Abstract — Omnidirectional depth recovery in a novel omnidirectional stereo vision system was addressed. The stereo vision based on a common perspective camera coupled with two hyperbolic mirrors. Given two images of a same point acquired the system, the depth is calculated by means of triangulation. The calculation process aims at having corresponding incident rays intersecting at the same 3D point. As the hyperbolic mirrors ensure a single viewpoint (SVP) the incident light rays are easily found from the points of the image. The two hyperbolic mirrors are aligned coaxially and separately, share one focus that coincides with the camera center. So the geometry of the system naturally ensures matched epipolar lines in the two images of the scene. The separation between the two mirrors provides a large baseline and eventually leads to a precise result. The geometrical property are derived and the experiments with real images are presented.

Index Terms - Omnidirectional stereo vision, catadioptric sensor, depth recovery

I. INTRODUCTION

Range imaging has a wide variety of applications including computer aided design, data acquisition for automated design, and manufacturing of customized orthopedic devices for the handicapped. Various strategies have been proposed for 3D range imaging. Some of the techniques include: projection of light patterns on objects, photo counting, and synchronized scanner. At the same time stereo vision has the significant advantages that it achieves high resolution and simultaneous acquisition of the entire range image without energy emission for mobile robot navigation compared to other navigation sensors such as acoustics, radar, or scanning lasers.

But the conventional cameras are limited in their fields of view (FOV). An effective way to enhance the FOV is to construct an omnidirectional vision system using mirrors in conjunction with perspective cameras. These systems normally referred as catadioptric have been applied to robot localization and navigation by several researchers [1], [2], [3], [4], [5]. A common constraint upon the omnidirectional sensors modeling requires that all the imaged rays pass through a unique point called single viewpoint (SVP) [13]. The reason is that a single viewpoint is so desirable that it is a requirement for the generation of pure perspective images from the sensed images. These perspective images can subsequently be processed using the vast array of techniques developed in the field of computational vision that assume perspective projection. The mirrors popularly used to construct wide FOV catadioptric are hyperbolic mirrors or parabolic mirrors. But the latter must be coupled with telecentric optics which makes them restrictive for certain applications in panoramic vision.

Obtaining range information using binocular omnidirectional stereo vision has been reported in [6], [7], [8], [9]. Such two-camera stereo systems can be sorted into horizontal stereo systems and vertical stereo systems according to their cameras’ configuration. In [9], the cameras are configured horizontally and the baseline of triangulation is in the horizontal plane. This configuration results in two problems: One is that the epipolar line becomes curved line which causes the computational cost increases; the other is that the accuracy of the 3D measurement depends on the direction of a landmark [14]. In [6], [7], [8] , two omnidirectional cameras are vertically arranged. Such configuration escapes the shortcomings brought by horizontal stereo system, whereas the cables of power and data introduce occlusion to the images captured by this configuration. In addition, two-camera stereo systems are costly and complicated besides they have the problem which their cameras required precise positioning.

Single camera stereo has several advantages over two-camera stereo. Because only a single camera and digitizer are used, system parameters such as spectral response, gain, and offset are identical for the stereo pair. In addition, only a single set of internal calibration parameters needs to be determined. Perhaps the most important is that single camera stereo simplifies data acquisition by only requiring a single camera and digitzter and no hardware or software for synchronization [15]. Omnidirectional stereo based on a double lobed mirror and a single camera was developed in [10], [11], [12], in which a double lobed mirror is a coaxial mirror pair, the centers of both mirrors are collinear with the camera axis and the mirrors have a profile radially symmetric.
around this axis. This arrangement has the advantage to produce two panoramic views of the scene in a single image. Thus it is extremely compact and naturally ensures the alignment of the two images of the scene. But since the two mirrors are so close together, the disadvantage of this method is that it provides the relatively small baseline (not more than twenty millimetres).

In this work we demonstrate how depth can be estimated in an omnidirectional scene using two hyperbolic mirrors and a single camera in order to obtain omnidirectional stereo imagery.

II. MATHEMATICS OF THE SYSTEM

Fig 1 is a diagram of our stereo sensor. The two hyperbolic mirrors share one focus which coincides with the camera center and are separated by a baseline, bl. A hole in the mirror bellow permits imaging via the mirror above. We define the coordinate system so that the focal point of the camera, O, is at the origin (in 3-space), and the optical axis is pointing along the z-axis. Then the hyperbolic mirrors can be represented in this coordinate as equation (1)

\[
\frac{(z-c)^2}{a^2} - \frac{(x^2 + y^2)}{b^2} = 1 \quad c = c_a, c_b
\]  

The catadioptric sensors have the advantage of wide angle of view. But the images acquired by such system have a relatively low spatial resolution. Especially for those objects which are more than a few meters away, their images captured by catadioptric sensors are severely distorted. In order to obtain images of a relatively good quality, we set our system which has a map range of 5 meters when mounted 750mm high above ground. Designing the two hyperbolic mirrors according to this specification, we get the parameters of the mirrors.

In Fig 2 points \(F_a=(0, 0, 2c_a)\) and \(F_b=(0, 0, 2c_b)\) are the focus of the mirror above and mirror below respectively. The incident rays pass the point in space \(P=(x, y, z)\) reach the mirrors at points \(M_a\) and \(M_b\) and project onto the image at points \(P_a=(u_a, v_a, -f)\) and \(P_b=(u_b, v_b, -f)\) respectively. As the coordinates of \(P_a\) and \(P_b\) are known, the rays \(O M_a, O M_b\) which intersect the mirrors at \(M_a\) and \(M_b\) respectively, can be represent by (2) and (3).

\[
\begin{align*}
\frac{x}{u_a} &= \frac{y}{v_a} = \frac{z}{(-f)} \\
\frac{x}{u_b} &= \frac{y}{v_b} = \frac{z}{(-f)}
\end{align*}
\]

As point \(M_a\) satisfies the equation of mirror above and the equation of line \(O M_a\), point \(M_b\) satisfies the equation of mirror below and the equation of line \(O M_b\), the coordinates of \(M_a=(x_a, y_a, z_a)\) and \(M_b=(x_b, y_b, z_b)\) can be solved subsequently by equation group (3) and (4).

\[
\begin{align*}
\begin{cases}
\frac{x}{u_a} &= \frac{y}{v_a} = \frac{z}{(-f)} \\
\frac{(z-c_a)^2}{a_a^2} - \frac{(x^2 + y^2)}{b_a^2} &= 1
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
\frac{x}{u_b} &= \frac{y}{v_b} = \frac{z}{(-f)} \\
\frac{(z-c_b)^2}{a_b^2} - \frac{(x^2 + y^2)}{b_b^2} &= 1
\end{cases}
\end{align*}
\]

As the coordinates of \(M_a\) and \(M_b\) are solved, the equations of rays \(F_aP\) and \(F_bP\) are

\[
\begin{align*}
\begin{cases}
x &= y = \frac{(z-2c_a)}{z_a - 2c_a} \\
x_a &= y_a = \frac{(z_a - 2c_a)}{z_a - 2c_a} \\
x &= y = \frac{(z-2c_b)}{z_b - 2c_b}
\end{cases}
\end{align*}
\]

Having solved the equation group (6), we can get the coordinate of space point \(P\), and from x and y we can gain the range and orientation information of point \(P\).
III. EPIPOLAR CONSTRAINT

Conventional stereo vision uses two cameras to observe the environment, find the same object in each image, and measure range to the object by triangulation; that is, it works by intersecting the lines of sight from each camera to the object. To find the same object in each image, which is called stereo matching, is the fundamental computational task underlying stereo vision. A constraint is necessary to reduce and simplify the search for the matching features in the other pair, thus reducing the computational cost. One of the most used constraints is the epipolar geometry. The epipolar constraint determines that the matching feature is located in a determined geometric locus, thus decreasing the search dimension from two to one.

In the case of catadioptric images, one characteristic of them is that they are severely distorted. So it is difficult to establish the correspondence for such a pair unless some special treatments are used. But on the other hand, the omnidirectional images provide rich information about a scene and allow the matching to be made in a global sense which usually gives a better result considering a spatial relationship among the scene objects.

The arrangement of our omnidirectional stereo vision system naturally ensures the vertical alignment of the two hyperbolic mirrors and based on a single perspective camera, one image generates the two views of the scene, so the image process and matching both are done in a single image. Furthermore, the configuration of the system also ensures matched epipolar lines are straight and emit radially from the epipole to the boundary of the image (see Fig 3). Matched epipolar lines assure efficient searching methods can be used to perform stereo correspondence, thus, allowing fast and real-time 3D reconstruction.

Fig 3 Omnidirectional image captured by our system

IV. EXPERIMENTAL RESULTS

The catadioptric sensors have the advantage of wide angle of view. But the images acquired by such system have a relatively low spatial resolution. Especially where objects more than a few meters away their images captured by catadioptric sensors are severely distorted. For obtaining images with a relatively good quality, we set our system have a map range of 5 meters when mounted on a 750mm high mobile robot. Designing the two hyperbolic mirrors according to this specification, we get the parameters of the mirrors (See Table I).

An external calibration process is first undertaken to obtain the parameters of the system. Several experiments were conducted using different images. The results presented in TABLE II are four different points in an image (See fig 3, point A, B, C, D respectively).

V. CONCLUSION

In this work we presented a mathematical model of an omnidirectional stereo vision system using two hyperbolic mirrors and a single perspective camera and demonstrated its application for depth estimation. Experimental results shows the error in Z axis direction is smaller compared to X and Y axis directions. So if it installs on mobile robots for obstacles detection, if the mobile robots with the capability of conquering 300mm high obstacles this error is completely acceptable. Further more this stereo vision system can be used to give omnidirectional range measurements with the benefits of stereo vision need for only one image. Compared with the single-camera omnidirectional stereo vision systems previously reported, our stereo vision system has such significant advantages that its geometry calculates easily and fast, and it can simultaneous achieve precise range information without high cost or system complexity. And as the separation between the two hyperbolic mirrors provides a large baseline, the range information obtained from this method has much improved precision. This property and the perfect alignment of the images ensuring matched epipolar lines make this system suitable for real-time stereo calculation.

<table>
<thead>
<tr>
<th>Points</th>
<th>Distance</th>
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<th>Error</th>
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<td>y</td>
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<td>z</td>
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<td>x</td>
<td>210.0mm</td>
<td>206.0mm</td>
<td>4.0mm</td>
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REFERENCES