A Integrated Contract Strategy in a Three-Echelon Supply Chain with Capacity Limitation under the Forecast Update

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Abstract

In this paper, a model with one supplier, one manufacturer and one retailer is constructed to analyze the situation that a retailer makes orders twice before and after the forecast update from the supply chain coordination perspective. The manufacturer’s capacity is insufficient according to the second order of the retailer, so the manufacturer may recover the deficit by capacity outsourcing from the supplier at a certain option price at first and then supplying the very product back to the retailer by exercising the options. There are two decision points in this model. At the beginning of the planning horizon, the retailer decide how much to order for the first time, and the manufacturer knows her limited capacity and decides to purchase options from the supplier to assure the retailer’s demand to be completely satisfied. At the second decision point, with forecast update, the retailer makes his second order, and the manufacturer exercises the options to meet the retailer’s order. A numerical example is presented to illustrate the efficacy of the developed model in the end.

1. Introduction

Matching supply with demand is essential in achieving an effective supply chain. On the one hand, capacity limitation toward a manufacturer will create opportunities for other competitors, and therefore outsourcing is often adopted to complement demand when in-house capacity is insufficient [1]. On the other hand, the retailer prefers delaying the order to the beginning of the selling season so that he can update the demand information and obtain a relatively accurate forecast. On the contrary, the manufacturer prefers the retailer to place a firm order earlier in order to have enough time to produce. Due to the obvious conflict above, it is impossible to match supply and demand well. An initiative in this situation is to propose a flexible supply contract.

Various coordination mechanisms have been proposed to resolve the conflict between demand and supply [2-5]. Quantity flexibility, return policies, and options are the ones appeared in recent literatures. Cachon [6] provides a review of the growing literature on supply contracts. In this paper, we model a scenario where a retailer makes orders twice before and after the forecast update.

How to effectively use the forecast information to help the construction of the supply chain models is of great interest for both practitioners and researchers in the SCM field. A number of papers examine the impact of order timing on profit improvements in a supply chain established by a flexible contract [7-8]. Wang et al presents a flexible supply contract with call options model for an inflexible supply chain with forecast update.

However, unlike papers above, this paper studies a model composed of one supplier, one manufacturer and one retailer. The retailer places two orders before and after forecast update before the selling season. The manufacturer’s capacity is insufficient according to the second order, so the manufacturer may recover the deficit by advanced-purchasing options from the supplier (i.e. an original equipment manufacturer) at a low price and then exercising the options at a special price before the selling season to meet the retailer’s demand. This study reveals the coordination policy between each other of the three agents in the supply chain under certain circumstance that the manufacturer’s capacity is insufficient when the retailer renews the order quantity with forecast update.

The rest of the paper is organized as follows. We make assumptions, introduce the parameters used in the model, and formulate the supply chain agents’ profit functions in section 2. In section 3, we present
the result of the developed model based on options. In section 4, a numerical example is proposed to illustrate the efficacy of our model. Finally, we conclude our findings in this work and also present suggestions for future research in section 5.

2. Assumption and Notation

Consider a supply chain consisting of one supplier with option contract, one manufacturer with limited capacity and one retailer with forecast update. The retailer orders twice under the random demand of one product within one single period. In the development of the mathematical model for the problem, the following assumptions and notations (see Table 1) are considered.

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<th>Table1. Notations</th>
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Assumptions:

1) The wholesale price and retail price is exogenously determined [9];

2) There are two agents (i.e. the supplier and the retailer) may hold surplus, and any unsold products may be salvaged;

3) The manufacturer’s capacity and production cost are fixed, and needs not pay the fee for holding the product due to instantaneous interactions between two adjacent agents.

4) All agents in the supply chain share complete information under the risk-neutral hypothesis.

3. Model based on options

The basic business process in the supply chain proceeds as follows. 1) The retailer makes an order $Q^1$ for the first time with an advanced-purchase price $p_m^a$, and the manufacturer may procure $q^o$ options from supplier to guarantee her supply. The supplier commits to produce up to the quantity $q^o$ that all the options can be exercised. 2) After the forecast update, the retailer makes a second order $Q^2$ to improve his service level that reduces the possibility of stock-out. The manufacturer exercises her options $q^e$ according to her capacity absence. Then she delivers $Q^1 + Q^2$ to the retailer. The sequence of these events can be shown in Figure 1.

![Figure 1. The sequence of decisions for the options-based model](Image)
3.1. The retailer’s problem

The retailer makes decisions about $Q^1$ and $Q^2$. The retailer’s profit function is given by the following expression.

$$
\pi_r = \text{Gross revenue} - \text{Purchase cost} - \text{Ordering cost} + \text{salvage value} + \max \left\{ \left[ \left(Q^1 + Q^2\right) - D \right], 0 \right\}
$$

(1)

Depending on the value of the $X$ with the demand $D$, we can derive this situation in three cases.

Case 1: $x - m > Q^1 + Q^2$

In this case, the total order quantity $Q^1 + Q^2$ cannot fulfill the demand. The retailer’s expected profit can be written as

$$
\pi^2_r = \int_{x-m}^{x+m} \left[ p_r \left(Q^1 + Q^2\right) - k \left(\rho - \left(Q^1 + Q^2\right)\right) \right] f(\rho) d\rho - p_n^m Q^1 - p_n Q^2
$$

(2)

Case 2: $Q^1 + Q^2 > x + m$

In this case, the advanced-purchasing quantity is far more than enough to fulfill the demand, so the buyer is surplus. The expected profit of the retailer is

$$
\pi^2_r = \int_{x-m}^{x+m} \left[ p_r + v_r \left(Q^1 - \rho\right) \right] f(\rho) d\rho - p_n^m Q^1 - p_n Q^2
$$

(3)

Case 3: $Q^1 + Q^2 + m \geq x \geq Q^1 - m$

The situation is between the two cases above, where either surplus or shortage exists. Then, the expected profit function can be described as

$$
\pi^3_r = \int_{x-m}^{x+m} \left[ p_r \left(Q^1 + Q^2\right) - k \left(\rho - \left(Q^1 + Q^2\right)\right) \right] f(\rho) d\rho + \int_{Q^1-Q^2}^{Q^1+Q^2} \left[ p_r + v_r \left(\rho \right) \right] f(\rho) d\rho - p_n^m Q^1 - p_n Q^2
$$

(4)

To maximize her profit, the retailer solves the following dynamic optimization problem:

$$
\max \pi_r \left(Q^1, Q^2\right) = \text{E}_x \pi^{1,2,3}_r \left(Q^1, Q^2 / X\right) - p_n^m Q^1 - p_n Q^2
$$

s.t. $Q^1 \geq 0, Q^2 \geq 0$

(5)

3.2. The manufacturer’s problem

The manufacturer makes decisions at two decision points (i.e. A and B in Figure 1.). At A, the manufacturer decides how much options to procure from the supplier according to the retailer’s first order (i.e. $q^o$). At B, due to the second order from the retailer, the manufacturer decides how much options to be exercised in order to satisfy the retailer’s demand. We also make a division of this situation so as to analyze the manufacturer’s problem accurately.

Case 1: $x - m > Q^1 + Q^2$

In this case, the manufacturer may exercise the determinate options, therefore, her profit function is written as

$$
\pi^1_m = p_n^m Q^1 + p_n Q^2 - w \left(\frac{Q^1 + Q^2}{u} - u\right)
$$

(6)

Here, the notation $u$ represents the manufacturer’s constant capacity.

Case 2: $Q^1 > x + m$

In this case, no option would be exercised. Thus, the manufacturer’s profit function is showed as

$$
\pi^2_m = p_n^m Q^1 - w q^e
$$

(7)

Case 3: $Q^1 + Q^2 + m \geq x \geq Q^1 - m$

In this case, the manufacturer’s decision cannot be denoted exactly, due to the complexity of the situation. Therefore, we can use a numerical example to analyze the results later.

3.3. The supplier’s problem

In this section, we analyze the supplier’s behavior. Since the quantity of the options is relatively confirmed according to the manufacturer, the supplier’s obligation is the commitment of the production up to the procurement options set by manufacturer. Therefore, the profit function of the supplier can be marked as

$$
\pi^e_s = w q^e + w' q^e + v r \left(q^e - q^o\right)
$$

(8)

The calculation process is also according to the three cases. The optimal number of options to be exercised depends not only on the quantities (i.e. the retailer’s orders and the limited capacity of the manufacturer), but also on all the prices, especially the options exercising price $w^e$.

4. Numerical example

In this section, we use a numerical example to analyze the efficacy of the model presented above, which uses the uncoordinated model (i.e. Newsvendor Model) and the centralized model as the benchmark. The basic parameters can be set as follows: $p_r = 10$, $p_n = 5$, $c_m = 4$, $c_s = 3.5$, $v_r = 1$, $v_y = 2.5$, $u = 1200$, $\rho = 1000$, $m = 50$, $n = 500$. The results of the numerical analysis are presented in Table 2.

Table 2 presents the optimal solutions. While using the coordination mechanism, i.e., the options-based
model we designed, not only the whole chain’s profit, but also the profit of each agent in the supply chain is improved. It is suggested that the mechanism presented above is attractive and can also reduce the double marginalization between two agents in the supply chain.

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5. Conclusion

This paper develops an options-based model from the manufacturer’s perspective in a supply chain that consists of one supplier, one manufacturer, and one retailer with forecast update. The results of this study show that an option-based contract is effective in improving the whole chain’s efficiency, and coordinating with the supplier enhances the manufacturer’s capacity to meet the retail’s demand and the improvement of both his and system’s expected profit. The possibility of matching demand and supply can be improved via outsourcing with a flexible contract.

More research works such as considering the cooperation of different kinds of contracts, information asymmetry, multi-sourcing, and various products in complex supply chains will be carried on in the future. The uncertainty of the supply side can be considered in the future research. Furthermore, the influence of the various parameters on the decisions of each agent in the supply chain will be studied more deeply.

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References


