Error modeling for Tailored Blank Laser Welding machine

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ABSTRACT

This paper introduces the research on error modeling of tailored blank laser welding machine which has four linear axes. The error models are established based on multi-body system (MBS) theories which are developed in this paper. The number arrays of low-order body are used to describe the topological structures which are taken to generalize and refine MBS, and the characteristic matrices are employed to represent the relative positions and gestures between any two bodies in MBS. Position error associated function which can reflect the influence of each error origin on the positioning error of the machine tool is given to describe transmission error of the machine in detail. Based on this method, the paper puts forward the error model of the tailored blank laser welding machine. The measurement and evaluation of their error parameters start, after complete error modeling of the machine. Leica Laser Tracker is introduced to measure the errors of the machine and to check the result of the error model.

Keywords: multi-body system theory, Tailored Blank Laser Welding, error modeling, position error associated function

1. INTRODUCTION

Laser processing is a modern manufacturing technique. It is a high energy beam process that continues to spread into modern industries and new applications due to its numerous advantages such as deep weld penetration and minimizing heat inputs [1-2]. Tailored laser welding technology has become an important manufacturing process for automobile industry. Tailor welded blanks are defined as two or more sheets of blanks with equal or different mechanical properties or surface coatings welded together by laser before stamping. With the development of this technique, the focus attention has been changed into solving the problems of reducing automobile weight, oil consumption and air pollution from the initial application of wide blanks with equal thickness by laser welding. Tailor welded blanks has become a hot issue in the automobile industry and steel industry for its high quality, high profit and low cost of production [3].

Mechanical precision plays an important role in laser welding processing because of the small scale of the laser focus. Defocus and offset of the laser focus point changes during welding for the existing of the kinematic errors. If the errors are too much, bad welding quality will happen.

The purpose of this study is to put forward an error modeling method for tailored blank laser welding machine that has four linear axes. First, basic principles of the error modeling are introduced. Then the error model of the tailored blank laser welding machine is established based on the basic principles, and then, Leica Laser Tracker is used to measure the errors of the machine. At last, the conclusion of this error modeling method is drawn.

2. BASIC PRINCIPLES

Most machines are designed with the intention that all of the axes will be either prismatic or rotary, but it is physically impossible to construct a joint that will perfectly generate this type of motion. For example, most prismatic joints consist of a carriage constrained to move along a bar. Since the bar will be subject to some slight curvature or irregularities along its surface, the generated motion will not be purely prismatic as expected. These irregularities will create roll, pitch and yaw error while moving with ideal movement [4]. In this paper, we try to model the errors containing geometric and other errors in terms of mathematics.

2.1 Linear and Rotation transformations

When a tool moves along (or around) an axis in space (Fig 1), its location can be uniquely identified by its travel distance (or angle rotated around the axis) referring to the origin of a coordinate system.

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The transformation $H$ corresponding to a translation by a vector $(P_x, P_y, P_z)$ is \[^4,5\]:

$$H = \text{Trans}(x,y,z) = \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (1)

The transformation corresponding to rotations about the X, Y and Z axes by $\alpha$, $\gamma$ and $\beta$ are respectively:

$$\text{Rot}(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{Rot}(y,\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{Rot}(z,\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (2)

2.2 Homogenous coordinate transformation

A homogeneous coordinate transformation system transfers a rigid body position and direction from one coordinate system to another coordinate system\[^6\]. For example, if the tailored blank laser welding machine moves along the linear axis, we can express the laser focus position of the machine by homogeneous coordinate transformations.

If the rotational/linear axes are not perpendicular to each other, although they should be, there are rotation/squareness errors around/along an arbitrary axis in the space. Though the variation is small, we cannot neglect it, because small variations in angular error give large laser focus deviation from the desired position, depending on the length of laser focus and spindle pivot distance. Homogeneous coordinate transformations have been applied to this research by considering small angle approximation.

2.3 Homogenous coordinate transformation: ideal and real case

For ideal machine tool with no geometric error, the transformation of coordinate $O_i-x_iy_iz_i$ to $O_j-x_jy_jz_j$ through rotating and moving can be expressed as:

$$T_{ij} = T_{ij}(R)T_{ij}(M) = \begin{bmatrix} c\beta c\gamma & -c\beta s\gamma & s\beta & x_j \\ c\alpha s\beta c\gamma + s\alpha s\beta s\gamma & -c\alpha c\gamma - s\alpha s\beta b\gamma - s\alpha c\beta & s\alpha c\beta & y_j \\ s\alpha s\beta s\gamma - c\alpha s\beta c\gamma & c\alpha c\gamma - s\alpha s\beta s\gamma & c\alpha c\beta & z_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (3)

where $T_{ij}$ is transformation matrix with respect to frame $i$ and frame $j$, $\alpha_j, \beta_j, \gamma_j$ express roll, pitch and yaw with respect to frame $i$ and frame $j$ respectively.

In (3), we assume that the movements and rotation are ideal, but in a real machine there are a number of geometric errors as described. There are six freedoms between two parts, so when they have relative motion there are 6 errors: $\Delta x_j$, $\Delta y_j$ and $\Delta z_j$ along $X$, $Y$ and $Z$ axes respectively; $\Delta \alpha_j$, $\Delta \beta_j$ and $\Delta \gamma_j$ about $X$, $Y$ and $Z$ axes respectively. Then the error matrix can be expressed as (4), here, small terms are neglected.

$$\Delta T_{ij} = \begin{bmatrix} 1 & -\Delta \gamma_j & \Delta \beta_j & \Delta x_j \\ \Delta \gamma_j & 1 & -\Delta \alpha_j & \Delta y_j \\ -\Delta \beta_j & \Delta \alpha_j & 1 & \Delta z_j \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ \hspace{1cm} (4)

Fig. 2a and Fig. 2b show six errors produced during the movement along $X$ axis and rotation around $X$ axis respectively. We can give the expression of the transformations in real case from (3) and (4): $T_{ij\text{real}} = T_{ij} \Delta T_{ij}$ \hspace{1cm} (5)
2.4 Error analysis

Equation (4) gives the general expression of the transformation error. In this part, a new expression which can reflect the
details of the error origin and correlations between errors is detailed.

1) Squareness error

If the $X$ axis is not perpendicular to $Y$ and $Z$ axes, there will be position error in the orthogonal direction, which can be
expressed by the direction cosine of the axis (with small angle assumption). Fig. 3 shows the actual axis versus ideal axis.
From Fig. 3, we can see that the $Z$ axis is assumed to be ideal and the $X$ and $Y$ axes are allowed to be out of square. How
these affect the positioning error. For example, when the laser head moves along $X$ axis with the error as shown in Fig. 3,
it will produce error in the $Y$ and $Z$ directions. If the laser head moves along $X$ axis by the distance of $x$, the error matrix
can be expressed as:

$$
\Delta T_x = \begin{bmatrix}
1 & -\Delta y_y & \Delta \beta_y & \Delta x_x + x \cdot k_{xy} \\
\Delta y_y & 1 & -\Delta \alpha_y & \Delta y_y + x \cdot k_{xy} \\
-\Delta \beta_y & \Delta \alpha_y & 1 & \Delta z_z + x \cdot k_{xz} \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

where $\Delta x_x$, $\Delta y_y$, and $\Delta z_z$ are basic positioning error, $k_{xy}$ is squareness between $X$ and $Y$ axes and $k_{xz}$ is squareness
between $Y$ and $Z$ axes. $x \cdot k_{xy}$ is the error in $Y$ axis produced by the squareness between $X$ and $Y$ axes. $x \cdot k_{xz}$ is the error in $Z$
axis produced by the squareness between $X$ and $Z$ axes. $E$ can obtain the similar error matrix for the movements in $Y$
direction. Because the real and ideal $Z$ axes are the same, there are no errors produced by the squareness.

2) Other errors

Actual machine tools have different kinds of errors including thermal, elastic deformation of axis, vibration caused by
frequent acceleration/deceleration and so on. All the errors concerned above contribute to the total positioning error in
the $X$, $Y$ and $Z$ directions. The error matrix considering deformation of axis is given next to describe how the
deformations of axis affect the position error.

In Fig. 4, if the $X$ axis is bent by gravity or other forces defined as $F$ for all, the error matrix can be shown as (7).

$$
\Delta T_y = \begin{bmatrix}
1 & -\Delta y_y & \Delta \beta_y & \Delta x_x + \Delta x_{\text{deformation}} \\
\Delta y_y & 1 & -\Delta \alpha_y & \Delta y_y + \Delta y_{\text{deformation}} \\
-\Delta \beta_y & \Delta \alpha_y & 1 & \Delta z_z + \Delta z_{\text{deformation}} \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

where $\Delta x_x$, $\Delta y_y$, and $\Delta z_z$ are positioning errors beside errors caused by deformation; $\Delta x_{\text{deformation}}$, $\Delta y_{\text{deformation}}$ and
$\Delta z_{\text{deformation}}$ are deformation errors which can be obtained by theory of mechanics of materials. Then the deformation
errors can be given by the following equations:

$$
\begin{align*}
\Delta x_{\text{deformation}} &= g_x(F) \\
\Delta y_{\text{deformation}} &= g_y(F) \\
\Delta z_{\text{deformation}} &= g_z(F)
\end{align*}
$$

Here $g_x(F)$, $g_y(F)$ and $g_z(F)$ are the deformation functions. Positioning errors caused by other errors can also be described
as functions in detail. Next, we will give conception of the position error associated function.
3) Error Associated Function

In 1) and 2), we give the expression of the positioning errors produced by squareness and deformation of axis. The aim of this research is to establish a general method to express the error effect.

Position error associated functions described as \( f_x, f_y, \) and \( f_z \) in (9) are introduced to consummate the error matrix. Equation (9) is the general expression of the error matrix.

\[
\Delta T_{ij} = \begin{bmatrix}
1 & -\Delta y_{ij} & \Delta \beta_{ij} & f_x \\
\Delta y_{ij} & 1 & -\Delta \alpha_{ij} & f_y \\
-\Delta \beta_{ij} & \Delta \alpha_{ij} & 1 & f_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(9)

The specific expression of \( f_x, f_y, \) and \( f_z \) which can reflect the influence of the error origins mentioned in 1) and 2) on the positioning error can be determined by the effect of the error origins. For example, \( f_y \) equals to \( \Delta y_{xy} + x \cdot k_{xy} \) in (6), \( \Delta y_{xy} + g_y(F) \) in (7). The full expressions of the position error associated functions can be given as (10).

\[
f_x(y, z) = \text{effect of squareness} + \text{effect of thermal} + \text{effect of elastic deformation of axis} + \text{effect of vibration} + \ldots
\]  

(10)

3. ERROR MODELING FOR TAILORED BLANK LASER WELDING MACHINE

The error model of the Tailored Blank Laser Welding machine shown in Fig. 5 is established based on the principles given in part 2.

Table 1. The number arrays of low-order body

<table>
<thead>
<tr>
<th>Typical body ( j )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L^b(j) )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>( L(j) )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>( \hat{L}(j) )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( \hat{L}(j) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \hat{L}(j) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{L}(j) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. The DOF of each body

<table>
<thead>
<tr>
<th>Adjacent body</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1-2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2-3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3-4</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4-5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 5 Tailored blank laser welding machine

Fig. 6 The topological structure and the generalized coordinate
3.2 Error modeling for Tailored Blank Laser Welding machine

1) Static and Kinematic Characteristic Matrices of Body 0 and Body 1

The first joint of the machine is a linear axis along $Y$ axis, and then the ideal static and kinematic characteristic matrix can be expressed as:

$$
T_{01p} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(11)

The error characteristic matrix can be given as:

$$
\Delta T_{01pe} = \begin{bmatrix}
1 & -\Delta Y_{01} & -\Delta \beta_{01} & f_{1x} \\
\Delta Y_{01} & 1 & -\Delta \alpha_{01} & f_{1y} \\
-\Delta \beta_{01} & \Delta \alpha_{01} & 1 & f_{1z} \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(12)

If the $Y$ axis is assumed to be ideal, the $X$ and $Z$ axes are allowed to be out of square. When the laser head moves along $Y$ axis, there will be no effect of squareness error in the $X$ and $Z$ directions. But, the $Y$ axis is bent in the $Z$ direction by gravity when the slide block moves along the $Y$ axis which is 3600mm in length. We use CATIA to help determine the deformation error of the beam in $Y$ axis. First, the 3D model of the beam is built in CATIA with the sectional dimension of the beam as 200mm×376mm. And then, we use the finite element analysis modular to analyze the deformations of the beam when the slide block moves from the beginning to the middle of the guide rail with the interval distance about 190mm. Last, the results of the finite element analysis are shown as Fig. 7. And then, the whole deformation of the beam can be described as a fitting curve from the analysis results. The expression of the $f_{1x}$, $f_{1y}$ and $f_{1z}$ can be described as follows:

$$
f_{1x} = \Delta x_{11}; \quad f_{1y} = \Delta y_{11}; \quad f_{1z} = \Delta z_{\text{deformation}1} + \Delta z_{11}
$$

where $\Delta z_{\text{deformation}1}$ is the result of the finite element analysis above. $\Delta x_{11}$, $\Delta y_{11}$ and $\Delta z_{11}$ are basic position errors independent of geometric error, i.e. this error depends on ball screw, thermal, measuring system etc.

The real position between body 0 and body 1 can be expressed by (13).

$$
T_{01real} = T_{01p} \Delta T_{01pe}
$$

(13)

Fig. 7 The finite element analysis of the beam deformation

2) Static and kinematic characteristic matrices of body 1 and body 2

The second joint of the machine is a linear axis along $X$ axis, and then the ideal static and kinematic characteristic matrix can be expressed as:

$$
T_{12p} = \begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(14)

The error characteristic matrix can be given as:
The moving range of $X$ axis is so small that the bent error can be neglected. The $X$ axis is not perpendicular to $Y$ and $Z$ axes, and then there will be position error in orthogonal direction, which can be expressed by the direction cosine of the axis (with small angle assumption). The position error associated function $f_{x_2}, f_{y_2} \text{ and } f_{z_2}$ can be described as follows:

$$f_{x_2} = \Delta x_{x_2}; \quad f_{y_2} = \Delta y_{y_2} + x \cdot k_{y_2}; \quad f_{z_2} = \Delta z_{z_2} + x \cdot k_{z_2}$$

where $k_{y_2}$ is cosine of angle between $X$ axis and $Y$ axis. $k_{z_2}$ is cosine of the angle between $X$ axis and $Z$ axis. $\Delta x_{x_2}, \Delta y_{y_2}$ and $\Delta z_{z_2}$ are basic position errors independent of geometric error, i.e. this error depends on pitch, thermal, measuring system etc.

The real position between body 0 and body 1 can be expressed by (16).

$$T_{12\, \text{real}} = T_{12\, p} \Delta T_{12\, pe} \quad (16)$$

3) Static and kinematic characteristic matrices of body 2 and body 3

The third joint of the machine is a linear axis along $Z$ axis, and then the ideal static and kinematic characteristic matrix can be expressed as:

$$T_{23\, p} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (17)$$

The error characteristic matrix can be given as:

$$\Delta T_{23\, pe} = \begin{bmatrix}
1 & -\Delta y_{z_3} & \Delta \beta_{z_3} & f_{z_3} \\
\Delta y_{23} & 1 & -\Delta \alpha_{23} & f_{y_2} \\
-\Delta \beta_{23} & \Delta \alpha_{23} & 1 & f_{y_2} \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (18)$$

For the $Z$ axis is not perpendicular to $X$ and $Y$ axes, then there will be position error in orthogonal direction, which can be expressed by the direction cosine of the axis (with small angle assumption). The position error associated function $f_{3x}, f_{y_3}$ and $f_{z_3}$ can be described as follows:

$$f_{x_3} = \Delta x_{x_3} + z \cdot k_{x_3}; \quad f_{y_3} = \Delta y_{y_3} + z \cdot k_{y_3}; \quad f_{z_3} = \Delta z_{z_3}$$

where $k_{x_3}$ is cosine of angle between $X$ axis and $Z$ axis. $k_{y_3}$ is cosine of the angle between $Y$ axis and $Z$ axis. $\Delta x_{x_3}, \Delta y_{y_3}$ and $\Delta z_{z_3}$ are basic position errors independent of geometric error, i.e. this error depends on pitch, thermal, measuring system etc.

The real position between body 2 and body 3 can be expressed by (19).

$$T_{23\, \text{real}} = T_{23\, p} \Delta T_{23\, pe} \quad (19)$$

4) Static and kinematic characteristic matrices of body 3 and body 4

The last joint of the machine is a linear axis along $X$ axis adjusting the position of laser head in $X$ direction during welding according to the laser tracking system, and then the ideal static and kinematic characteristic matrix can be expressed as:

$$T_{34\, p} = \begin{bmatrix}
1 & 0 & 0 & x' \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (20)$$
The error characteristic matrix can be given as:

\[
\Delta T_{34\text{pe}} = \begin{bmatrix}
1 & -\Delta y'_{34} & \Delta \beta_{34} & f_{4x} \\
\Delta y'_{34} & 1 & -\Delta \alpha_{34} & f_{4y} \\
-\Delta \beta_{34} & \Delta \alpha_{34} & 1 & f_{4z} \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (21)

Because of the structure of the last axis, the main influence factor of the position error associated function is squareness error. Then \(f_{3x}, f_{3y}\) and \(f_{2z}\) can be described as follows:

\[
f_{3x} = \Delta x'_{44}; \quad f_{3y} = \Delta y'_{44} + x' \cdot k_{xy}; \quad f_{2z} = \Delta z_{44} + x' \cdot k_{xz}
\]

where \(k_{xy}\) is cosine of angle between \(X'\) axis and \(Y\) axis. \(k_{xz}\) is cosine of the angle between \(X'\) axis and \(Z\) axis. \(\Delta x'_{44}, \Delta y'_{44}\) and \(\Delta z_{44}\) are basic position errors independent of geometric error, i.e. this error depends on pitch, thermal, measuring system etc.

The real position between body 3 and body 4 can be expressed by (22).

\[
T_{23\text{real}} = T_{23p}\Delta T_{23\text{pe}}
\] (22)

5) Static and kinematic characteristic matrices of body 4 and body 5

The laser head (body 5) is fixed on body 4; there are only static errors which can be expressed as:

\[
\Delta T_{45\text{pe}} = \begin{bmatrix}
1 & -\Delta y'_{45} & \Delta \beta_{45} & \Delta x_{45} \\
\Delta y'_{45} & 1 & -\Delta \alpha_{45} & \Delta y_{45} \\
-\Delta \beta_{45} & \Delta \alpha_{45} & 1 & \Delta z_{45} \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (23)

The ideal static characteristic matrix can be expressed as:

\[
T_{45p} = \begin{bmatrix}
1 & 0 & 0 & P_x \\
0 & 1 & 0 & P_y \\
0 & 0 & 1 & P_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (24)

The real position between body 4 and body 5 can be expressed by (25).

\[
T_{45\text{real}} = T_{45p}\Delta T_{45\text{pe}}
\] (25)

6) Error model of the Tailored Blank Laser Welding machine

According to the characteristic matrices above, the laser focus position in \(O_x0-y0z0\) can be expressed by a series of multiplications \[^{10}\].

The real laser focus position \(P_{\text{real}}\) in \(O_x0-y0z0\) is displaced as (26).

\[
P_{\text{real}} = \left[ \prod_{j=0}^{n} T_{E^{i(5)}(5)j(5)\text{real}}} \right] = T_{01\text{real}}T_{12\text{real}}T_{23\text{real}}T_{34\text{real}}T_{45\text{real}}
\] (26)

The ideal laser focus position \(P_{\text{ideal}}\) in \(O_x0-y0z0\) is displaced as (27).

\[
P_{\text{ideal}} = \left[ \prod_{j=0}^{n} T_{E^{i(5)}(5)j(5)p} \right] = T_{01p}T_{12p}T_{23p}T_{34p}T_{45p}
\] (27)

Then the error of the system is presented as:

\[
E = P_{\text{ideal}} - P_{\text{real}}
\] (28)
4. SIMULATION AND MEASUREMENT OF THE POSITION ERROR

When the tailored blank laser welding machine is measured with Leica Laser Tracker (Fig. 8), we are able to record the total linear positioning error. Fig. 9 are the linear errors in $X$, $Y$ and $Z$ axes directions measured by Laser Tracker and built by using error model in section 2 with the parameters in Table 3. The position errors obtained by the error modeling method agree well with the measurement, except that the magnitudes of the errors are not exactly the same due to the fact of thermal error, measuring error and so on.

Table 3 Motion parameters of each axis

<table>
<thead>
<tr>
<th>Axis</th>
<th>X (mm)</th>
<th>Y (mm)</th>
<th>Z (mm)</th>
<th>X'(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start point</td>
<td>3</td>
<td>47</td>
<td>20</td>
<td>350</td>
</tr>
<tr>
<td>End point</td>
<td>2</td>
<td>19.4</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

Fig. 9 Error comparison between measurement and simulation

5. CONCLUSION

This paper develops a method of error modeling for tailored blank laser welding machine. The error model of the tailored blank laser welding machine is built based on this method. The obtained conclusion can be summarized as below:

1) The application of multi-body system theories is applied in error modeling of tailored blank laser welding machine.
2) The development of position error associated function which can reflect the influence of each error origin on the positioning error of the machine tool can improve the precision of the error model.
3) The specific expression of the position error associated function can be determined by theory or empirical formula, finite element analysis and measurement.
4) The method introduced in this paper is proved to be effective according to the comparison between the measurement using Laser Tracker and simulation by the error model.

REFERENCES