A MATHEMATICAL MODEL OF REPEATABILITY FOR INDUSTRIAL ROBOTS

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SUMMARY: In this paper the influences of Joint's error, motion history, speed and robot posture on repeatability are analysed. Based on this the mathematical expressions of the quantity, direction and distribution of the stochastic positional error of end effector are derived. Using this model the manipulator's positional error after any motion can be preestimated and compensated.

KEYWORDS: Robot, Repeatability, Evaluation

1. INTRODUCTION

Industrial robots today have become generally accepted elements in many tasks, such as loading and unloading machine tools, assembly, welding, deburring, laser-cutting, measuring, etc. The regular task in these applications is positioning of an object or making it follow a prespecified trajectory in space. The performance of industrial robot depends largely upon its positional accuracy.

The position error of a robot is defined as the difference between the actual position and the programmed position of its end effector. With the exception of dynamic error or path error, digitisation error and calibration error, the main components consisted in position error are deterministic kinematic error and stochastic kinematic error. The deterministic kinematic errors are caused by differences between the actual kinematic parameters (length of links ...) and the ones used in the kinematic model used for inverse kinematic imperfections. This part of position error can be preestimated and compensated using a suitable geometric error model and control algorithm. The stochastic kinematic errors are due to imperfections, like friction or clearance, meaning that, within an encoder resolution, a robot link can assume an arbitrary position, depending on its motion history. The magnitude of this error directly affects the robot repeatability. Clearly, the magnitude and direction of this part of position error can not be preestimated for every positioning.
The probabilistic characteristic of this stochastic error, however, is able to be analysed and estimated, from which we can find out the new way to furtherly improve the positional accuracy of robots.

This paper will give the mapping relationship from the probabilistic characters of the stochastic motion error of all joints to the ones of robot end effector, present the measured results on the stochastic motion error of joints, and the way how can improve the repeatability of robot motion.

2. RELATIONSHIP OF STOCHASTIC MOTION ERRORS BETWEEN JOINTS AND END EFFECTOR

The robot manipulator generally is an open kinematic chain composed of a sequence of elements or links connected by joints providing either a rotational or translatory degree of freedom. The position and pose of end effector are depended upon the geometric parameters of links and the magnitude of variables of joints. the motion velocity of end effector is related to the ones of joints also. From kinematic analysis, this relationship can be shown mathematically to be:

\[ \dot{\mathbf{x}} = \mathbf{J} \dot{\mathbf{q}} \]  

(1)

Where,

\[ \dot{\mathbf{x}} \] --- the velocity vector in reference coordinate space;
\[ \dot{\mathbf{q}} \] --- the velocity vector in joint space;
\[ \mathbf{J} \] --- the Jacobian matrix.

Substituting the difference for differential the Equation (1) becomes:

\[ \Delta \mathbf{x} = \mathbf{J} \Delta \mathbf{q} \]  

(2)

In Equation (2), \( \Delta \mathbf{q} \) is the positional error vector of joint, \( \Delta \mathbf{x} \) is the position and pose error of the end effector. If the \( \Delta \mathbf{q} \) is stochastic, the \( \Delta \mathbf{x} \) is stochastic also. Therefore, the repeatability of end effector is a linear function of stochastic positional errors of joints. And the Jacobian matrix, which represents the linear relationships between \( \Delta \mathbf{x} \) and \( \Delta \mathbf{q} \), will change with the changing of positioning point in working space of robot.

The stochastic positional errors of joints can be considered as some independent continuous random variables. In many cases of application the understanding of distribution function for a variable is not always necessary, but its probabilistic characters, specially the mathematical expectation and variance, are more interested.

If the mathematical expectation and variance of variable vector \( \Delta \mathbf{q} \) are \( \mathbb{E}(\Delta \mathbf{q}) \) and \( \text{D}(\Delta \mathbf{q}) \) respectively, the mathematical expectation of \( \Delta \mathbf{x} \) is:

\[ \mathbb{E}(\Delta \mathbf{x}) = [\mathbf{J}] \mathbb{E}(\Delta \mathbf{q}) \]  

(3)

and the variance of \( \Delta \mathbf{x} \) is:

\[ \text{D}(\Delta \mathbf{x}) = [\mathbf{J}^2] \text{D}(\Delta \mathbf{q}) \]  

(4)

where,

\[ [\mathbf{J}^2] = \begin{bmatrix}
\mathbf{J}_1^2 & \mathbf{J}_2^2 & \cdots & \mathbf{J}_n^2 \\
\mathbf{J}_1 \mathbf{J}_2 & \mathbf{J}_2 \mathbf{J}_2 & \cdots & \mathbf{J}_n \mathbf{J}_n \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{J}_1 \mathbf{J}_m & \mathbf{J}_2 \mathbf{J}_m & \cdots & \mathbf{J}_n \mathbf{J}_m
\end{bmatrix} \]
Equations (3) and (4) defines the relationships between probabilistic characters of the stochastic positional errors of end effector and joints. When the Jacobian matrix, the $[J^2]$ matrix, $M(AQ)$, and $D(AQ)$ are known, the $M(AX)$ and $D(AX)$ can be determined easily. So that the probabilistic preestimating and compensating on the repeated positional error of robot is possible.

3. STOCHASTIC POSITIONAL ERROR OF JOINT AND ITS MEASUREMENT

The stochastic positioning error range of a joint is determined by the encoder on each robot axis. And its magnitude can be considered as equal to this encoder resolution. To understand the distribution of the stochastic positioning error of joints in this range, the measurement on joint positional error has been measured using a measuring setup. The measured robot was PUHA 562, and the measuring head is a 3-sensors noncontacted measuring system. The accuracy of these measuring sensors was 0.005mm. The artifact held on the end effector was a precision metal ball with diameter of 25.4 mm.

Fig.1 shows two of the measuring results on the positional error of joint 1 under same conditions except the motion direction of joint. Although the distribution of resolution error for an encoder is symmetrical in its error range, the actual distribution of this error is asymmetrical. The direction of its partial peak is determined by the motion direction of joint. The distance between partial peak and target position is decided by the motion velocity and applied load to this joint. Evidently, the mathematical expectation and variance of $k$th joint positional error are the functions as shown follows:

$$M(\Delta q_k) = \text{Sign}(D_k) f_v(v_k, p_k)$$ (5)

and

$$D(\Delta q_k) = f_a(v_k, p_k)$$ (6)

where,

$Sign(D)$ --- when the direction of joint motion is positive, this sign will be plus, and conversely will be minus.

$V$ --- the velocity of $k$th joint motion;

$P$ --- the applied load on this joint.

As long as the relationship of Equations (6) and (6) are determined in advance by measuring test, the probabilistic characters of each joint under various motion conditions can be preestimated in control software in real time.

Except the measurement, the characters $M(AQ)$ and $D(AQ)$ can be calculated from measured data of $M(AX)$ and $D(AX)$ also.

That is

$$M(AQ) = [J]^T M(AX)$$ (7)

and

$$D(AQ) = [J^2]^T D(AX)$$ (8)

As long as we measure and determine out the mathematical expectations and variances of the stochastic positional
and direction of stochastic positional error component of robot end effector are dependent on the ones of each joint. And the effects of the positional errors of every joints on end effector positional error are different and will change with changing of the target position in robot working space. Generally, the joint close to the base element of robot will have more effect than that far from base.

As a result of the asymmetrical distribution of the positional error of each joint, the distribution of stochastic positional error of robot end effector is not a normal distribution as general thinking but an asymmetrical one. Only in some cases it is approximate to the normal distribution.

Fig.2 shows the measured distribution of the stochastic positional error components in the directions of the x, y and z axes for a robot end effector.

In consideration of the axial symmetry of the distribution range of $\Delta x$, $\Delta y$ and $\Delta z$, the all actual position of end effector centre reached after many times motion will be inter a triaxial ellipsoid which centre is the target point. The mathematical expectation point $M$ of these actual position, however, will deviate from the target point $O$, this is the centre of error space as shown in Fig.3 because of the asymmetrical distribution of components $\Delta x$, $\Delta y$ and $\Delta z$.

The point $P_i$ in Fig.3 is the actual position reached after $i$th motion. The distance between target and reached point is the module of vector $OP$ or $[\Delta x_i, \Delta y_i, \Delta z_i]^T$.
mathematical expectation of vectors $\overline{OP}$; when other probabilistic characters are similar.

To improve the robot repeatability, therefore, we can preestimated the magnitude of $\overline{OM}$ before next positioning based on Equation (3) in which the $M(\Delta Q)$ can be determined by relationship (5) using the programmed $V$ and $P$ of this positioning. Then the compensation can be accomplished by pre-feed control of robot control software during programming the work path and positioning.

Otherwise, from Equation (3) a new thinking to improve repeatability can be drawn out that we can optimize the motion parameters of each joint.
with the object of shortest divergency of $M(\Delta X)$ from target position during control programming.

5. CONCLUSIONS

1). The stochastic positional error of robot end effector depends on that of each joint of this robot, and the probabilistic characters, such as mathematical expectation and variance, of these stochastic errors of end effector are depended directly on that of its joint also.

2). The distribution of the stochastic positional error of joint is not symmetrical. The direction and position of its partial peak is determined by the motion direction and velocity of joint and applied load on this joint.

3). The deviation of mathematical expectation point of all actual reached position of end effector in space from the target point due to the asymmetrical distribution of positional error of joints causes robot repeatability to reduce.

4). As a new way to improve the repeatability, the magnitude and direction of deviation of mathematical expectation for given joints motion conditions and manipulator pose can be preestimated and compensated by improving the function of robot control software.

5). Optimizing the motion history to get a symmetrical distribution of stochastic positional error of end effector is another way to improve robot repeatability.

REFERENCES


