$

and therefore the time derivation of V_2 satisfies

$$\dot{V}_2 = -\lambda_0\lambda_1\xi_1^2 - \lambda_0\xi_1\xi_2 + \xi_2\left(u_1 + \lambda_0\left(\lambda_1 + 1\right)\left(z_2 + d - z_1\right) + \dot{d}\right). \tag{28}$$

The control input u_1 can be chosen as

$$u_1 = -\lambda_0\left(\lambda_1 + 1\right)\left(z_2 + d - z_1\right) - \dot{d} + \lambda_0\xi_1 - \lambda_2\xi_2, \tag{29}$$

where $\lambda_2 > 0$ is also a positive design parameter to be determined. With the control input u_1 of Eq. (29) the time derivation of V_2 can be computed as

$$\dot{V}_2 = -\lambda_0\lambda_1\xi_1^2 - \lambda_2\xi_2^2 \le 0. \tag{30}$$

Finally we re-write the control input Eqs. (16) and (29) designed in this section as

$$\begin{cases} u_0 = -\lambda_0 x_0 \\ u_1 = -\lambda_0\left(\lambda_1 + 1\right)\left(z_2 + d - z_1\right) - \dot{d} + \lambda_0\xi_1 - \lambda_2\xi_2 \\ \quad = -\lambda_0\left(\lambda_1 + 1\right)\left(x_2 + k\lambda_0^2 x_0^2 - x_1 / x_0\right) + 2k\lambda_0^3 x_0^2 + \\ \quad \lambda_0 x_1 / x_0 - \lambda_2\left(x_2 - \left(\lambda_1 + 1\right)x_1 / x_0 + k\lambda_0^2 x_0^2\right) \end{cases}. \tag{31}$$

With the designed control inputs of Eq. (31), the global exponential stability of the system is guaranteed by the following theorem.

Theorem 1: The control input of Eq. (31) can make the kinematic model Eq. (4) of the tracked robot with slipping parameter globally exponentially stabilized to the origin, that is, the condition defined in Eq. (12) is satisfied.

Proof: Choose the Lyapunov function candidate as

$$V = \left(x_0^2 + \xi_1^2 + \xi_2^2\right) / 2 = \left\|\boldsymbol{\xi}\right\|^2, \tag{32}$$

where $\boldsymbol{\xi}$ is defined as $\boldsymbol{\xi} = (x_0, \xi_1, \xi_2)^{\mathrm{T}}$. By using control input of Eq. (31), the time derivation of V is given as

$$\dot{V} = -\lambda_0 x_0^2 - \lambda_0\lambda_1\xi_1^2 - \lambda_2\xi_2^2. \tag{33}$$

Note that the design parameters should satisfy $\lambda_i > 0$ (i = 0, 1, 2), a new positive parameter $\lambda = \min\{\lambda_0, \lambda_0\lambda_1, \lambda_2\} > 0$ can be defined and Eq. (33) can be transformed to

$$\dot{V} \le -2\lambda\left\|\boldsymbol{\xi}\right\|^2 = -2\lambda V. \tag{34}$$

By integrating Eq. (34) we have $V \le V\left(0\right)e^{-2\lambda t}$, which implies

$$\left\|\xi\left(t\right)\right\| \le \left\|\xi\left(0\right)\right\|e^{-\lambda t}. \tag{35}$$

It can be seen from Eq. (35) that $\boldsymbol{\xi}$ is globally exponentially stabilized to origin. As for the solution Eq. (19) of x_0 sub-system, the state x_0 is also globally exponentially stable. Similar conclusion can be made for $\boldsymbol{z} = (z_1, z_2)^{\mathrm{T}}$ through Eq. (30). From the transformation Eqs. (21) and (24) as well as the property Eq. (25) of α_1, we can induce that

$$\left\|z\left(t\right)\right\| \le \gamma\left(\left\|\left(x_0\left(0\right), z\left(0\right)\right)\right\|\right)e^{-\lambda t}, \tag{36}$$

where γ is a K-class function. Eq. (36) shows global exponential stability of the z system. Moreover, based on Eqs. (13), (14), (15) and (16), the original kinematic system of Eq. (4) is also globally exponentially stabilized to origin. The proof of Theorem 1 is then completed.

After the auxiliary control input $\boldsymbol{\eta}$ is designed, the real control input $\boldsymbol{u}$ can be obtained from Eq. (7), that is, $\boldsymbol{u} = \boldsymbol{T}^{-1}\boldsymbol{\eta}$.

4 Simulation results

In this section, we perform some simulations on the kinematic model of the tracked vehicle with slipping parameter, using the methods described in previous section. The experimental platform used in study is a tracked vehicle with two arms as shown in Fig. 2.

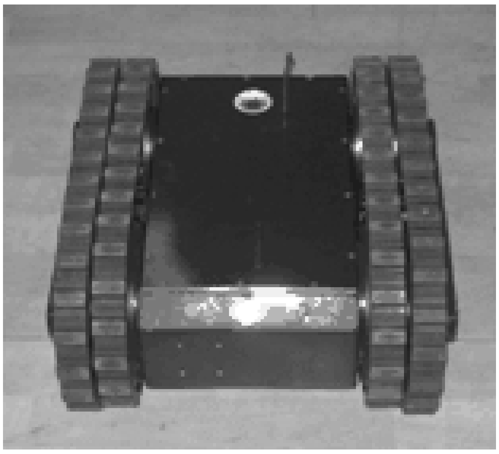

Fig. 2 The experimental tracked vehicle.

In the simulation, the angular velocities of the two wheels that drive the two main tracks are considered as input variables. The physical parameters in the model are chosen as follows: b = 0.65 m, r = 0.35 m. In the simulation, we assume some different slipping parameters which stand for some grounds with homogeneous texture or soil such as dry sand, sandy loam, clayey soil, dry clay, and so on, as mentioned above. The initial point of the robot is also set to different values to test the convergence and robustness of the proposed method. The design parameters contained in our controller are set to $\lambda_i = 1$ (i = 0, 1, 2). The total time of simulation is chosen as t_s = 20 s and the time-step is set to 0.1 s.

Fig. 3 shows the trajectories of the tracked robot with different initial states $\boldsymbol{q}(0)$ as shown in Table 1.

Table 1 Initial state setting for different trajectories

Trajectory index	Initial state $\boldsymbol{q}(0)$
Trajectory A	$[1\ \ 1\ \ -\pi/2]^T$
Trajectory B	$[-1\ \ 1\ \ -\pi/4]^T$
Trajectory C	$[-1\ \ -1\ \ \pi/3]^T$
Trajectory D	$[1\ \ 1\ \ \pi/4]^T$

It can be seen from Fig. 3 that for different starting points $\boldsymbol{q}(0)$ all four trajectories tend to the origin as time increases, which shows our method is not sensitive to initial points. It is also interesting to observe that the robot sometimes heads back to the target point after going ahead for a while, which reminds us of the parking problem of vehicles in real world.

Fig. 4 shows the trajectories of the robot with different slipping coefficient, k = 0.1, 0.5, 1, 2, of the time-varying slipping parameters, d. The initial states of all four trajectories are set as trajectory B. It can be seen that all trajectories tend to zero as time increases, which indicates the robustness of the designed controller that is not sensitive to slipping coefficient that stands for different soil textures of the ground.

Fig. 5 shows the state trajectory of the chained system Eq. (15), and Fig. 6 shows the position of the tracked robot in both x and y directions and the heading angles ψ. The initial state is set the same as trajectory B. In Fig. 5 all the states of chained system tend to zero with exponential convergence, which is consistent to Eq. (35). As for Fig. 6, it can be easily seen that the system states all converge to the target point quickly, which also shows the exponential stability of the proposed method and the conclusion of Theorem 1 is verified. Results for other trajectories are all similar to Fig. 6.

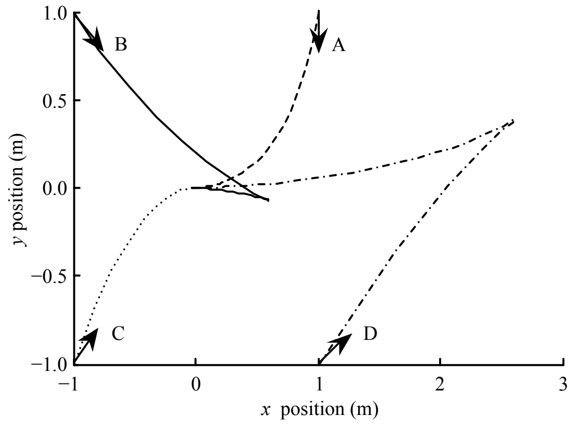

Fig. 3 Trajectories of x via y with different initial states.

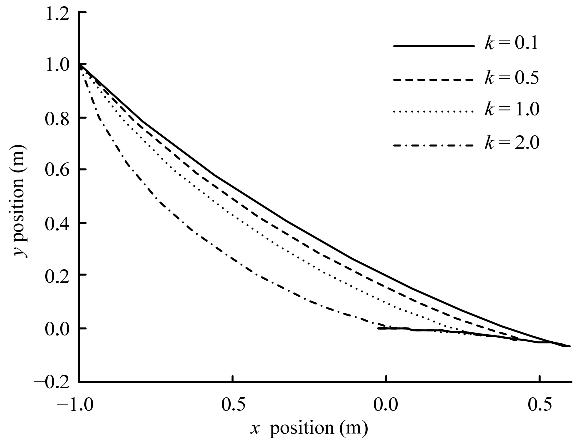

Fig. 4 Trajectories of x via y with different slipping coefficient.

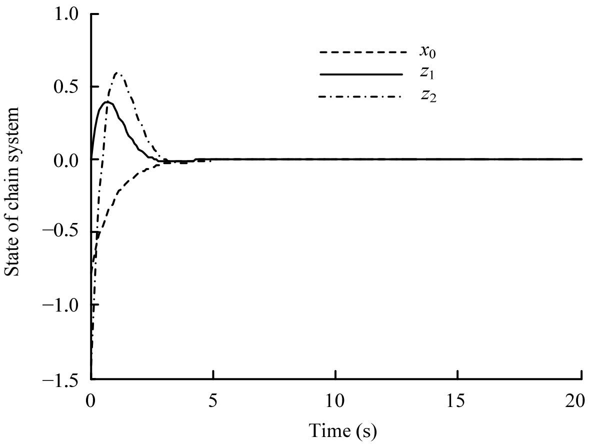

Fig. 5 State of chain system for trajectory B.

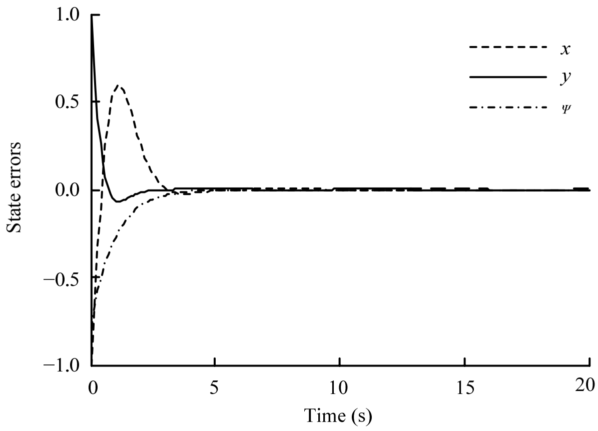

Fig. 6 State errors for trajectory B.

In Fig. 7 control inputs for trajectory B, the angle velocities ω_L and ω_R of the two driving wheels of the tracked vehicle are given. The dash line stands for input ω_L and the solid line stands for input ω_R respectively. It can be also seen that the two inputs both tend to zero as time increases, which is consistent to the convergence of the results.

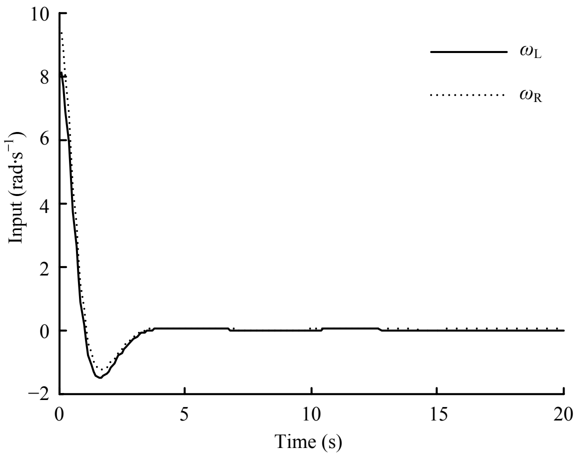

Fig. 7 Control inputs of trajectory B.

5 Conclusion

The stabilization problem of a tracked vehicle with slipping efficiency is considered in this paper. The slipping here is modeled as a slipping parameter, so that the kinematic model is created as a system that violates the pure nonholonomic constraint. The backstepping method combined with state scaling technique is applied to construct the stabilization controller for the nonholonomic system in chained form that is transformed form the original system via some nonsingular coordination transformations. The Lyapunov analysis guarantees the global exponential stability of the final system with the designed control law. Some simulation results are given to demonstrate the effectiveness of the proposed methods. The aim is to give some pioneer investigation on the control problem of field robot with slipping. More works will be done to investigate the physical mechanism of slipping, and the stabilization experiments on several kinds of terrain will be studied in future.

Acknowledgments

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