Asymmetric S-curve Trajectory Planning
for Robot Point-to-point Motion

Fengshan Zou\textsuperscript{1,2}, Daokui Qu\textsuperscript{1,3}, Fang Xu\textsuperscript{1,3}

1) Shenyang Institute of Automation, Chinese Academy of Science Shenyang 110016
2) Graduate School of Chinese Academy of Sciences Beijing 100039
3) SIASUN Robot and Automation Co.,LTD. Shenyang 110168

Abstract—In this paper, we develop an asymmetric s-curve profile method with jerk bounded to obtain high-precision motion and reduce the residual vibration. All possible instances for asymmetric s-curve profile planning are analyzed. The algorithm and its implementation are proposed with a direct and complete method. Simulation results are presented to verify the effectiveness and reliability of the proposed approach.

Keywords—point-to-point motion; asymmetric s-curve; trajectory planning

I. INTRODUCTION

High-speed and high-precision motion control is a very important issue in modern manufacturing industry, especially in cluster tool robots for handling work pieces such as semiconductor wafers, flat panel displays and other substrates. The main problem is how to achieve fast and precise motion with the reduction of the residual vibration.

In many industrial robots, the method for point-to-point motion profile called trapezoidal velocity profile is widely used for its fast motion. But sudden changes in the acceleration will cause a large force to excite residual vibrations. As a modified form, s-curve velocity profile is proposed to reduce residual vibrations. In recent years, the symmetric s-curve velocity profiles have been widely used in motion control fields [1]-[3]. In [1], the research gave a method to estimate the ramp-up and the ramp-down time to improve the kinematics and dynamic performance of a punching machine. The main aim is to reduce the residual vibration while keeping speed and precision.

It has been known the suppressing jerk is essential for reducing the residual vibration[4],[5]. Macfarlane and Croft [6] developed an online method with jerk-bounded trajectories which was described as no oscillation and near time optimal paths compare with the conventional method. In the method a sine-wave template is used to compute control way-points, which are then connected smoothly by 5th order polynomials. In [7] HuaiZhong Li presents a low-vibration motion profile generation method to reduce the residual vibration. The acceleration profile is designed by using a level-shifted sinusoidal waveform in order to control its change rate.

To achieve a high precise motion control with lower residual vibration, it is highly required that the motion profile has a fast start and a slow smooth finish. So when the velocity is asymmetric, the residual vibration at target position can be alleviated. In [8, 9] the basic idea of asymmetric s-curve velocity profiles has been developed but not all possible trajectory instances are taken into account.

In this paper, based on the previous approaches, the asymmetric s-curve motion profile with jerk bounded is discussed using a complete method which takes all possible instances into account. In Section II, a typical asymmetric third-order trajectory and constraints are given, and, in Section III, the possible shapes of trajectory are analyzed and the algorithm is also developed. In Section IV, the results of trajectory planning are shown. Finally, the conclusions are given.

II. TRAJECTORY CONSTRAINTS

Point-to-point motion needs high positioning accuracy without concerning about the process. Fig. 1 is a typical asymmetric s-curve trajectory. By giving different constraints time intervals $t_1 - t_2$, $t_3 - t_4$ and $t_5 - t_6$ will disappear.

Let us assume the following constraints:

C1. Time intervals are denoted as $t_0$ to $t_7$, $t_0$ implies the starting time and $t_7$ implies the finishing time. And $t_{au} = t_2 - t_1$, $t_{ad} = t_6 - t_5$, $t_j = t_4 - t_3$, $t_{ja} = t_3 - t_2 = t_1 - t_0$, $t_{jd} = t_5 - t_4 = t_7 - t_6$.

C2. The velocity and the acceleration are zero at $t_0$ and $t_7$.

C3. $P(t_0) = 0$ and $P(t_7) = s$, $s$ is the target distance.

C4. The velocity is bounded with $v_m$.

C5. The acceleration and jerk in ramp-up phase are bounded with $a_{uu}$ and $j_{uu}$.

C6. The deceleration and jerk in ramp-down phase are bounded with $a_{dd}$ and $j_{dd}$.

C7. $t_{ju} = Rt_{ju}$.

C8. $t_{ad} = Rt_{ad}$, the parameter R can be adjusted according to applications. When $R=1$, the trajectory becomes symmetric s-curve.

978-1-4244-4775-6/09/$25.00 © 2009 IEEE.
III. TRAJECTORY INSTANCE AND ALGORITHM

A. Limitation values definitions

1) When acceleration reaches its maximal value, the corresponding velocity and position are defined as $v_{u_a}$ and $s_{u_a}$.

2) When deceleration reaches its maximal value, the corresponding velocity and position are defined as $v_{d_d}$ and $s_{d_d}$ in the process of deceleration.

3) When velocity reaches its maximal value, the total distance of acceleration process and deceleration process is defined as $s_{v_1} (v_m < v_{a_u})$ or $s_{v_2} (v_m > v_{a_u})$.

B. Implementation of Algorithms

1) Calculate limited values.

The limited values defined above are expressed as:

$$v_{u_a} = a_{mu}^2 \cdot j_{mu}$$

$$s_{u_a} = v_{a_u} \cdot \sqrt{\frac{v_{a_u}}{j_{mu}}}$$

$$v_{d_d} = a_{md}^2 \cdot j_{md}$$

$$s_{d_d} = v_{a_d} \cdot \sqrt{\frac{v_{a_d}}{j_{md}}}$$

$$s_{v_1} = v_m \cdot \sqrt{\frac{v_m}{j_{md}}} + v_m \cdot \sqrt{\frac{v_m}{j_{mu}}}$$

$$s_{v_2} = 0.5 \cdot v_m \cdot \left( \frac{a_{mu}}{j_{mu}} + \frac{v_m}{a_{mu}} \right) + 0.5 \cdot v_m \cdot \left( \frac{a_{md}}{j_{md}} + \frac{v_m}{a_{md}} \right)$$

2) Relationship between $A_{mu}, J_{mu}, A_{md}, J_{md}$.

$$a_{mu} = j_{mu} \cdot t_{fu}$$

$$a_{md} = j_{md} \cdot t_{fd}$$

and Constraint C7 $t_{jd} = R_{fu}$ and C8 $t_{ad} = R_{mu}$

The velocity at $t_j$ is zero. So we can obtain:

$$v_{u_a} = v_{d_d}$$

Substituting (1) and (2) into (3), the maximal deceleration $a_{md}$ and the slope of deceleration $j_{md}$ can be expressed as:

$$j_{md} = \frac{j_{mu}}{R \cdot R}$$

$$a_{md} = \frac{a_{mu}}{R}$$

3) Possible Trajectory Instances and Formula Deduction.

The essential of asymmetric s-curve trajectory planning is to calculate the time interval $t_{fu}, t_{mu}, t_{jd}$ and $t_{ad}$. According to aforementioned conditions, there are four instances for different constraints.

a) $v_m \leq v_{u_a}$ and $s > s_{v_1}$

As shown in Fig. 2, the trajectory of acceleration is triangle shape. The negative triangle and the positive triangle are not directly connected. In the instance, the trajectory can reach the maximal velocity $v_m$ and can’t reach the maximal acceleration $a_{mu}$ and the maximal deceleration $a_{md}$.

$t_{fu}$ and $t_{jd}$ are determined by the maximal velocity $v_m$.

And the time interval $t_{a}$ is determined by the distance $s$.
So we can obtain all time intervals expressed as the following:

\[
\begin{align*}
 t_{ju} &= \frac{v_m}{\sqrt{f_{mu}}} \\
 t_{jd} &= \frac{v_m}{\sqrt{f_{md}}} = R \cdot \frac{v_m}{\sqrt{f_{mu}}} \\
 t_{au} &= t_{ad} = 0 \\
 t_v &= s - s \cdot v \cdot \frac{1}{v_m} 
\end{align*}
\]

b) \( v_m \leq v - a_u \) and \( s < s \cdot v \cdot 1 \) or \( v_m > v - a_u \) and \( s \leq s \cdot a_u + s \cdot a_d \)

The acceleration shape is triangular. But the negative triangle and the positive triangle are directly connected. So there is not a const velocity phase in this instance. It is shown in Fig. 3. Acceleration and Velocity all can't reach their maximal values. \( t_{ju} \) and \( t_{jd} \) are determined by the distance \( s \).

The distance can be expressed as:

\[
j_m \cdot t_{ju}^3 + j_m \cdot t_{jd}^3 = s \quad (5)
\]

Substituting (2) and (4) into (5). All time intervals can be represents as:

\[
\begin{align*}
 t_{au} &= t_{ad} = 0 \\
 t_v &= 0 \\
 t_{ju} &= \frac{s}{\sqrt{f_{mu} \cdot (1 + R)}} \\
 t_{jd} &= \frac{s \cdot R}{\sqrt{f_{md} \cdot (1 + R)}}
\end{align*}
\]

c) \( v_m > v - a_u \) and \( s \cdot a_u + s \cdot a_d < s \leq s \cdot v \cdot 2 \)

In the case, the acceleration shape which is shown in Fig. 4. is trapezoid without a const velocity phase. the trajectory can't reach the maximal velocity \( v_m \), but can reach the maximal acceleration \( a_mu \) and the maximal deceleration \( a_mld \). So \( t_{ju} \) and \( t_{jd} \) are determined by \( a_mld \) and \( a_mu \). The distance equal to

\[
\frac{1}{2} \cdot a_{mu} \cdot (t_{ad} + t_{ju}) \cdot (2 \cdot t_{ju} + t_{au}) +
\]

\[
\frac{1}{2} \cdot a_{md} \cdot (t_{ad} + t_{jd}) \cdot (2 \cdot t_{jd} + t_{au}) = s.
\]

From above formulation, we can deduce the following result.

\[
\begin{align*}
 t_{ju} &= \frac{a_{mu}}{j_{mu}} \\
 t_{jd} &= \frac{a_{md}}{j_{md}} = R \cdot t_{ju} \\
 t_{au} &= \sqrt{\frac{s}{a_{mu} \cdot (1 + R)^2} - \frac{t_{ju}^2}{4}} - \frac{t_{ju}}{2} \\
 t_{ad} &= \sqrt{\frac{s \cdot R}{a_{md} \cdot (1 + R)^2} - \frac{t_{jd}^2}{4}} - \frac{t_{jd}}{2} = R \cdot t_{au} \\
 t_v &= 0
\end{align*}
\]

d) \( v_m > v - a_u \) and \( s \cdot v \cdot 2 < s \)

In the instance, the trajectory has a const velocity phase with trapezoid form acceleration as shown in Fig. 5. Acceleration and Velocity all can reach their maximal values. All time intervals can be calculated with latter formulas.
IV. EXPERIMENT RESULTS

An experiment is performed in order to verify validity and reliability of the proposed asymmetric motion profile planning method. The maximal jerk is $100 m/s^3$ and the maximal acceleration is $10 m/s^2$. The parameter R can be adjusted arbitrarily according applications. When R=1, the trajectory becomes symmetric s-curve. In our experiment the adjustable parameter is R=2. Other parameters for various trajectory constraints are summarized in Table 1. Results are shown in Figs. 6~10 below.

<table>
<thead>
<tr>
<th>$v_m$ (m/s)</th>
<th>$s$ (m)</th>
<th>$v_{au}$ (m/s)</th>
<th>$s_{ad}$ (m)</th>
<th>$s_{v-1}$ (m)</th>
<th>$s_{v-2}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.15</td>
<td>0.64</td>
<td>0.1536</td>
<td>0.0493</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.07</td>
<td></td>
<td></td>
<td>0.0757</td>
<td>0.175</td>
</tr>
<tr>
<td>0.7</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td>0.216</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td>0.3075</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 6. Motion Profile ($v_m = 0.3, s = 0.15$)**

**Fig. 7. Motion Profile ($v_m = 0.4, s = 0.07$)**

**Fig. 8 Motion Profile ($v_m = 0.7, s = 0.15$)**
V. CONCLUSIONS

In order to reduce the residual vibration in high-precision and high-speed motion control, we proposed the asymmetric s-curve profile method with jerk bounded which gives a complete solution for all possible profile instances. The simulation results testify the proposed method is a effective and universal point-to-point motion profile planning approach.

Future work will be devoted to apply the proposed approach to our high-precision vacuum cluster robot which is used in the vacuum chambers for handling semiconductor wafers.

REFERENCES