Modelling a small-size unmanned helicopter using optimal estimation in the frequency domain

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Abstract: In this paper, a semi-decoupled state-space model of a small-size helicopter is developed for hovering conditions in order to simplify the model identification process. An enhanced identification algorithm in frequency domain is proposed and implemented to estimate the parameters in the state-space model using real flight data collected from a SERVOHELI-40 small-size unmanned helicopter. The accuracy of the identified model is verified by simulation in time domain, using a different set of hovering flight data. The results have shown the accuracy of the developed semi-decoupled model and the effectiveness of the proposed optimal estimation algorithm in frequency domain.

Keywords: small-size unmanned helicopter; helicopter model; optimal estimation; hover; frequency domain.
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1 Introduction

Unmanned helicopters are increasingly popular platforms for unmanned aerial vehicles (UAVs). With abilities to hover, to take off and to land vertically, unmanned helicopters extend the potential applications of UAVs. Compared with traditional full-size helicopters, small-size helicopters tend to be naturally more manoeuvrable and more responsive. However, only a modest part of the helicopter’s inherent qualities are exploited in the relevant literature, and there are few suitable state-space models available for flight control design.

Helicopter system identification is a highly versatile procedure for extracting dynamic model of a helicopter from the measured response to specific control inputs. Modelling of full-size helicopters based on first principles has already been reported in the literature, these include tilt-rotor aircraft XV-15 (Tischler, 1985), helicopter BO-105 (Tischler and Cauffman, 1992), UH-60 (Fletcher, 1995) and SH-2G (Tomashofski and Tischler, 1999). However, there are only several reported applications of system identification techniques to modelling of small-size helicopters, including the model identification of YAMAHA R-50 (Mettler et al., 1999) and X-Cell (Gavrilets et al., 2001) for flight control, and a six-DoF dynamic modelling of Raptor 50 V2 for simulations (Subodh and Richard, 2005). The difficulties in modelling small-size helicopters lie in high frequency dynamics and noise from sensors. The environment also needs to be considered due to their structural characteristics, and a large amount of flight data are necessary for system identification.

A popular dynamic state-space model for small-size unmanned helicopters was first presented in Mettler et al. (2004). The model explicitly accounts for the stabiliser bar and rotor system, based on first principles, which has a strong influence on the flight dynamics characteristics of small-size unmanned helicopters. The frequency response method (Tischler and Cauffman, 1992) is used to identify the parameters. However, some parameters in the model cannot be measured from the input or output data because of too
many coupling parameters in the system matrix, and the strong correlation among parameters makes the initial estimation, which is necessary for ordinary frequency identification, difficult to select. Some constraints must be added during identification and the initial estimation of parameters must be adjusted by hand based on a large number of flight experiments, which is costly for rotorcraft. Because of the non-linear search algorithm in the identification process, irrelevant initial estimation of parameters may result in parameters divergence. There is also no parameters convergence criterion in the ordinary frequency response method, and in practice excessive iteration may make numerical value unstable.

In this paper, for effective hovering identification, the original model in Mettler et al. (2004) is decomposed into three groups (longitudinal, lateral and yaw-heave coupling), and a semi-decoupled model is obtained. Each group has a decoupled system matrix, and the coupling characteristics are presented only in the control matrix. Thus, the number of unknown parameters and control inputs are reduced and the control loops are semi-decoupled. To identify the unknown parameters in the MIMO semi-decoupled model, a new cost function is proposed to make the traditional method of SISO system frequency estimation (Tischler and Cauffman, 1992) applicable to the MIMO state-space models. The proposed cost function is presented in the additional form of the frequency error of every input–output pair for transfer matrix, and the parameters are identified by minimising the cost function. An enhanced frequency identification process is proposed to make the traditional identification process more efficient for small-size helicopter identification. The proposed identification process first uses linear regression in time domain to get the crude state estimation, and then runs the frequency response method (Tischler and Cauffman, 1992) to obtain the free parameters. Based on the value of the free parameters, a norm convergence criterion, which is about the parameter matrix error between adjacent iterations, is defined to avoid excessive iteration. The simplified model and proposed identification method free the selection of initial estimation and constraint is not required. The improved model and method of identification are applied to the SERVOHELI-40 small-size helicopter in hovering conditions. The data were collected from the experimental platform by applying frequency sweeping input, and the model’s accuracy is verified by simulations in time domain, using the data from another flight experiment. The conclusion is obtained from the simulation results that the developed semi-decoupled model is feasible in practice and the proposed enhanced identification algorithm in frequency domain is effective. Successful identification of model parameters has been achieved using the data of a one-min hovering flight experiment, with model accuracy revising for simulation in time domain.

2 The dynamic model of small-size helicopter for hovering

2.1 Complete coupling parameterised model for hovering

In Mettler et al. (2004), a parameterised state-space model is presented that explicitly accounts for the stabiliser bar and rotor system based on first principles. The model can be expressed in the following form
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\[
\begin{bmatrix}
\Delta u \\
\Delta v \\
\Delta p \\
\Delta q \\
\Delta \phi \\
\Delta \theta \\
\Delta \tau_x \\
\Delta \tau_y \\
\Delta \tau_z \\
\Delta \delta_l \\
\Delta \delta_h \\
\Delta \delta_r \\
\Delta \delta_v
\end{bmatrix} = \Delta X = \\
\begin{bmatrix}
X_x & 0 & 0 & -w_0 & 0 & -g & X_x & 0 & 0 & X_x & 0 & 0 & 0 \\
Y_x & 0 & w_0 & 0 & 0 & 0 & Y_x & 0 & 0 & Y_x & 0 & 0 & 0 \\
L_x & 0 & 0 & 0 & 0 & 0 & L_x & 0 & 0 & L_x & 0 & 0 & 0 \\
M_x & 0 & M_x & 0 & 0 & 0 & M_x & 0 & 0 & M_x & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_l \\
\Delta \delta_h \\
\Delta \delta_r \\
\Delta \delta_v
\end{bmatrix} + \begin{bmatrix}
A_{\delta_l} / \tau_x \\
A_{\delta_h} / \tau_y \\
A_{\delta_r} / \tau_z \\
A_{\delta_v} / \tau_w
\end{bmatrix} = AX + BU
\]

(1)

where \( u, v, \) and \( w \) are longitudinal, lateral and vertical velocity, \( p, q, \) and \( r \) are roll, pitch and yaw angle rates, \( \phi, \) and \( \theta \) are the angles of roll and pitch, respectively, \( a \) and \( b \) are the first harmonic flapping angle of main rotor, \( c \) and \( d \) are the first harmonic flapping angle of stabiliser bar, \( r_{fb} \) is the feedback control value of an angular rate gyro, \( \delta_{lat} \) is the lateral control input, \( \delta_{lat} \) is the longitudinal control input, \( \delta_{col} \) is the yawing control input and \( \delta_{col} \) is the vertical control input. \( X \) is the \( 13 \times 1 \) vector of states, \( A \) is the \( 13 \times 13 \) system matrix, \( B \) is the \( 13 \times 4 \) control matrix and \( U \) is the \( 4 \times 1 \) control inputs. All the symbols except gravity acceleration \( g \) in \( A \) and \( B \) are unknown. Thus, all of the states and control inputs in Equation (1) are physically meaningful and defined in body-axis, which is described in Figure 1.

The measurement equation for the state-space model can be determined according to the type of sensors on the helicopter platform. For our SERVOHELI-40 small-size unmanned helicopter platform, which will be described in further detail in the following section, all velocities, angular rates, Euler angles and accelerations are available for identification and control, so we select the measurement equation as

\[
y = \begin{bmatrix}
\Delta u \\
\Delta v \\
\Delta p \\
\Delta q \\
\Delta \phi \\
\Delta \theta \\
\Delta \tau_x \\
\Delta \tau_y \\
\Delta \tau_z \\
\Delta \delta_l \\
\Delta \delta_h \\
\Delta \delta_r \\
\Delta \delta_v
\end{bmatrix}^T \\
= \begin{bmatrix}
I_{m \times m} & 0_{m \times n} & 0_{m \times n} & \Delta Y \\
0_{n \times m} & I_{n \times n} & 0_{n \times n} & \Delta \bar{Y}
\end{bmatrix}
\]

(2)

where \( I_{m \times m} \) is the \( m \times m \) unit matrix and \( 0_{m \times n} \) is the \( m \times n \) zero matrix.
2.2 State-space semi-decoupled model for identification

Equation (1) has 13 states and 4 coupling control channels. There are 44 unknown parameters in the system matrix and control matrix. Thus, the matrix computation is complex, and some parameters cannot be directly identified from the input–output data because they are unmeasured. This is known as over-parameterised (Ljung, 1987) and therefore constraints must be used during identification. At the same time, the strong correlation among parameters also makes the initial estimation difficult to select because of the coupling control loops and the large number of control inputs. The identification process using Equation (1) directly will be complex and costly.

To simplify the identification process, we consider the fact that lateral and longitudinal dynamics are approximately decoupled when hovering, and decompose the model into three parts: lateral dynamics, longitudinal dynamics and coupling dynamics of altitude with yaw. Each part is coupled with others through the control matrix. We set lateral and longitudinal coupling parameters in the matrix \( A \) to zero, and add some free control coefficients in matrix \( B \) to compensate the coupling dynamics. Then, each part involves only two control inputs and five states at most, and their parameters are simple to identify. The semi-decoupled state equations for the three parts are

\[
\begin{pmatrix}
\Delta \dot{u} \\
\Delta \dot{q} \\
\Delta \dot{\theta} \\
\dot{\delta}_{kn}
\end{pmatrix}
= \begin{pmatrix}
X_u & 0 & -g & X_u & 0 \\
M_u & 0 & 0 & M_u & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & -1/\tau_f & A/\tau_f \\
0 & -1 & 0 & 0 & -1/\tau_f \\
\end{pmatrix}
\begin{pmatrix}
\Delta u \\
\Delta q \\
\Delta \dot{\theta} \\
\Delta \dot{\delta}_{kn}
\end{pmatrix}
+ \begin{pmatrix}
X_{kn} & X_{lt} \\
M_{kn} & M_{lt} \\
0 & 0 \\
A_{kn} & A_{lt} \\
C_{kn} & C_{lt}
\end{pmatrix}
\begin{pmatrix}
\delta_{kn} \\
\dot{\delta}_{lt}
\end{pmatrix}
\]  

(3)
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\[ \begin{align*}
\Delta \psi &= \Delta X_{\text{at}} = \begin{pmatrix} Y_u & 0 & g & Y_u & 0 \\ L_u & 0 & 0 & L_u & 0 \\ \Delta \psi + 0 & 0 & 0 & \delta_{\text{at}} \end{pmatrix} \\
\Delta \phi &= \begin{pmatrix} \delta_{\text{kn}} \\ \delta_{\text{at}} \end{pmatrix} \\
\dot{b} &= \begin{pmatrix} 0 & -1 & 0 & -1/\tau_f & B_d/\tau_f & b \\ 0 & -1 & 0 & -1 & D_{\text{dn}} & D_{\text{at}} \end{pmatrix}
\end{align*} \]

\[ \lambda_{\text{at}} = (I_{5 \times 5} - 0_{5 \times 2}) \Delta X_{\text{at}} \]

\[ \begin{align*}
\Delta \psi &= \Delta X_{\text{yaw- vertical}} = \begin{pmatrix} Z_w & Z_z & 0 \\ N_w & N_z & -N_{\text{pal}} \\ \Delta \phi + N_{\text{pal}} & N_{\text{col}} & \delta_{\text{pal}} \end{pmatrix} \\
\Delta \phi_b &= \begin{pmatrix} 0 & K & -K_{\text{b}} \end{pmatrix} \\
\lambda_{\text{yaw-vertical}} &= (I_{2 \times 2} - 0_{2 \times 2}) \Delta X_{\text{yaw-vertical}}
\end{align*} \]

3 Optimal estimation in the frequency domain for state-space model

3.1 Parameter identification in the frequency domain for SISO systems

In the frequency domain, the input and output of a system are related through the frequency-response function (Ljung, 1987):

\[ Y(j\omega) = H(j\omega)U(j\omega) \]

where \( U(j\omega) \) and \( Y(j\omega) \) are the Fourier transforms of the input and output signals \( u(t) \) and \( y(t) \), respectively, and \( H(j\omega) \) is the Fourier transform of the impulse-response function. In the experiments, \( u(t) \) and \( y(t) \) are finite-length discrete samples. The Fourier transform of the input and output signals \( u(t) \) and \( y(t) \) can be obtained through a discrete Fourier transform (DFT) as follows (Ljung, 1987):

\[ \begin{align*}
Y(j\Omega_k) &= \sum_{n=0}^{N-1} y(t_n) e^{-j\Omega_k t_n}, \quad k = 0,1,2,\ldots,N-1 \\
U(j\Omega_k) &= \sum_{n=0}^{N-1} u(t_n) e^{-j\Omega_k t_n}, \quad k = 0,1,2,\ldots,N-1
\end{align*} \]

where \( \Omega_k = k\Omega_s \) is the discrete frequency points, \( \Omega_s = \frac{2\pi}{NT_s} \) is the frequency sampling interval, \( T_s \) is the time sampling interval, \( N = T_d / T_s \) is the number of sample points and \( T_d \) is the length of a data segment. Then, for \( N_d \) experiments, the input auto-spectral density function \( G_{uu}(j\omega) \) and input–output cross-spectral density function \( G_{vy}(j\omega) \) can be estimated as follows (Chen, 2004):

\[ \hat{G}_{uu}(j\Omega_k) = \frac{2}{N_d T_d} \sum_{n_d=1}^{N_d} U_{n_d}(j\Omega_k) U_{n_d}^*(j\Omega_k) \]

\[ \hat{G}_{vy}(j\Omega_k) = \sum_{n_d=1}^{N_d} U_{n_d}(j\Omega_k) Y_{n_d}(j\Omega_k) \]
Finally, the estimates of the frequency response at the discrete frequency points $\omega_k$ are computed from

$$\hat{G}_{uy}(j\omega_k) = \frac{2}{N_d T_d} \sum_{n_d=1}^{N_d} Y_{n_d}(j\omega_k) U_{n_d}^*(j\omega_k)$$  \hspace{1cm} (10)$$

A magnitude squared coherence function $\gamma_{uy}^2$ is defined here to represent the correlation metric between $u(t)$ and $y(t)$:

$$\gamma_{uy}^2 = \frac{|G_{uy}|^2}{|G_{uu}| |G_{yy}|}$$  \hspace{1cm} (12)$$

where $G_{yy}(j\omega)$ is output auto-spectral density function.

In Tischler and Cauffman (1992), a cost function is defined as

$$J = \sum_{i=1}^{n_k} \epsilon(\omega_i, \Theta)^T W(\omega_i) \epsilon(\omega_i, \Theta)$$  \hspace{1cm} (13)$$

where $\omega_i$ is the frequency point, $\epsilon(\omega_i, \Theta)$ is the vector of magnitude and phase error between predicted response based on parameters $\Theta$ and the sampling data in frequency domain, and $W(\omega_i)$ is a 2×2 weight matrix related with the magnitude squared coherence $\gamma_{uy}^2$. So, $\Theta$ can be obtained by minimising the cost function (13) for SISO systems. Thus, the process of identification for rotorcraft (Tischler and Cauffman, 1992) is illustrated in Figure 2. The cost function $J$ in Equation (13) is a complex non-linear function of unknown parameters $\Theta$, and we can minimise it through secant method (Bendat and Piersol, 1993) to obtain $\Theta$. Thus, the method for SISO systems is applied to an MIMO state-space model.

**Figure 2** Flowchart of the frequency-domain identification process for SISO system (see online version for colours)
3.2 Optimal estimation in the frequency domain for MIMO state-space systems

For a MIMO linear parameterised state-space model

\[
\begin{align*}
\dot{X}(t) &= A(\Theta)X(t) + B(\Theta)U(t) \\
Y(t) &= C(\Theta)X(t)
\end{align*}
\tag{14}
\]

where \( \Theta \) is the unknown parameters, \( X \) is the \( n \times 1 \) vector of states, \( U \) is the \( r \times 1 \) control inputs, \( Y \) is the \( p \times 1 \) outputs, \( A \) is the \( n \times n \) system matrix, \( B \) is the \( n \times r \) control matrix and \( C \) is the \( p \times n \) measurement matrix. However, the SISO cost function (13) is not applicable here because Equation (14) is a MIMO state-space model. So, we have to consider a new cost function for all of the input-output pairs.

For the MIMO linear parameterised state-space model (14), the impulse-response matrix can be obtained as:

\[
T(j\omega, \Theta) = C(\Theta) [j\omega I - A(\Theta)]^{-1} B(\Theta) + D(\Theta)
\tag{15}
\]

If frequency points are selected as \((\omega_1, \omega_2, \ldots, \omega_n)\), we define a new cost function as follows:

\[
J_M = \frac{1}{n_T} \sum_{l=1}^{n_T} \left\{ \sum_{k=1}^{n} W_g \left[ \Delta| \cdot |_l \right]^2 + W_p \left[ \Delta \angle l \right]^2 \right\}
\tag{16}
\]

where \( \Delta| \cdot |_l \) is the magnitude error between prediction and real frequency of \( l \)th impulse-response function at \( \omega_k \), \( \Delta \angle l \) is the phase (degree) error between prediction and real frequency of \( l \)th impulse-response function at \( \omega_k \), \( W_g \) is a weighting function dependent on the value of magnitude squared coherence function \( \gamma_{yy}^2 \) (Tischler and Cauffman (1992) uses \( W_g = 2.25(1 - e^{-\omega^2})^2 \) to emphasise the most reliable data), \( W_g \) and \( W_p \) are the relative weights for magnitude and phase squared errors and \( n_T \) is the number of elements in \( T(j\omega, \Theta) \).

3.3 Enhanced identification process in frequency domain for rotorcraft

The cost function \( J \) in Equation (20) is a complex non-linear function of unknown parameters \( \Theta \), and we can minimise it through secant method (Ljung, 1987) to obtain \( \Theta \). The process of identification is illustrated in Figure 2, showing that the method for SISO systems is applied to an MIMO state-space model.

In practice use, the above frequency response identification process (Figure 2) has the following two problems: first, the initial estimation of parameters must be adjusted for secant search method by hand based on a large number of flight experiments, which is costly for rotorcraft; then, no parameters convergence criterion exists to stop the iteration process, which may make numerical value unstable.

To get the initial estimation of the unknown parameters, we define the following one-step prediction error criterion
\[ \sum_{i=1}^{N} \left\| \hat{X}_{t+1} - X_{t+1} \right\|^2 \]  

(17)

where \( \hat{X}_{t+1} = X_t + dt(A(\theta)X_t + B(\theta)U_t) \) is one-step prediction at time \( t \), \( dt \) is the sampling time. Let column vector \( \hat{\theta}_i \) stand for the unknown parameters in the \( i \)th row of matrix \( [A(\theta) \ B(\theta)] \), then we have

\[ \hat{X}_{i,t+1} = \phi_{i,t} \hat{\theta}_i + F_{i,t} \]  

(18)

where \( \phi_{i,t} \) is the state and input mixed row vector at time \( t \), whose coefficient is unknown in the \( i \)th row of matrix \( [A(\theta) \ B(\theta)] \), and \( F_{i,t} \) is the known term for calculating the \( i \)th element of the one-step prediction vector \( \hat{X}_{t+1} \). Let \( \hat{Q}_i = \hat{X}_{i,t+1} - F_{i,t} \), then we have

\[
\begin{align*}
\hat{Q}_i &= \phi_{i,t} \hat{\theta}_i \\
\| \hat{Q}_i - \hat{Q}_i \| &= \| \hat{Q}_i - \phi_{i,t} \hat{\theta}_i \| \\
\Rightarrow \sum_{i=1}^{N} \left\| \hat{X}_{i,t+1} - X_{i,t+1} \right\|^2 &= \sum_{i=1}^{N} \| \hat{X}_i - X_i \|^2 = \sum_{i=1}^{N} \| \hat{Q}_i - \phi_{i,t} \hat{\theta}_i \|^2 = \sum_{i=1}^{N} J_i 
\end{align*}
\]

where \( \hat{Q}_i = (\hat{Q}_{i,1}, \hat{Q}_{i,2}, \ldots, \hat{Q}_{i,N}) \), \( \hat{X}_i = (\hat{X}_{i,1}, \hat{X}_{i,2}, \ldots, \hat{X}_{i,N}) \) and \( F_i = (F_{i,1}, F_{i,2}, \ldots, F_{i,N}) \).

To minimise \( \sum_{t=1}^{N} \left\| \hat{X}_{t+1} - X_{t+1} \right\|^2 \), \( \hat{\theta}_i \) must set \( \partial J_i / \partial \hat{\theta}_i = 0 \) for every \( i \), thus we can obtain \( \hat{\theta}_i \) through linear regression as

\[
\hat{\theta}_i = (\phi_i^T \phi_i)^{-1} \phi_i^T Q_i , i = 1, 2, \ldots, n
\]

(20)

Thus, using linear regression, the initial estimation can be obtained in time domain by minimising cost function (17).

To diagnose the parameters convergence, we propose the following criterion

\[
\left\| A^{(i+1)} - A^{(i)} \right\| + \left\| B^{(i+1)} - B^{(i)} \right\| < \varepsilon
\]

(21)

where \( A^{(i)} \) is the \( i \)th estimation of \( A(\theta) \), \( B^{(i)} \) is the \( i \)th estimation of \( B(\theta) \) and \( \varepsilon \in R \) is the threshold value set by hand. To maintain non-linear optional search algorithm (Bendat and Piersol, 1993) stable, we must avoid parameters’ sudden change between two iterations. We use the following equations to achieve this

\[
A^{(i+1)} = (1 - \alpha)A^{(i)} + \alpha \bar{A}
\]

(22)

\[
B^{(i+1)} = (1 - \alpha)B^{(i)} + \alpha \bar{B}
\]

(23)

where \( \alpha \in (0,1) \) and \( A \) and \( B \) are the current parameter estimation by minimising cost function (16).
Thus, an enhanced frequency identification process is proposed as the following steps:

1. Use Equation (20) to minimise cost function (17) to calculate the initial estimation of parameters \( A(0) \) and \( B(0) \), and set \( i = 1 \).

2. Calculate the frequency response based on Equations (7)–(10), and select the interesting frequency points \( (\omega_1, \omega_2, \ldots, \omega_n) \).

3. Run the frequency identification process in Figure 2 with the proposed cost function (16), and let \( J_M(\theta) = \min \theta J_M(\theta) \).

4. If \( |A(i^{(i+1)}) - A(i)| + |B(i^{(i+1)}) - B(i)| > \epsilon \) then \( i = i + 1 \) and go back to step 2 otherwise, let \( \hat{A} = A(i^{(i+1)}) \) and \( \hat{B} = B(i^{(i+1)}) \).

**Theorem:** The parameter estimation vector \( \hat{\theta} \), obtained by minimising cost function (17), is unbiased and consistent estimate for the real parameter vector \( \theta \), and the enhanced frequency identification process has \( \lim_{i \to \infty} \left[ \begin{array}{c} A(i) \\ B(i) \end{array} \right] = \left[ \begin{array}{c} A \\ B \end{array} \right] \).

**Proof:**

\[
\hat{\theta}_i = \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T Q_i \Rightarrow E\left\{ \hat{\theta}_i \right\} = E\left\{ \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T Q_i \right\} = E\left\{ \left( \phi_i^T \phi_i \right)^{-1} (\phi_i^T \phi_i) \theta_i \right\} = \theta_i
\]

and

\[
\text{Cov}\left\{ \theta_i(N) \right\} = E\left\{ \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T (\theta_i - \hat{\theta}_i)(\theta_i - \hat{\theta}_i)^T \phi_i \left( \phi_i^T \phi_i \right)^{-1} \right\}
\]

\[
= \sigma^2 \left( \phi_i^T \phi_i \right)^{-1}
\]

\[
\Rightarrow \lim_{N \to \infty} \text{tr Cov}\left\{ \theta_i(N) \right\} = \lim_{N \to \infty} \text{tr} \sigma^2 \left( \phi_i^T (N) \phi_i(N) \right)^{-1}
\]

\[
= \lim_{N \to \infty} \frac{1}{N} \sigma^2 \left( \frac{1}{N} \phi_i^T (N) \phi_i(N) \right)^{-1} = 0
\]

Because \( \theta = (\theta_1 \ldots \theta_k) \), \( \hat{\theta} \) is unbiased and consistent estimate for the real parameter vector \( \theta \). For cost functions about linear combination of magnitude and phase, the traditional frequency method in Figure 2 is unbiased and consistent estimate for unbiased and consistent initial estimate and has \( \lim_{N \to \infty} \left[ \hat{A}(N) \right] = [A \ B] \) (Chen, 2004), where \( N \) is the number of the selected frequency points. Thus, if we select different frequency points for every iteration \( i \) and considering \( E[(1-\alpha)\theta^{(0)} + \alpha\theta] = \theta \), then we have \( \lim_{i \to \infty} \left[ \begin{array}{c} A(i) \\ B(i) \end{array} \right] = \left[ \begin{array}{c} A \\ B \end{array} \right] \).
4 Experiment

This section describes how flight data were collected in the hovering flight experiment. The entire experiment was implemented on the SERVOHELI-40 small-size helicopter platform (Figure 3).

The platform, which is fixed with a 3-axis gyro, a 3-axis acceleration sensor, a compass and a GPS, can save the data of velocities, angular rates, Euler angle accelerations and positions into a SD card through an ARM processor. There is a CPLD used for sampling control inputs from the remote control of the pilot. The rotor speed is controlled by a Governor. Table 1 shows the physical characteristics of SERVOHELI-40 small-size helicopter.

Figure 3 SERVOHELI-40 small-size helicopter platform (see online version for colours)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Physical characteristics of SERVOHELI-40 small-size helicopter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>2.12m</td>
</tr>
<tr>
<td>Height</td>
<td>0.73m</td>
</tr>
<tr>
<td>Main rotor diameter</td>
<td>2.15m</td>
</tr>
<tr>
<td>Stabiliser bar diameter</td>
<td>0.75m</td>
</tr>
<tr>
<td>Rotor speed</td>
<td>1,450 rpm</td>
</tr>
<tr>
<td>Dry weight</td>
<td>20kg</td>
</tr>
<tr>
<td>Engine</td>
<td>2-stroke, air cooled</td>
</tr>
<tr>
<td>Flight time</td>
<td>45 min</td>
</tr>
</tbody>
</table>
For each flight, the pilot applied a frequency sweep sequence into one of the four control inputs via the remote control unit. While doing so, the pilot used the other three control inputs to maintain the helicopter close to hovering. The real frequency sweeping inputs of roll is shown in Figure 4 as an example, x-axis is the time and y axis is the percentage of the maximum roll input of the control unit. By frequency sweeping inputs, the pilot inputs some sine-waves of different frequency (1–20 Hz) and the dynamics of the system is excited, and the response provides a large frequency bandwidth for identification. According to the different inherent frequency between rotor and fuselage in Bramwell (2001), the low frequency component, which is below 3 Hz, is used for fuselage dynamics identification, and the high frequency component, which is above 10 Hz, is used for rotor dynamics identification. The selection of frequency points for identification can be realised by the proposed optimal estimation.

During the experiment, according to Bendat and Piersol (1993), all control inputs and all vehicle state variables were recorded and sampled at 50 Hz (> $4\pi/T_d$, $T_d = 1$). The data (of Euler angles, angular body rates, body accelerations and body velocities) were filtered (~3dB at 10 Hz) to remove effects of structural vibrations. The length of the data segments collected for the hovering conditions were 60 sec. We also collected 5 sec hovering data, which is not in frequency sweep mode, to be used for real time verification.

**Figure 4** Frequency sweeping of roll for identification (60 sec) (see online version for colours)
5 The results of identification

It should be noted that the actuator dynamics are not considered here, and we apply the method of optimal estimation in the frequency domain described in Section 3 to identify the parameters in each decomposed part. The results of identified parameters are listed in Table 2.

The ultimate model accuracy verification is done in time domain to compare the response from model with the hovering flight data, which is shown in Figures 5 and 6. All of the calculations were done in MATLAB. For the initial states $\Delta X_0$, we consider that all of the differential initial state $\Delta X_0$ should be zeros when hovering. Thus, we can obtain the initial state for simulation as

$$\Delta X_0 = -A^{-1}B\Delta U_0$$  \hspace{1cm} (24)

To verify the precision of the model by numerical evaluation further, the prediction accuracy of the model output is defined by the following root mean square criterion:

$$V = \sqrt{\frac{1}{TN} \sum_{t=1}^{T} \sum_{i=1}^{N} (y_i(t) - \hat{y}_i(t))^2}$$  \hspace{1cm} (25)

where $y_i(t)$ and $\hat{y}_i(t)$ are the $i$th true and estimated output, respectively, and $N$ is the relative dimensions of the output. The criterion $V$ represents the average error of the model, and the value for the simplified model is listed in Table 3.

Table 2 The values of identified parameters

<table>
<thead>
<tr>
<th>Longitudinal parameters</th>
<th>Value</th>
<th>Lateral parameters</th>
<th>Value</th>
<th>Vertical parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Value</td>
<td>Parameter</td>
<td>Value</td>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>$X_u$</td>
<td>0.2446</td>
<td>$Y_v$</td>
<td>–0.0577</td>
<td>$Z_w$</td>
<td>1.666</td>
</tr>
<tr>
<td>$X_a$</td>
<td>–4.962</td>
<td>$Y_b$</td>
<td>9.812</td>
<td>$Z_r$</td>
<td>–3.784</td>
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<tr>
<td>$X_{lat}$</td>
<td>–0.0686</td>
<td>$Y_{lat}$</td>
<td>–1.823</td>
<td>$Z_{ped}$</td>
<td>2.304</td>
</tr>
<tr>
<td>$X_{lon}$</td>
<td>0.0896</td>
<td>$Y_{lon}$</td>
<td>2.191</td>
<td>$Z_{col}$</td>
<td>–11.11</td>
</tr>
<tr>
<td>$M_u$</td>
<td>–1.258</td>
<td>$L_v$</td>
<td>15.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_a$</td>
<td>46.06</td>
<td>$L_b$</td>
<td>126.6</td>
<td>$M_{lon}$</td>
<td></td>
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<tr>
<td>$M_{lat}$</td>
<td>–0.6269</td>
<td>$L_{lat}$</td>
<td>–4.875</td>
<td>$N_w$</td>
<td>–0.027</td>
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<tr>
<td>$M_{lon}$</td>
<td>3.394</td>
<td>$L_{lon}$</td>
<td>28.64</td>
<td>$N_r$</td>
<td>–1.087</td>
</tr>
<tr>
<td>$A_e$</td>
<td>0.1628</td>
<td>$B_{lat}$</td>
<td>–1.654</td>
<td>$N_{th}$</td>
<td>–1.845</td>
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<tr>
<td>$A_{lat}$</td>
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<td>$B_{lat}$</td>
<td>0.04732</td>
<td>$N_{ped}$</td>
<td>1.845</td>
</tr>
<tr>
<td>$A_{lon}$</td>
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<td>$B_{lon}$</td>
<td>–9.288</td>
<td>$N_{col}$</td>
<td>–0.972</td>
</tr>
<tr>
<td>$C_{lat}$</td>
<td>2.238</td>
<td>$D_{lat}$</td>
<td>–0.7798</td>
<td>$K_r$</td>
<td>–0.040</td>
</tr>
<tr>
<td>$C_{lon}$</td>
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<td>$D_{lon}$</td>
<td>–5.726</td>
<td>$K_{th}$</td>
<td>–2.174</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>0.5026</td>
<td>$\tau_x$</td>
<td>0.5054</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Modelling a small-size unmanned helicopter

Figure 5  Control input for simulation (5 sec) (see online version for colours)

![Graph showing control input](image)

Figure 6  Comparison between the response predicted by the identified model and the response obtained during Flight test in hovering conditions (5 sec) (see online version for colours)

![Graph showing comparison](image)
Table 3  Performance of the simplified model

<table>
<thead>
<tr>
<th>Model part</th>
<th>Lateral accuracy</th>
<th>Longitudinal accuracy</th>
<th>Yawing-vertical accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria $V$</td>
<td>0.0698</td>
<td>0.0434</td>
<td>0.1832</td>
</tr>
</tbody>
</table>

6 Conclusion

With linear regression-based selecting of initial estimation of the unknown parameters and proposed converge criterion, this paper developed an enhanced frequency method for unmanned helicopter identification. The iteration convergence of the proposed identification process was verified. To simplify the small-size helicopter modelling, the hovering dynamics was decoupled into three parts. The results from the MATLAB simulations above show that the structure of the semi-decoupled model and identified parameters are accurate enough to describe the characteristics of a small-size unmanned helicopter in hovering conditions. The method used in the experiment and data processing are effective, and the simplification of the semi-decoupled model structure for identification is very feasible in practice. The results also show that the modification and application of the method of the optimal estimation of parameters in the frequency domain for the MIMO state-space modelling of a small-size helicopter are effective and have the following advantages:

1. Output measurement noise or process noise that is uncorrelated with the control inputs does not bias the frequency-response estimates. This type of disturbances is automatically separated from data.

2. It is possible to individually specify the frequency range used for fitting each input–output pair, and it is particularly effective for faster rotor dynamics. This is particularly effective in addressing the frequency disparity between dynamic modes, such as rigid body dynamics and faster rotor dynamics.

3. Selection of specific frequency ranges is also effective at separating information that is relevant for the identification of helicopter dynamics from irrelevant information, which is often characterised by their frequency content.

4. The formulation of the cost function in the frequency domain requires fewer data points than in the time domain, resulting in more efficient parameter identification.

Acknowledgements

This paper is supported by National Natural Science Foundation of China ‘Control of Mobile Robots Based on Study of Hand Tele-operation’ (60705028). The authors wish to thank the contribution of the UAV group in Shenyang Institute of Automation, Chinese Academy of Sciences.
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Reference


