Abstract

In this paper we present an approach to decoupled force/position control of the end-effector along the same direction for redundant robots, and an approach to nonholonomic cart pushing with mobile manipulators. The mobile manipulator is considered as a redundant robot, and a unified dynamic model for an integrated mobile platform and on-board manipulator is developed. The dynamic model is decoupled and linearized using the nonlinear feedback technique in a unified frame. Combining the event-based planning and control method with singularity analysis of the robot arm, a task level action controller is designed and an online kinematic redundancy resolution scheme is developed. The system is stable during normal operation as well as at the occurrence of unexpected obstacles. In addition, explicit force/position control along the same task direction for redundant robots is proposed. The kinematic redundancy of mobile manipulators enables independent control of force and position along the same task directions. To verify the decoupled force/position scheme, an integrated task planning and control approach is further proposed for the mobile manipulator to complete complicated tasks by regulating its output force. A cart pushing task, which requires both force and position control along the same task direction, is discussed. The cart manipulation task fully integrates trajectory and force planning of the cart, and planning and control of the mobile manipulators. The approaches have been tested on a mobile manipulator consisting of a Nomadic XR4000 and a Puma 560 robot arm. The experimental results demonstrate the efficacy of the approach for the mobile manipulation of a nonholonomic cart.
mobile manipulator require the robot to interact with its environment dynamically while providing motion, such as pushing, pulling, cutting, excavating. Implementation of these tasks demands the mobile manipulator to provide both output force control and motion control along the same task direction. The kinematic redundancy of mobile manipulators makes possible the independent control of force and position along same task directions. Thirdly, an integrated task planning and control approach to cart pushing with a mobile manipulator is introduced. This task involves the trajectory and force planning of the object, the trajectory planning and control of the mobile manipulator, synchronization of the motion planners for the cart and the mobile manipulator. The integrated task planning and control approach allows the mobile manipulator to accomplish complicated tasks by regulating its output force and position independently, and to coordinate the motion of the object and mobile manipulator by a common motion reference. The above approaches are discussed based on a unified model of mobile manipulators. A quick survey of the ongoing research in related fields is given below.

1.1. Kinematic Redundancy Resolution

The existing approaches to coordinate the platform and the robot arm, or, in other words, to resolve the kinematic redundancy, are generally based on additional tasks or based on the optimization of certain criteria. The additional task method resolves the kinematic redundancy of mobile manipulators by performing secondary tasks (Bayle et al. 2002; Fourquet and Renaud 2000; Seraji 1998). Seraji (1998) proposed an online coordinated motion control method by treating the mobile manipulator as a redundant mechanical system. Secondary tasks for redundancy resolution are selected based on task specifications. Bayle et al. (2002) developed the additional task method to provide solutions to arbitrary numbers of additional user-defined tasks. Egerstedt and Hu (2000) proposed a coordinated virtual vehicle solution to the trajectory following problem for mobile manipulators. A trajectory for the mobile platform is planned in such a manner that the desired end-effector position is within the specified workspace of the arm.

The kinematic redundancy can also be resolved by optimizing application specific requirements, such as minimum torque output, minimum distance, etc. Huang, Sugano, and Tanie (1998) proposed a method for coordinating the motion of the platform and the manipulator while considering the stability of the platform and manipulator workspace. Foulon, Forquet, and Renaud (1998) presented strategies for planning paths between two points in the generalized and operational spaces. Redundancy is resolved by optimizing the path of the nonholonomic mobile platform. Chen and Zalzala (1997) discussed trajectory generation while considering the dynamics of the system. The optimal trajectory generation problem with nonholonomic constraints and obstacle avoidance were formulated as a nonlinear multi-criteria optimization problem. Fourquet and Renaud (2000) compared the additional task method and the criteria-based method.

When a mobile manipulator performs multiple tasks in a sequence, the final configuration of each task becomes the initial configuration of the subsequent task. The configuration between two tasks is called commutation configuration (Pin, Culioli, and Reister 1994). In the path planning for a sequence of tasks, the motion trajectories together with commutation configuration need to be considered simultaneously. Most of the redundancy resolution approaches for executing a sequence of tasks are criteria based (Lee and Cho 1997; Pin, Culioli, and Reister 1994; Zhao, Ansari, and Hou 1992). A variety of optimization criteria such as obstacle avoidance, reach and maneuverability are developed and the algorithms for optimization problems are discussed.

1.2. Modeling and Control

To achieve effective and practical control, various dynamic models and control schemes are proposed in the literature. The control strategies are generally classified into two categories. One is separated control of the mobile platform and the manipulator arm. The controller for each part is constructed separately and then the interaction between the platform and the manipulator is considered. The other is an integrated approach for the control of both the mobile platform and the robot arm. In this case, the mobile manipulator is treated as a redundant robot.

By considering the mobile robot and the manipulator as two subsystems, Liu and Lewis (1990) developed a decentralized robust controller for a mobile manipulator. The reaction forces are considered in the dynamic models of the two subsystems. Chung and Velinsky (1998, 1999) also derived the dynamic model of nonholonomic mobile robots and manipulators separately. The kinematic redundancy is resolved by decomposing the mobile manipulator into two subsystems. Yamamoto and Yun (1994) studied a two-linked planar mobile manipulator subject to nonholonomic constraints. The dynamics model is derived and reformulated according to the nonholonomic constraints. Output feedback linearization is used to simplify the nonlinear model and derive the controller. The mobile platform and the manipulator are coordinated based on the concept of a preferred operating region. Colbaugh, Trabatti, and Glass (1999) considered the dynamics model for redundant nonholonomic mechanical systems and reformulated it into two parts, a reduced dynamics model and a kinematic relationship which considers the nonholonomic constraints. The mobile platform and the arm are coordinated such that the distance between the end-effector and the mobile platform does not exceed a pre-specified value.

A variety of integrated control of the mobile platform and onboard robot manipulator approaches are available in the literature. Bayle et al. (2002) and Bayle, Fourquet, and Renaud
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(2000) proposed a pseudo-inversion scheme for coordinating the evolution the mobile platform and the robot arm. Kinematic control of mobile manipulators is solved to avoid singularities and maximize the arm manipulability. Khatib (1999) and Khatib et al. (1996, 1999) developed a general framework for the dynamic coordination and control of mobile manipulators based on the operational space model (Khatib 1987). The dynamic model is formulated in operational space for task-orientated robot motion and force control. The dynamic bandwidth is considered to coordinate the robot arm and the mobile platform. An augmented object model enables the manipulation of object in multiple mobile manipulation systems. Brock and Khatib (2000) proposed an elastic strips framework, which enables the integration of various motion behaviors, such as task-consistent obstacle avoidance, posture control, transition between different motion behaviors (Brock, Khatib, and Viji 2002). Tanner and Kyriakopoulos (2000) investigated the path planning for nonholonomic mobile manipulators based on a navigation function. A discontinuous feedback control law, which guarantees convergence to the desired final position and singularity avoidance, is generated based on the navigation function. Ogren et al. (2000) presented a trajectory tracking algorithm for mobile manipulators such that the mobile platform motion was generated to coordinate with the gripper. Obstacle avoidance behavior and manipulability are considered into the coordination behavior.

1.3. Force Control Schemes

Robot manipulator force control schemes such as external force control, hybrid position/force control, impedance control have been applied to the force control of the mobile manipulators. Umeda and Nakamura (1999) discussed the hybrid position/force control of mobile manipulators. In order to obtain desired hybrid control with respect to the end-effector’s tasks, the concept of performance indices based on an equivalent mass matrix is introduced. Antonelli, Sarkar, and Chia-Verini (1999) presented a force control scheme for underwater vehicle manipulator systems. The environment is modeled as a frictionless and elastically complaint plane. External control structure is used and kinematic redundancy is considered. Khatib (1999) introduced a virtual linkage model for the control of internal forces for multi-grasp manipulation based on the operational space model of mobile manipulators. Inoue, Murakami, and Ohnishi (2001) discussed a control strategy of mobile manipulators when external force is imposed on the end-effector. The dynamic bandwidth and the singular configuration are taken into consideration.

Recently, research interests in coordination of multiple mobile manipulators for object handing and human/mobile manipulator cooperation have been growing (Chung 2002; Khatib 1999; Kosuge, Oosumi, and Seki 2000; Yamamoto, Eda, and Yun 1996). Multiple mobile manipulators can work together to handle heavy and oversized objects in large workspace. Furthermore, human and mobile manipulators can also work together to integrate human intelligence with robot strength and mobility. Interactive force control is necessary for object handling and human/robot cooperation tasks. Due to the kinematic redundancy, the force control and position control of mobile manipulators can be decoupled along the same task direction. In this paper we discuss the decoupled force/position control based on a linearized and decoupled model. The decoupled force/position control scheme enables independent control of force and position along the same task direction. It can be applied to a variety of human/mobile manipulator cooperation tasks (Tan et al. 2003) and object handling tasks. A cart pushing task with mobile manipulators is discussed in this paper.

1.4. Mobile Manipulation Using Force Control

The integrated task planning and control of mobile manipulators performing tasks of interacting with moving objects has received relatively less attention. Based on the decoupled force/position control of mobile manipulator, in this paper we present an integrated task planning and control approach for the task of the mobile manipulator pushing a nonholonomic cart. As shown in Figure 1, the mobile manipulator and nonholonomic cart system is similar to the tracker–trailer system. In a tracker–trailer system, there is generally a steering mobile robot and one or more passive trailer(s) connected together by rigid joints. The tracking control and open loop motion planning of such a nonholonomic system has been discussed in the literature. The trailer system is controlled to track certain trajectory using a linear controller based on the linearized model (Desantis 1994). Exact linearization and nonlinear control of

Fig. 1. The mobile manipulator and nonholonomic cart system.
the tracker–trailer system have been discussed in Sampei et al. (1995) and experiments on backward steering the trailer have been reported in Kim and Oh (1998). Instead of pulling the trailer, the tracker pushes the trailer to track certain trajectories in the backward steering problem. Fire truck steering is another example for pushing a nonholonomic system. Based on the chained form (Bushnell, Tibury, and Sastry 1995), motion planning for steering a nonholonomic system has been investigated in Murray and Sastry (1993) and Tibury, Murray, and Sastry (1995).

The planning and control of the mobile manipulator and nonholonomic cart system is different from a tracker–trailer system. They are linked not by a rigid joint, but by the robot arm. Therefore, the mobile manipulator has more flexibility and control while maneuvering the nonholonomic cart. In a tracker–trailer system, control and motion planning are considered based on the kinematic model, and the trailer is steered by the motion of the tracker. In a mobile manipulator and nonholonomic cart system, the mobile manipulator can manipulate the cart by a regulated output force. The nonholonomic cart can be controlled at a dynamic level by the output force of the mobile manipulator. However, this requires full integration of the path planning and force planning of the cart, and the planning and control of mobile manipulators.

The rest of the paper is organized as follows. In Section 2, a unified model of a mobile manipulator is developed and linearized using nonlinear feedback control techniques. The model is considered in a common coordinated frame such that path tracking control and force control can be easily considered. In Section 3 we present a redundancy resolution scheme based on the event-based coordinating approach and the analysis of the robot arm singular configurations. This online approach is suitable for both a single task and multiple segment tasks in a sequence. In Section 4 we discuss force/position control along the same task direction. The kinematic redundancy of mobile manipulators is used to achieve explicit force and position control along the same task direction. Based on the decoupled force/position control scheme, in Section 5 we present an integrated task planning and control approach while the mobile manipulator is interacting with moving objects. This approach integrates the motion and force planning of the moving object, the planning and control of the mobile manipulator, and the estimation of the cart configuration. The proposed integrated task planning and control approach enables the mobile manipulator to perform complicated manipulation tasks by regulating its output force. We discuss the experimental results in Section 6.

2. Unified System Model

A mobile manipulator usually consists of a mobile platform and a robot arm. Figure 2 shows the associated coordinate frames of both the platform and the manipulator. The world coordinate frame $\Sigma_w$ is an inertial coordinate frame where robot tasks are generally defined. The moving coordinate frame $\Sigma_m$ is a frame attached on the mobile platform. The mobile platform and robot manipulator are generally designed separately, the mathematical models are developed in coordinate frames $\Sigma$ and $\Sigma_b$, respectively. A virtual moving coordinate frame, $\Sigma'_m$, is introduced here such that the mobile platform model in coordinate frame $\Sigma$ and the robot manipulator model in coordinate frame $\Sigma_b$ can be easily integrated to develop the mobile manipulator model. The virtual moving coordinate frame $\Sigma'_m$ shares the same origin with $\Sigma_b$ and parallels to $\Sigma$.

Since the tasks of a mobile manipulator are generally given in task space, i.e., in the world coordinate frame, a unified model in the world coordinate frame is preferable for control and motion planning of the mobile manipulator. The models are developed based on PUMA type robot arms and mobile platforms with three DOF. However, the procedure is applicable to mobile manipulators consisting of generic robot arms and mobile robots. For easy references, all the definitions of the state variables for the mobile manipulator, the mobile platform, and the robot arm are listed as follows:

- $q = \{q_1, q_2, q_3, q_4, q_5, q_6\}^T$ is the joint angles of the robot arm;
- $Y_b = \{x_b, y_b, \theta_b\}^T$ is the configuration of the mobile platform in the world frame;
- $q' = \{q_1 + \theta_b, q_2, q_3, q_4, q_5, q_6\}^T$ considers the orientation of mobile platform as an addition of the first joint angle of the robot arm;
- $\theta_b = \{0, 0, 0, 0, 0\}^T$ is the orientation vector of the mobile platform;
- $p = \{q_1, q_2, q_3, q_4, q_5, q_6, x_b, y_b, \theta_b\}^T$ is defined as the joint variables of the mobile manipulator;
- $y'_b = \{x_b, y_b, h_b, 0, 0, 0\}^T$ is the origin of $\Sigma_b$ and $\Sigma'_b$ where $h_b$ is the height of the mobile robot;
- $y_m = \{p_{x_m}', p_{y_m}', p_{z_m}', O_{m}', A_{m}', T_{m}'\}^T$ defines the end-effector position and orientation in frame $\Sigma'_b$;
- $y'_m = \{p_{x_m}', p_{y_m}', p_{z_m}', O_{m}', A_{m}', T_{m}'\}^T$ defines the end-effector position and orientation in the virtual moving frame $\Sigma'_m$;
- $y = \{p_s, p_r, p_o, O, A, T\}^T$ defines the end-effector position and orientation in the world frame, which is also the output of the mobile manipulator;
- $f = \{f_x, f_y, f_z\}^T$ defines force applied on the end-effector of the mobile manipulator.

2.1. Kinematic Model

From Figure 2, it can be seen that the relationship between the virtual moving coordinate frame $\Sigma'_m$ and the moving coordinate frame $\Sigma_b$ can be defined by a transformation matrix.
$T^b_\Sigma$. Only translation is involved between coordinate frame $\Sigma$ and the virtual moving coordinate frame $\Sigma'_b$. The kinematics of robot arms in coordinate frame $\Sigma_b$ and the kinematics of mobile platforms in the coordinate frame $\Sigma$ are used to build the kinematics of mobile manipulators in frame $\Sigma$.

In the moving frame coordinate $\Sigma_b$, the kinematics of the robot manipulator can be described as $y_m = h(q), y_n = \dot{J}q$, where $h$ is a nonlinear transformation and Jacobian $\dot{J}$ denotes the relation between joint space and task space velocities in coordinate frame $\Sigma_b$. The kinematics of the manipulator in frame $\Sigma_b$ can be transformed into frame $\Sigma'_b$ using the transformation $T^b_\Sigma$. The end-effector in the moving frame $\Sigma'_b$ can be proven to be $y'_m = h(q'), y'_n = \dot{J}'q'$, where $h$ is the same as the transformation in coordinate frame $\Sigma_b$. $\dot{J}$ denotes the velocity relation between task space and joint space in coordinate frame $\Sigma_b$. The robot manipulator model in frame $\Sigma'_b$ can be easily obtained by replacing $q$ using $q'$. The derivation of the robot manipulator model is simplified by using the virtual coordinate frame. The existing models of the robot arm and the mobile platform can be used.

Since only translation is involved between coordinate frames $\Sigma$ and $\Sigma'_b$, the end-effector position and orientation in coordinate frame $\Sigma$ can be expressed as $y = y'_m + y'_n$. The velocity and acceleration relation in the world frame $\Sigma$ can then be derived as $\dot{y} = \dot{y}'_m + \dot{J}\dot{q}'$ and $\ddot{y} = \ddot{y}'_m + \ddot{J}\ddot{q}' + \dddot{J}\dot{q}'$. Actually, by relating the task space velocity and the joint space velocity of the mobile manipulator in matrix form, the kinematics of the mobile manipulator in coordinate frame $\Sigma$ can be written as

$$\begin{bmatrix}
1 & 0 & \dot{J}_{11} \\
0 & 1 & \dot{J}_{21} \\
0 & 0 & \ddot{J}_{31} \\
0 & 0 & \dddot{J}_{41} \\
0 & 0 & \dddot{J}_{51} \\
0 & 0 & \dddot{J}_{61}
\end{bmatrix} \begin{bmatrix}
\dot{p} \\
\ddot{p}
\end{bmatrix} = J_b \dot{p},$$

(1)

where $J_b$ is defined as the Jacobian of the mobile manipulator.

It can be seen from $J_b$ that the robot manipulator Jacobian is augmented by a submatrix whose rank is 3 ($J_{61}$ equals 1). The Jacobian can be full rank even when $\dot{J}$ is singular. Thus the mobile platform increases the capability of manipulation.

### 2.2. Dynamics Model

The dynamics of the robot arm in frame $\Sigma_b$ can generally be described by

$$M_1(q)\ddot{q} + c_1(q, \dot{q}) + g_1(q) = \tau,$$

(2)

where $\tau$ is the $6 \times 1$ vector of applied torques, $M_1(q)$ is the $6 \times 6$ positive definite manipulator inertia matrix, $c_1(q, \dot{q})$ is the $6 \times 1$ centripetal and Coriolis torques, and $g_1(q)$ is the $6 \times 1$ vector of gravity term. Substituting the acceleration relationship of the mobile manipulator into the dynamics equation (2), the robot dynamics described by variables in the world coordinate frame can be obtained:

$$M_1\ddot{\bar{x}}_b - \bar{J}\ddot{\bar{\theta}}_b - \dddot{\bar{\theta}}_b + \dddot{\bar{\theta}}_b + \dddot{\bar{J}}\dot{\bar{q}},$$

(3)

where $N_1, \ddot{x}_b$ combines terms $-M_1\dddot{\bar{\theta}}_b$ and $-M_1\dddot{\bar{\theta}}_b$.

Following the work of Liu and Lewis (1990), Yamamoto and Yun (1994), and Morel and Dubowsky (1996), the dynamics of a holonomic mobile platform can be represented by

$$N_2(p)\ddot{x}_b + M_2(p)\dddot{x}_b + c_2(p, \dot{p}) = \tau_2,$$

(4)

where $N_2(p)$ represents the interaction between the mobile platform and the robot arm, $M_2(p)$ is the inertia matrix of the mobile platform, and $\tau_2 = \{\tau_x, \tau_y, \tau_z\}$ is the generalized input torques applied to the mobile platform. The dynamic model...
of mobile manipulators with holonomic mobile platform can then be described by
\[
\begin{bmatrix}
M_1 \ddot{\vec{x}} - N_1 \\
N_2 \\
M_2
\end{bmatrix} + \begin{bmatrix}
\ddot{\vec{x}}_1 \\
\ddot{\vec{x}}_2
\end{bmatrix} + \begin{bmatrix}
-M_1 \ddot{\vec{\theta}} + c_1 + g_1 \\
c_2
\end{bmatrix} = \begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}.
\]
(5)

It is worth noting that both the kinematic model and the dynamic model are described in a unified coordinate frame. The unified model in the world coordinate frame eases the kinematic redundancy resolution and the decoupled end-effector force/position control.

3. Motion Control and Redundancy Resolution

3.1. Tracking Controller

Based on the unified model of mobile manipulators in eq. (5), robot motion tracking controllers can be designed by using the existing robot control approaches. Applying the following nonlinear feedback control to eq. (5)
\[
\tau = \begin{bmatrix}
M_1 \ddot{\vec{x}} - N_1 \\
N_2 \\
M_2
\end{bmatrix} \cdot u + \begin{bmatrix}
-M_1 \ddot{\vec{\theta}} + c_1 + g_1 \\
c_2
\end{bmatrix},
\]
(6)
the dynamics model of the mobile manipulator can be linearized and decoupled as
\[
\ddot{x} = \dot{u},
\]
(7)
where \(\tau = \{\tau_1, \tau_2\}^T\) is the generalized torque input of the mobile manipulator, \(x = \{p_x, p_y, p_z, O, A, T, x_b, y_b, \theta_b\}^T\) is the mobile manipulator output and \(u\) is a linear control vector. Equation (7) forms a unified model for the mobile manipulator. Vector \(x\) consists of task level variables \(y\) of the end-effector in coordinate frame \(\Sigma\) and the configuration of the mobile platform \([x_b, y_b, \theta_b]\) in frame \(\Sigma\). Since \(y\) is defined in task space, direct control of the end-effector position and orientation has been implemented. At the same time, the joint level redundancy can also be resolved easily by considering the variable in the same coordinate frame.

Given \(x^d(t) = \{p_{x}^d(t), p_{y}^d(t), p_{z}^d(t), O^d(t), A^d(t), T^d(t), x_b^d(t), y_b^d(t), \theta_b^d(t)\}^T\) as the desired position and orientation of the mobile manipulator, the linear system feedback for model (7) can be designed as
\[
u = \ddot{x} + k_x(\ddot{x} - \ddot{x}) + k_y(x - \dot{x}).
\]
(8)
It is easy to prove that the above control algorithm (8) is asymptotically stable. However, the task of a mobile manipulator is generally given in the form of \(y^d = \{p_{x}^d(t), p_{y}^d(t), p_{z}^d(t), O^d(t), A^d(t), T^d(t)\}^T\). The desired mobile robot position and orientation \(Y^d = \{x_b^d(t), y_b^d(t), \theta_b^d(t)\}^T\) should be obtained by considering the mobile manipulator configuration, and bandwidth of the mobile platform and the arm. By properly positioning the mobile platform, singular arm configurations (Cheng et al. 1997) can be avoided, i.e., the coordinated motion of the mobile platform and the robot arm can maximize the capability of manipulation. In the following subsection, a coordination scheme considering the robot arm manipulation capability is given.

3.2. Singularity Analysis and Redundancy Resolution

Ample literature is available for the analysis of the singularity and manipulation capability of a variety of robot manipulators (Bedrossian 1990; Cheng et al. 1997). At a robot singular configuration, the Jacobian matrix loses rank, and the end-effector cannot generate motion along certain directions. The robot has to be far away from its singular configurations to keep certain dexterity. The effect of singular configurations to robot dexterity can also be seen from its task level controllers. For example, it can be seen from eq. (6) that no matter how the linear controller \(u\) is designed, the feasibility of the mobile robot controller (6) depends on the existence of matrix \(J^{-1}\). The proximity to a singular configuration can be analyzed based on the Jacobian matrix and mechanical structure of the robot arm (Tan and Xi 2002). For a mobile manipulator with a PUMA type robot arm, the following variables can be obtained to measure the proximity to singular configurations (Tan and Xi 2002)
\[
\gamma_s = \left| da_s - a_3 s_3 \right|,
\gamma_r = \left| d a_3 + a_2 c_2 + a_3 c_2 \right|,
\gamma_w = \left| - s_3 \right|,
\]
(9)
where \(d_s, a_1, a_2, a_3, s_s\) are robot arm parameters and \(s_s\) and \(c_s\) denote \(\sin q_s\) and \(\cos q_s\), respectively. Here the values of \(\gamma_s, \gamma_r, \gamma_w\) represent the proximity to arm singular configuration, interior singular configuration and wrist singular configuration, respectively. While the robot arm approaches a singular configuration, the value of at least one of the three expressions becomes smaller. The value of one variable is zero at certain singular configuration. If the manipulator is put on a fixed platform, singular configurations cannot be avoided for some tasks. However, singular configurations of the arm can be avoided by appropriately positioning the mobile platform for a mobile manipulator. The objective of coordinated control of the mobile platform and the robot manipulator is to maximize the capability of manipulation, which also means maximizing the values of the expressions given by eq. (9). However, it is not necessary to keep the capability of manipulation at its maximum value. If the variables in eq. (9) are always maximized, the manipulator will be fixed at a certain configuration and lose its dexterous manipulation capability. As long as the values of \(\gamma_s, \gamma_r, \gamma_w\) are greater than certain pre-specified minimum values, the manipulator will certainly have the ability to move.
The desired position and orientation of the platform are determined by the trajectory of the end-effector and the values of the variables given by eq. (9), which are also the system output. An event-based planning approach (Xi, Tarn, and Bejczy 1996), as shown in Figure 3, is used for online coordination control of the mobile platform and the manipulator. The basic idea of event-based planning is to introduce a new motion reference variable $s$, which is different from time and directly related to the sensory measurement of system output. As shown in Figure 3(b), the control input is parametrized by the motion reference instead of time in Figure 3(a). Since the motion reference is a function of the real time measurement, the values of the desired robot motion are functions of the measured data. This creates a mechanism to adjust or modify the plan based on the measurements. Thus, the planning becomes a closed-loop real-time process. The planner generates the desired values of the system, according to the online computed motion reference parameter $s$. The motion information of both the arm and the manipulator is passed to the planner by the motion reference $s$. For coordinated control of the mobile platform and robot arm, the planners take the common motion reference as an input. Thus the robots are coordinated by a common motion reference.

Defining the motion of the end-effector as the motion reference, the desired trajectory of the mobile platform is a continuous function of $s$ and $\gamma_b, \gamma_i, \gamma_w$:

\[
\begin{align*}
    x_b &= g_x(s, \gamma_b, \gamma_i, \gamma_w) \\
    y_b &= g_y(s, \gamma_b, \gamma_i, \gamma_w) \\
    \theta_b &= g_\theta(s, \gamma_b, \gamma_i, \gamma_w).
\end{align*}
\]  

Functions $g_x, g_y, g_\theta$ are continuous and piecewise differentiable. Here the motion reference $s$ can represent the motion of both the platform and the manipulator, i.e., a function of the augmented output $x$. Letting $g$ represent $g_x, g_y, g_\theta$, and $\gamma$ represent $\gamma_b, \gamma_i$ or $\gamma_w$, the requirement of the definition of coordination can be represented by

\[
\begin{align*}
    \frac{\partial g}{\partial s} &= 0 \quad \text{if } |\gamma| > \gamma_{\text{min}}, \text{ or } \frac{\partial \gamma}{\partial s} > 0 \\
    \frac{\partial g}{\partial s} &> 0 \quad \text{if } |\gamma| \leq \gamma_{\text{min}} \text{ and } \frac{\partial \gamma}{\partial s} < 0
\end{align*}
\]

where $\gamma_{\text{min}}$ stands for the allowable minimum value of $\gamma$. If the value of $\gamma$ is greater than $\gamma_{\text{min}}$, or if it is increasing with respect to $s$, the platform can choose to keep the current position. The mobile platform should be commanded to a position such that the value of $\gamma$ will increase. The functions $g_x, g_y, g_\theta$ can be defined to be adaptive to the motion reference $s$ and the values of three variables $\gamma_b, \gamma_i, \gamma_w$. An example implementation of $g$ considering only the arm singular condition $\gamma_b$ is shown here and the parameters in the implementation can be obtained adaptively

\[
\begin{align*}
    x_b^\prime &= x_b + \delta_x \cdot r \\
    y_b^\prime &= y_b + \delta_y \cdot r \\
    \theta_b^\prime &= \theta_b + \delta_\theta \cdot r,
\end{align*}
\]

where $\delta_x = x^0(s) - x_1$, $\delta_y = y^0(s) - y_1$, $\delta_\theta = \tan^{-1}(x_2 - y_2, x_1 - x_2)$ and

\[
r^\prime = \begin{cases} 0, & \text{if } |\gamma| > \gamma_{\text{min}} \text{ or } \frac{\partial \gamma}{\partial s} > 0 \text{ and } r^- = 0 \\ \alpha_1, & \text{if } |\gamma| > \gamma_{\text{min}} \text{ and } r^- > 0 \\ \alpha_2, & \text{if } |\gamma| \leq \gamma_{\text{min}} \text{ and } \frac{\partial \gamma}{\partial s} < 0.
\end{cases}
\]

Here the system is designed as a hybrid system and $r$ is piecewise continuous from the right, $r^-$ and $r^+$ denote the values of $r$ before and after a time instant, and $\alpha_2 > \alpha_1 > 0$ are constants. The values of $\alpha_2$ and $\alpha_1 > 0$ are obtained adaptively in the experiment. $\delta_x$ and $\delta_y$ are the desired and actual values along the $x$- and $y$-direction, respectively. The value of $r$ determines how much of the difference should be compensated by the mobile platform. $\delta_\theta$ is designed such that the rotation of the mobile platform can be used to compensate the motion of the end-effector. $r$ can take more discrete values in implementation. By this design, the end-effector will go into a certain range of $\gamma$ no matter what the initial value of $\gamma$.

The trajectory of the mobile platform is planned based on the constraints in eq. (9), and therefore resolves the kinematics redundancy. It is also a function of the motion reference, $s$. Using this strategy, coordination can be achieved even when obstacles block one of them. When the end-effector is blocked by obstacles but the mobile platform is free to move, $s$ stops evolving and maintains a constant value. The desired trajectory of the mobile platform will therefore not change and the whole system is stable. If the mobile platform is blocked but the arm is not, the arm can still continue performing its task without the help of the platform. If now a singular configuration is reached, the desired trajectory of both the arm and the platform will remain constant. Thus, the system stability is not affected by obstacles or singularities.

4. Decoupled Output Force/Position Control

Besides trajectory tracking, object manipulation is another kind of task for mobile manipulators. The mobile manipulator needs to interact with the environment, and output force control is necessary in this case. There are generally two ways for the end-effector to interact with the environment. One is interacting with static environments or objects, such as force tracking along hard surface. Force control schemes such as hybrid force/position control, impedance force control, explicit force control and many others have been proposed for fixed base manipulators. The force control schemes are highly dependent on the environments of the robot. In many cases, the environment is assumed as static and the directions for force control and position control are separated. The end-effector can also interact with dynamic environments, such as interacting with moving objects. For some objects that are
too heavy or large for the mobile manipulator to carry, pushing or pulling is an alternative choice to manipulate objects. Since the mobile manipulator can provide both motion in large workspace and dexterous manipulation capability, we discuss the force/position control of mobile manipulators while pushing an object. This requires the mobile manipulator to provide both output force and motion along the same task direction. In this paper, the kinematic redundancy of the mobile manipulator is utilized to decouple the force control loop and motion control loop along the same task direction. For comparison, the force control schemes based on a decoupled nonredundant robot model are revisited first.

4.1. Force Control Schemes Revisited

Hybrid force/position control was first proposed by Raibert and Craig (1981). The workspace is divided into two orthogonal subspaces as shown in Figure 4. A selection matrix $S$ determines the subspaces for which force or position are to be controlled. The control laws for position and force control can be designed independently to satisfy different control requirements of force and position. Generally, the force control law is designed to interact with a static environment. However, the motion of the environment should also be considered when the robot interacts with a moving object. To improve the performance of the force control law, Schutter (1988) proposed an approach to feed forward the object motion parameters such as object velocity $\dot{x}_o$ and acceleration $\ddot{x}_o$, as shown by the dashed lines in Figure 4. The desired output force $f^d$ along the motion can be tracked. However, it can been seen that the motion control along the same direction is open loop.

For an object or environment, it is assumed that the end-effector position and the contact force with the environment along one task direction cannot be controlled independently. The force can then be regulated by controlling the impedance, or compliance of the robot, as shown in Figure 5 (Hogan 1985). The basic idea of this approach is to design a control law which will function in accordance with $f = M\ddot{x} + B\dot{x} + Kx$, where the constant matrices $M$, $B$, and $K$ represent inertia, damping and stiffness matrices of the interactive system, respectively.

Combining the hybrid control and impedance control, hybrid impedance control has been proposed (Anderson and Spong 1988). The environment dynamics was also represented in an impedance form and the interaction between the environment and the manipulator was studied. Since the robot may encounter different environments for various applications, control gains of the robot should be tuned in accordance with the environmental characteristics. This scheme also has a slow response to force perturbations and the performance of the implicit force control is restricted by the bandwidth of the position controller (Vukobratović 1997).

4.2. Output Force/Position Control of Redundant Robots

For a nonredundant robot arm, the directions for force control and position control have to be orthogonal (Khatib 1987), as shown in Figure 4. Therefore, the force and the position cannot be controlled independently along the same task direction. This is caused by the equal number of control inputs and desired system outputs. For the hybrid force/position control scheme, the force and position control directions are generally separated by a selection matrix $S$. For many cases of
Due to the kinematic redundancy, there are more control inputs than desired outputs. For the decoupled unified system model (7) of the mobile manipulator, the linear control input is a $9 \times 1$ vector, while the desired end-effector position and orientation, $\{p^e, p^e, p^e, O^e, A^e, T^e\}^T$, is only a $6 \times 1$ vector. The redundant DOF, which correspond to the extra linear control inputs, can be utilized to accomplish secondary tasks. For a path tracking task, the redundant DOF can be used to position the mobile platform such that singular configurations of the arm are avoided. For a task to interact with the environment, the output force of the end-effector can be chosen as a secondary task. For instance, the desired output force and position of the end-effector can be chosen as $\{f^e, f^e, f^e, O^e, A^e, T^e, x^e, y^e, \theta^e\}^T$, where $f^e, f^e$ and $f^e$ are the desired output forces. As shown in Figure 6, a selection matrix $S$ is not necessary for the redundant robot. It is worth noting that along the $x$-direction of the world frame $\Sigma$, both the desired position of the mobile base, $x^d$, and the desired output force of the end-effector, $f^e$, are chosen. The desired based position, $y^d$, and end-effector output force, $f^e$, are chosen simultaneously in the $y$-direction.

System (7) is decoupled, it can be divided into two subsystems: position control subsystem and force control subsystem. The state variable space of the position control subsystem, $x_p$, is a subspace of the state space $x$ of system (7). Let $x_p = \{O, A, T, x_b, y_b, \theta_b\}^T$ and denote its corresponding linear control input by $u_p$. The force control subspace $x_f$ is chosen as $x_f = \{p^e, p^e, p^e\}^T$ and the corresponding linear control input is denoted by $u_f$. System (7) can therefore be rewritten into two subsystems:

$$\ddot{x}_p = u_p$$
$$\ddot{x}_f = u_f.$$  \hspace{1cm} (11)

The linear feedbacks for the two subsystems can be designed as

$$u_p = k_{p_p}(x^d - x_p) + k_{p_p}(\dot{x}^d - \dot{x}_p) + \ddot{x}^d$$
$$u_f = \ddot{x}_f + k_{f_p}(f^d - f) + k_{f_p}\int_0^t (f^d(\sigma) - f(\sigma))d\sigma.$$

(12)

In the controller (12), the force control loop and position control loop along the same task direction are decoupled due to the redundancy of the control inputs. And explicit force/position control of the end-effector can be designed.

From eq. (12), it is seen that force and position control along the same task directions are decoupled. The desired position of the mobile based and desired output force of the end-effector along the same task direction can be controlled independently. The redundant mobile manipulator provides the capability of independently controlling position and force along the same task direction. However, environmental constraints should be considered to plan the desired force $f^d$ and
desired trajectory $x_d^e$. There exist two circumstances of the environments. In a passive environment, the desired output force and trajectory are coupled by the dynamics of the object that interacts with the end-effector. Force planning and trajectory planning along the same task direction are considered based on the dynamics of the environments. In an active environment, force and position along the same task directions can be planned separately. The environment determines the force imposed at the end-effector at will. For instance, while a robot cooperates with humans, the output force of a human, $f_m$, is independent of the robot. In multi-robot coordination tasks, more than one robot can share a task. The output force for one individual robot can be considered independently from its motion, as long as the composed force satisfies the constraints of the environment dynamics. In brief, environmental constraints are considered to determine the desired output force and desired trajectory. In either case, the decoupled force/position control of mobile manipulators makes possible the independent control of force and position along the same task direction. The interacting force with the dynamic environment can then be regulated explicitly by considering the environmental dynamics, as shown in Figure 6. The force does not have to be regulated implicitly as is done in the implicit force control scheme. This ensures the bandwidth of the force control loop, and ensures that the force planning for manipulating the cart can be tracked. A nonholonomic cart pushing task is used to demonstrate the proposed force/position control approach.

5. Task Planning and Control for Mobile Manipulation

Due to the large workspace and dexterous manipulation capability, mobile manipulators can be used in a variety of applications. For some tasks such as painting and soldering, only the motion planning and control of the mobile manipulator are involved. For some complicated tasks, such as pushing a nonholonomic cart, the mobile manipulator maneuvers the object by the output forces. Therefore, the task planning for manipulating a cart involves three issues: the trajectory and force planning of the cart; the trajectory planning and control of the mobile manipulator; synchronization of the motion planners for the cart and the mobile manipulator.

Figure 7 shows a flow chart for the integrated task planning and control for pushing a nonholonomic cart. First, for a given task, the geometrical path needs to be determined considering the nonholonomic constraint of the cart. Secondly, a trajectory should be planned based on the geometrical path, and the input force for the cart to track the trajectory should be planned. Thirdly, the trajectory planning and coordination of the mobile manipulator should be considered. Finally, the decoupled output force control and position control can be achieved based on the decoupled system model (7) and controller (12).

In the cart pushing task, the mobile manipulator and the cart compose a system. Therefore, the trajectory planning of the mobile manipulator and the cart should be synchronized. The event-based approach for the path planning and control of multi-robot coordination (Xi, Tarn, and Bejczy 1996) can be used in integrating the planning and control of the pushing task. A common motion reference $s$ is chosen for the trajectory planning of both the mobile manipulator and the cart. In other words, the evolution of the cart trajectory and the mobile manipulator trajectory is based on the same motion reference. It is worth noting that the common motion reference $s$ is computed based on sensory information of the mobile manipulator.

5.1. Path Planning of the Cart

The cart shown in Figure 8(b) is a nonholonomic system. The path planning for a nonholonomic system is to find a path connecting specified configurations that also satisfy nonholonomic constraints. The cart model is similar to a nonholonomic mobile robot except that the cart is a passive object. Therefore, many path planning algorithms for a nonholonomic system such as steering using sinusoids (Murray and Sastry 1993) and goursat normal form approach (Tibury, Murray, and Sastry 1995) can be used for the path planning of the cart. In this paper, the kinematic model is transformed into a chained form and the path planning is considered as the solution of an optimal control problem. We consider the kinematic model of the cart as...
5.2. Force Planning of the Cart

The nonholonomic cart shown in Figure 8(b) is a passive object. Assuming that the force applied on the cart can be decomposed into \( f_1 \) and \( f_2 \), the dynamic model of the nonholonomic cart in the world frame \( \Sigma \) can be represented by

\[
\begin{align*}
\dot{x}_c &= \frac{\lambda}{m_c} \sin \theta_c + \frac{f_1}{m_c} \cos \theta_c, \\
\dot{y}_c &= -\frac{\lambda}{m_c} \cos \theta_c + \frac{f_1}{m_c} \sin \theta_c, \\
\dot{\theta}_c &= \frac{f_2}{I_c},
\end{align*}
\]  

where \( x_c, y_c \), and \( \theta_c \) are the configuration of the cart, and \( m_c \) and \( I_c \) are the mass and inertia of the cart. \( \lambda \) is a Lagrange multiplier and \( \theta_c \) is the cart orientation.

As shown in Figures 8(a) and (b), the input force applied onto the cart, \( f_1 \) and \( f_2 \), corresponds to the desired output force of the end-effector, \( f_d^1, f_d^2 \). In other words, the mobile manipulator controls the cart to achieve certain tasks by its output force. Therefore, manipulating the cart requires the motion planning and control of the mobile manipulator based on the decoupled model (7), and the trajectory and force planning of the cart based on its dynamic model (17).

To track a given trajectory \( \{x_c(t), y_c(t), \theta_c(t)\}^T \), the input forces applied onto the cart, \( f_1 \) and \( f_2 \), also need to be planned. The input force planning of the cart is equivalent to the output force planning of the mobile manipulator. The control input of the nonholonomic cart is determined by its dynamic model (17), the nonholonomic constraint (16) and its desired trajectory \( \{x_c^*, y_c^*, \theta_c^*\} \).

Due to the lack of a continuous time-invariant stabilizing feedback for nonholonomic systems (Brockett 1983), output stabilization is considered in this paper. Choosing a manifold rather than a particular configuration as the desired system output, the system can be input–output linearized. By choosing \( x_c, y_c \) as the system output, the system can be linearized with respect to the control input \( f_1 \) and \( f_2 \). This can be explained by the following derivations. From eqs. (13) and (14), it is easy to see that \( v_1 = \dot{x}_c \cos \theta_c + \dot{y}_c \sin \theta_c \). Here \( v_1 \) is actually the forward velocity along the \( x \)-direction. Considering the velocity along the \( y \)-direction is \( \dot{y}_c \sin \theta_c - \dot{x}_c \cos \theta_c = 0 \), the following relation can be obtained:

\[
\begin{align*}
v_1 &= \dot{x}_c \cos \theta_c + \dot{y}_c \sin \theta_c, \\
v_2 &= \ddot{\theta}_c.
\end{align*}
\]

Suppose the desired output of interest is \( \{x_c, y_c\} \), the following input–output relation can be obtained by the derivative of eqs. (13, 14):

\[
\begin{align*}
\dot{x}_c &= \frac{1}{m_c} \cos \theta_c \cdot f_1 - v_1 \sin \theta_c \cdot v_2, \\
\dot{y}_c &= \frac{1}{m_c} \sin \theta_c \cdot f_1 + v_1 \cos \theta_c \cdot v_2.
\end{align*}
\]

Considering \( f_1 \) and \( f_2 \) as the control inputs of the system, the input–output can be formulated in a matrix form

\[
\begin{pmatrix}
\dot{x}_c \\
\dot{y}_c
\end{pmatrix} = G
\begin{pmatrix}
f_1 \\
v_2
\end{pmatrix},
\]

where

\[
G = \begin{pmatrix}
\cos \theta_c & -v_1 \sin \theta_c \\
\frac{m_c}{m_c} \sin \theta_c & v_1 \cos \theta_c
\end{pmatrix}.
\]

The nonholonomic cart can then be linearized and decoupled as

\[
\begin{pmatrix}
\dot{x}_c \\
\dot{y}_c
\end{pmatrix} = \begin{pmatrix}
w_1 \\
w_2
\end{pmatrix},
\]
where \( \{w_1, w_2\}^T = G\{f_1, v_2\}^T \). Given a desired path of the cart, \( x'_d, y'_d \), which satisfies the nonholonomic constraint, the controller can be designed as
\[
\begin{align*}
    w_1 &= \ddot{x}_d + k_v (x_d - x_c) + k_d (\dot{x}_d - \dot{x}_c), \\
    w_2 &= \ddot{y}_d + k_v (y_d - y_c) + k_d (\dot{y}_d - \dot{y}_c).
\end{align*}
\]

The angular velocity \( v_2 \) can then be obtained by \( \{f_1, v_2\}^T = G^{-1}\{w_1, w_2\}^T \). The physical meaning of this approach is that the control input \( f_1 \) generates the forward motion and \( v_2 \) controls the cart orientation such that \( x'_d \) and \( y'_d \) are tracked. However, control input \( v_2 \) is an internal state controlled by \( f_2 \), which is the tangent force applied onto the cart. The control input \( f_2 \) can then be computed by the backstepping approach (Khalil 1996) based on the design of \( v_2 \). Defining \( v_2 = \phi (x_c, y_c, \theta_c) \) and \( z = \dot{\theta}_c - \phi \), then eq. (15) can be transformed into
\[
\dot{z} = -\phi + \frac{L}{I_c} f_2. \tag{18}
\]

The control input \( f_2 \) can then simply be designed as
\[
f_2 = \frac{L}{I_c} (\dot{\theta}_c - \phi). \tag{19}
\]

It is worth noting that the desired cart configuration \( \{x'_d, y'_d\} \) is the result of trajectory planning. As shown in Figures 8(a) and (b), the output force of the mobile manipulator corresponds to the control input \( f_1 \). Given the force planning \( f_1 \) and \( f_2 \), the desired output force of the mobile manipulator would be \( f_2^* = f_1 \sin \theta_c - f_2 \cos \theta_c \), \( f_2^* = f_1 \cos \theta_c + f_2 \sin \theta_c \). Given the desired output force and trajectory, the decoupled force/position controller (12) can be applied to achieve force control of the end-effector and position control of the mobile base along the same task directions.

To implement the force control of the cart, the actual configuration \( \{x_c, y_c, \theta_c\} \) and corresponding velocities need to be estimated. Since the cart has no sensors equipped on it, sensors on the mobile manipulators, such as laser range finder, encoder and force/torque sensor are used to estimate the configuration of the cart. To estimate the orientation of the cart in the moving frame attached on the mobile platform, the laser range finder is used. Figure 8(c) shows a configuration of the cart in the moving frame of the mobile platform. The laser sensor provides a polar range map of its environment in the form of \((\alpha, r)\), where \(\alpha\) is an angle from \([-\pi/2, \pi/2]\) which is discretized into 360 units. The range sensor provides a ranging resolution of 50 mm. Obviously the measurement of the cart is mixed with the environment surround it. Only a chunk of the data is useful and the rest should be filtered out. The actual configuration \( \{x_c, y_c, \theta_c\} \) can be estimated based on the standard extended Kalman filter technique. To interact with a cart with different weight and length, the cart parameters, \(m_c, I_c\), and cart length \(L\) are estimated based on the force/torque sensor and accelerometer (Sun, Tan, and Xi 2002).

6. Experimental Implementation and Results

The path planning and control approaches have been implemented on a mobile manipulator consisting of a Nomadic XR4000 mobile robot and a Puma560 robot arm, as shown in Figure 1. For the mobile manipulator, there are two PCs in the mobile platform: one uses Linux as the operating system and runs the mobile robot control software and the other uses a real-time operating system QNX and runs the Puma 560 control software. The two computers are connected via an ethernet connection and communicate at a frequency of 300–500 Hz. The sampling period for the Puma 560 control software is 1 ms. The end-effector is equipped with a Jr3 force/torque sensor. For the nonholonomic cart, the mass is 45 kg and the cart length is 0.89 m. The gripper of the mobile manipulator holds the handle of the cart during the manipulation.

Fig. 8. A mobile manipulator and a nonholonomic cart.
6.1. Path Tracking Results

To test the path tracking controller and the event-based coordination scheme when tasks are not known a priori and an unpredictable obstacle is present, the results of teleoperation are presented in Figure 9. The desired position of the end-effector is generated by a joystick. It is worth noting that the actual and desired values overlap. Figures 9(a) and (b) depict the end-effector position in the $x$- and $y$-direction of the world frame, respectively. Figure 9(c) shows the position of the platform $x_b$ and $y_b$. The operation can be divided into four parts by $t_1 = 15$ s, $t_2 = 44.4$ s and $t_3 = 54.9$ s, as shown in the figure. Before $t_1$, the mobile manipulator operates normally. At time $t_1$, the platform meets a unexpected obstacle and can no longer move, as shown in Figure 9(c). The tasks assigned to the mobile manipulator can still be performed by the robot arm without the help of platform. It can be seen that the value of manipulation capability measurement becomes very small around time instance 38 s. At $t_2$, the obstacle is removed, the platform begins to cooperate and the manipulation capability adapts to a higher value. The robot arm autonomously adjust itself to best possible configurations and singular configurations are avoided. At time $t_3$, the platform is blocked again, the robot arm performs the task alone and the mobile manipulator stops evolving after the manipulation capability drops to a very low value. Using the event-based coordination scheme and path tracking controller based on the unified model, the mobile manipulator system is stable even when unexpected obstacles occurred. This capability is especially useful when the mobile manipulator works in a highly unstructured environment. This experiment also shows that the kinematic redundancy is resolved online and the manipulation capability is optimized during the operation.

6.2. Pushing Along A Straight Line

Figure 10 shows the results of pushing a cart along a straight line using the mobile manipulator. In this experiment, the task was to push the cart along the $y$-direction in frame $\Sigma$ from 1.675 to 2.675 m. Figure 10(a) shows the desired output force and actual output force in the $y$-direction. Figure 10(b) records the force tracking error. It is worth noting that the static friction force was also overcome in the first two seconds of the task. Figures 10(c) and (d) show the trajectory of the cart and the cart orientation, respectively. The position and the orientation of the cart were estimated by fusing the sensor information on the mobile manipulator. The laser range sensor, encoders of the mobile platform and the robot arm are used in the estimation. In this experiment, the motion in the $y$-direction was chosen as the motion reference to synchronize
the motion planning of the cart and the mobile manipulator. This experiment demonstrated that the desired force, the desired trajectory of the cart has been tracked with satisfactory precision.

For the same task, Figure 11 shows the experimental results when an unexpected obstacle blocked the mobile manipulator along the straight line. At about 7.5 s, the system is blocked by an obstacle, and the motion reference stops evolving. As a result, the desired output force was set to zero and the cart stopped moving. When the obstacle was removed at about 14 s, the motion reference started evolving again, the desired output force was computed based on the desired motion of the cart and the pushing task resumed. It can be seen that the motion of the cart and mobile manipulation system was synchronized by a common motion reference.

6.3. Turning The Cart At A Corner

Pushing the cart along a straight line is relatively easy since the desired end-effector position and output force are easy to plan. The second experiment considered a complex task. The mobile manipulator first pushed the cart forward 0.4 m along the y-direction for about 20 s, made a turn at about 35 s, and then pushed the cart forward again for 0.4 m along the x-direction. Figure 12(a) shows the trajectories of the cart, x, and y, and the end-effector trajectories, p, and p. Figure 12(b) is the cart orientation θ, with respect to time. The cart started from a configuration parallel to the y-direction, and turned to a configuration parallel to the x-direction. The output force is planned based on the desired trajectory of the cart. Figures 12(c) and (d) are the forces applied onto the cart, f, and f. It is worth noting that the force are recorded in the world frame Σ. It is seen that the force f pushed the cart along the x-direction in the last 20 s, and f pushed the cart along the y-direction in the first 20 s. This experiment has demonstrated that a complex task can be completed by the integrated task planning and control approach.

6.4. Pushing Along A Sine Wave

The task in this experiment was to push the cart forward along a sine wave denoted by $x_c = 0.2 \sin(1.8y_c)m$. In this experiment, the force $f_1$ generates the motion along the since wave, and the force $f_2$ regulates the cart orientation. Figure 13(a) shows the desired and actual trajectories of the cart, and the desired and actual trajectories of the end-effector. The noise of the actual trajectory was caused by the noisy estimation of cart orientation, which is shown in Figure 13(b). Figures 13(c) and (d) show the desired and actual force signals in the x- and y-direction, respectively. $f_1$ and $f_2$ were transformed to

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**Fig. 10.** Pushing the cart along a straight line.
Fig. 11. Pushing with an unexpected obstacle.

Fig. 12. Turning the cart at a corner.
frame $\Sigma$ and compared with the actual force signal in the same frame. It is seen the desired forces in both directions were tracked. At $t = 25$ s or $y_c = 1.9$ m, the cart slipped on the floor and the cart orientation was changed; the mobile manipulator can still manipulate the cart such that the desired trajectory is tracked. In this experiment, the motion of the cart in the $y$-direction, $y_c$, was chosen as the motion reference. The experimental results have shown that the integrated task planning and control approach enables the mobile manipulator to perform complicated tasks with satisfactory performance. In all the experiments, both the desired cart trajectory and desired output force were tracked.

7. Conclusion

In this paper, a unified dynamic model for the mobile manipulator is derived and nonlinear feedback is applied to linearize and decouple this model. The kinematic redundancy and dynamic properties of the platform and the robot arm have been considered in the unified model. Based on this model, mobile manipulator tasks such as path tracking control, force/position control can be easily designed. Based on the analysis of the robot arm, an online event-based coordination scheme is proposed for the cooperation of the platform and the robot arm. This online approach is suitable for a point-to-point task, multiple segment tasks and teleoperation. The mobile manipulator system is stable even in the case of the appearance of unexpected obstacles and the manipulation capability can adapt to a best possible configuration during the operation. Based on the decoupled and linearized model, in this paper we further discuss the decoupled force/position control along the same task direction utilizing the kinematic redundancy of mobile manipulators. Based on the analysis of the robot capability and the constraints of the environments, the concept of decoupled force/position control along the same task direction is first presented. For tasks that require both force and position control along the same task direction, such as pushing a nonholonomic cart, an integrated task planning and control for mobile manipulators is proposed. The approach for manipulating the nonholonomic cart integrates the planning and control of the mobile manipulator, the trajectory planning and force planning of the nonholonomic cart, and the estimation of the cart configuration based on the sensors of the mobile manipulators. The integrated task planning and control enables the robot to interact with moving objects and complete complicated tasks by the output force. The approaches discussed in this paper can find applications in the motion planning and control of mobile manipulators, which can find applications in manufacturing, hospitals, homes and offices. Experimental results have demonstrated the advantage of the integrated task planning and control approach.
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