Contracts-based Research on Competition between Symmetric Products' Manufacturers

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Abstract—It is common for a retailer to sell products from competing manufacturers. How then should the firms manage their contract negotiations? Different from market structures researched before, contracts serve as a tool for competition as well as coordinating the whole supply chain in our paper. We study our models between competing manufacturers who sell symmetric products, and a common retailer facing stochastic demand. We allow the manufacturers to compete for the retailer’s business using one of two types of contracts, a wholesale-price contract, and a buyback contract. We find that some key conclusions of our settings are inconsistent in other market structures discussed before. We show that in our market structure the buyback contract forces the manufacturers to compete more aggressively than when they only offer wholesale-price contracts, and this may leave them worse off and the retailer substantially better off.

Keywords—game theory; supply chain management; buyback contract; competing manufacturers; symmetric products

I. INTRODUCTION

The literature on supply chain coordination has studied several contractual forms in settings with a single manufacturer and one or more retailers. One of the key results from this literature is that wholesale-price contracts lead to suboptimal decisions for the supply chain (i.e., double marginalization) and more sophisticated contracts (like buyback contracts) can be employed to achieve both channel coordination (i.e., maximize the supply chain’s profit) and rent extraction (i.e., the ability to allocate a high share of the profits to the manufacturer) that leaves the retailer with her reservation profit. Hence, a retailer may have just cause to fear buyback contracts. But in our settings, a retailer may actually prefer buyback contracts. Our objective in this paper is to test this conclusion in a setting in which multiple manufacturers compete to sell their symmetric products through a single retailer such as fig. 1.

In our model, the retailer faces stochastic demand. Two manufacturers simultaneously offer to the retailer one of two types of contracts: a wholesale-price contract, a buyback contract (i.e., a per unit payment the retailer for unsold products). The retailer sets her demands and prices to maximize her total profit given the offered contracts and her costs.

As researched in most papers, the retailer’s reservation profit is assumed to be an exogenous constant that reflects the retailer’s bargaining power - an increase in the retailer’s bargaining power is modeled by increasing the retailer’s reservation profit. However, in our structure with two manufacturers, the retailer’s reservation profit is endogenous - the profit the retailer can earn if it were to reject manufacturer A’s offer depends on what manufacturer B offers, and vice-versa. This distinction is significant and, as we demonstrate, important for our findings. Furthermore, competition between the manufacturers serves to raise the retailer’s reservation profits.

Figure 1. Typical supply chain composed of multiple competing manufacturers and a common retailer

The present paper is foremost a commentary on the supply chain coordination literature[1].addition to the coordination scheme by using buyback contract, it may achieve collaborative forecasts under asymmetric demand information[2]. Compared to the large body of research on vertical interactions (through supply contracts) among supply chain partners, horizontal competition has received only limited attention. Under stochastic demand, horizontal competition has been studied in single-product distribution networks [1] [3], multiple-product distribution networks [4], [5] [6] analyzed assembly networks with a deterministic, price-quantity linear relationship and no capacity constraints, respectively focusing on supply network design and coordination mechanisms with supply contracts. [7] [8] study systems with multiple manufacturers and a common retailer, but [7]only considers wholesale-price contract and [8] considers quantity discounts under deterministic demand.

The rest of the paper is organized as follows. Section II describes the model. Section III presents our analysis of the retailer’s problem, Section IV the analysis of the wholesale
and buyback games of the manufacturers’ problem, and Section VI comparison of the equilibrium under the wholesale and buyback contracts, Section VII concludes the paper.

II. THE MODEL

There are two products in the market supplied by two different manufacturers. The products are partial substitutes and are sold through a common retailer. In the first stage of the game, the manufacturers simultaneously announce the payment schemes for their products. In the second stage, the retailer chooses prices, which determine the products demand rates, to maximize her profit. In addition to the payments to the manufacturer, the retailer incurs operating costs. The manufacturers incur constant marginal production costs.

Suppose products’ market stochastic demand \( D \geq 0 \) with cumulative distribution function (c.d.f.) \( F(.) \), probability density function (p.d.f.) \( f(.) \), and increasing generalized failure rate (IGFR) distribution. \( F(x) \) is continuous, differentiable, monotonically increasing. \( F(x)=0 \), \( \bar{F}(x)=1-F(x) \). \( \mu = E(x) = \int_{-\infty}^{\infty} xf(x)dx \) is D’s mean value. Manufacturer \( i \) per-unit product cost \( c_i \), per-unit inventory cost \( c_e \), per-unit stock-out loss cost \( c_u \), suppose salvage value is zero. The retailer order \( O_i \) from manufacturer \( i \) and product \( i \) ’s expected sales \( S(O_i) = \min(O_i, D) = O_i - \int_0^{O_i} F(x)dx \), the retailer’s expected out of stock is \( L(O_i) = E(x-O_i)^+ = \mu - S(O_i) \), retail price is \( p_i(O) = \theta - \beta_i O_i - \gamma_i O_j \), and \( \beta_i > \gamma_i > 0 \) for all \( i, j \). So the revenue for product \( i \) is \( R_i(O_i) = p_i(O_i) O_i \). Let \( G_i(O_i) \) be the retailer’s operational costs of product \( i \),

\[
G_i(O_i) = c_e I_i(O_i) + c_u L(O_i). 
\]

Let \( \pi_i \) denote the retailer’s profit from product \( i \) and \( \pi = \pi_i + \pi_j \), the retailer’s total profit. It follows that \( \pi_i = R_i(O_i) - G_i(O_i) - T_i(O_i) \), and \( T_i(O_i) \) is the payment made to the manufacturer based on the retailer’s demand rate and their agreed upon contract. Manufacturer \( i \)’s profit is

\[
\prod_i = T_i(O_i) - c_i O_i. \tag{1}
\]

We consider two different types of contracts. With a wholesale-price contract, the payment function is \( T_i(O_i) = w_i O_i \), where \( w_i \) is the wholesale price chosen by manufacturer \( i \).

We include the following set of buyback contracts:

\[
T_i(O_i) = \begin{cases}
T(F^{-1}(w_i - c_i/b)) + c_i(Q - F^{-1}(w_i - c_i/b)) & \text{otherwise}
\end{cases}
\]

III. RETAILER’S DECISION WITH BUYBACK CONTRACTS

In the first stage of the game, the manufacturers simultaneously announce their contract offers. We assume that the particular contractual form offered is established before the game begins, but we later discuss what happens when the manufacturers choose their contractual form and parameters simultaneously. In the second stage, given functions \( T_1 \) and \( T_2 \), the retailer chooses the demand rates to maximize her profit.

Consider the symmetric products condition. It is equivalent to the retailer’s problem if the manufacturers charge only their production cost. Define \( \hat{\pi}_1 = \pi_1(O_i(0), 0) \), \( \hat{\pi}_2 = \pi_1(O_i(0), 0) \), and \( \hat{\pi}_1 = \pi_1(O_i(0), 0) \). When \( T_i(O_i) = c_i O_i \) for \( i = 1, 2 \). These profit levels are respectively the maximum profit the system earns, if it were to carry only product 1, only product 2 and both products. We assume \( \hat{\pi}_2 > \hat{\pi}_1 > 0 \) for \( i = 1, 2 \), which implies that it is always optimal for the system to carry both products under symmetric game.

Define

\[
\hat{\pi}_i(O_i, \hat{O}_i) = \arg \max \{ \pi_i(O_i, \hat{O}_i) \} \{ \hat{O}_i(O_i) \}
\]

Hence, the optimal solution to the retailer’s problem is \( (O^*_1, \hat{O}^*_2) = (O^*_1, \hat{O}^*_2) \).

Let \( H_i \) denote the first derivative of \( \pi \) with respect to \( O_i \). We have

Where \( w_i \) is a flexible parameter chosen by the manufacturer, \( c_i \) is the manufacturer’s product cost per unit, and \( b_i \) is an exogenous constant, where \( b_i \in [0, w_i] \). Wholesale-price contracts are a subset of our buyback contracts-a buyback price with \( b_i = 0 \) is a wholesale-price contract.

For expositional simplicity, we discuss symmetric products condition in which the symmetric products are provided by competing manufacturers. So a symmetric game across manufacturers means that the data for the two products are identical, i.e., \( c_i, \beta_i, \gamma_i, b_i \) are the same for any \( i \). The subscript \( i \) will be dropped in those cases. In a symmetric solution, the decisions \( (\hat{O}_i, \hat{O}_i) \) at the retail level, \( w_i \) or \( T_i \) at the manufacturer level ) are identical across products.

In the following sections, we solve the problem using backward induction. We analyze the retailer’s decision first and then the game between the manufacturers.
The retailer’s profit function $\pi$ is jointly concave in $(Q_i, Q_j)$, so the unique solution to $\{H_i=0, i=1,2\}$ is the unique optimal solution. The next theorem shows that there can be at most one interior local maximum and that the optimal solution is that interior solution. Furthermore, in a symmetric problem, the unique interior maximum is a symmetric solution.

**Theorem 1** In a symmetric problem, the retailer’s optimal solution $(Q^*_i, Q^*_j) = (\widetilde{Q}_i, \widetilde{Q}_j)$ is the unique interior optimal solution to $\{H_i=0, i=1,2\}$ and $(\widetilde{Q}_i, \widetilde{Q}_j) = (\widetilde{Q}, \widetilde{Q})$, where $\widetilde{Q}$ is the larger of the two solutions to

$$-(\beta + \gamma)S(Q) + \overline{F}(Q)[\theta - (\beta + \gamma)Q] - G'(Q) - T'(Q) = 0$$

**Proof.** The proof is by contradiction. Suppose that there are two interior local maxima $(x', y')$ and $(x'', y'')$. The line that connects $(x', y')$ and $(x'', y'')$ can be characterized by $(x + \alpha t, y + \delta t)$, where $\alpha = x' - x$, $\delta = y' - y$ and $\alpha, \delta \in \mathbb{R}, t \in [0,1]$ represents the line segment between the two point, and $t \in \mathbb{R}$ represents the whole line. Define $\Delta(t)$ as the value of $\pi$ on that line, we have

$$\Delta(t) = \pi(x + \alpha t, y + \delta t)$$

Let $\Delta'(t)$ be the derivative of $\Delta(t)$ with respect to $t$. Then, we have

$$\Delta'(t) = \frac{\partial^2 \pi}{\partial Q_i^2} \frac{\partial Q_i}{\partial t} + \frac{\partial^2 \pi}{\partial Q_j^2} \frac{\partial Q_j}{\partial t} = \alpha^2 \frac{\partial^2 \pi}{\partial Q_i^2} + \delta^2 \frac{\partial^2 \pi}{\partial Q_j^2} < 0$$

$\Delta(t)$ is concave in $t$.

Recall that $\Delta(t) = 0$ and $\Delta'(t) < 0$ at $t \in [0,1]$. Given that $\Delta(t)$ is continuous in $t$, this can only occur if there is at least one segment in $t \in [0,1]$ such that $\Delta'(t) > 0$. However, we have established that $\Delta'(t) < 0$. Hence, a contradiction.

For a symmetric problem, it follows from the above argument that the unique interior optimal solution is on the $Q_i = Q_j$ line. (If $(Q_i, Q_j)$ with $Q_i \neq Q_j$ is an interior optimal solution, then there are at least two local maxima, because $(Q_i, Q_j)$ is also an optimal solution by the symmetry of the profit function. This contradicts with the result above.)

IV. MANUFACTURERS’ DECISION WITH BUYBACK CONTRACTS

In this section, each manufacturer chooses its own best response $T_j(T_j)$ given the other manufacturer’s contract $T_j$.

$$H_i = -\beta S(Q_i) + \overline{F}(Q_i)(\theta - \beta Q_i - \gamma Q_j) - \gamma_i S(Q_i), \quad j \neq i$$

$\widetilde{G}'(Q) - T'(Q)$

The second derivation of

$$T_i(T_j) = \arg \max_{T_i} \prod_i (Q^*_i)$$

for all $i$, where $Q^*_i = \arg \max \pi$

An equilibrium of the game is a pair of contracts $(T^*_i, T^*_j)$ such that neither manufacturer has an incentive to offer a different contract. The following theorem characterizes the unique symmetric equilibrium of the contract offer game.

**Theorem 2** consider a symmetric game in which the manufacturers offer buyback contracts. There exist a unique solution to

$$\frac{\partial \prod_i Q^*_i}{\partial w_i}(w_i, w_j) + (w_i - b_i \overline{F}(Q_i) - c_i) \frac{\partial Q^*_i}{\partial w_i} = 0$$

for all $i$, denoted $(w^*_i, w^*_j)$, which is the unique candidate to be a symmetric equilibrium.

**Proof.** Recall that $b_i = 0$ when the manufacturers employ wholesale-price contracts. Furthermore, while evaluating the equilibrium, we need to only consider the quadratic part of the payment functions. Suppose by contradiction, that the retailer chose $Q_i$ such that $Q_i > F^{-1}(w_i - c_i/b_i)$. The manufacturer can increase $w_i$ such that $Q_i = F^{-1}(w_i - c_i/b_i)$ and increase its profit without affecting the retailer’s or the other manufacturer’s decisions.

We show in the proof that $\prod_i$ is concave in $w_i$ at the symmetric solution to the first order condition stated in the theorem. Hence, we obtain the first order condition for a manufacturer as follows.

$$\frac{\partial \prod_i w^*_i}{\partial w_i}(w_i, w_j) + (w_i - b_i \overline{F}(Q_i) - c_i) \frac{\partial Q^*_i}{\partial w_i} = 0$$

Applying the Implicit Function Theorem on the retailer’s first order conditions $\{Y_i = 0, \text{ for all } i\}$, we can derive the impact of the wholesale prices on the optimal demand. Define $A_i = 2\beta_i \overline{F}(Q_i) + (\theta - \beta_i Q_i - \gamma_i Q_j) \overline{F}'(Q_i) + \overline{G}'(Q_i) + \overline{T}'(Q_j)$, $B = [\gamma_j \overline{F}'(Q_j) + \gamma_j \overline{F}(Q_j)]$, and $\Delta = A_i A_j - B^2$. Because demand is IGFR distribution, we have $(\overline{f}(Q_i) / \overline{F}(Q_i)) > 0$ Hence, $A_i > (\overline{f}(Q_i) / \overline{F}(Q_i)) > 0$

Note that $A > B > 0$, and

$$\frac{\partial^2 \pi (Q_i, Q_j)}{\partial Q_i^2} = -(A + B) < 0.$$
\[ \Pi_i \text{ yields } 2 \frac{\partial Q'_i}{\partial w'_i} - b_i f(Q_i) \left( \frac{\partial Q'_i}{\partial w'_i} \right)^2 + (w_i - b_i F(Q_i) - c_i) \frac{\partial^2 Q'_i}{\partial w'_i^2} \]

Which is negative when \( Q'_i = Q^*_i \), because we have

\[ A'_i = A'_2 > 0 \text{ and } \frac{\partial^2 Q'_i}{\partial w'_i^2} = \Delta^{-3}[-A'_i A'_j + A'_i B^3] < 0 \]

Hence, the first order condition above has a unique solution when \( Q'_i = Q^*_2 \). The solution \((w^*_1, w^*_2)\) satisfies the first and second order conditions for both manufacturers. That is, \( w^*_i \) is a local maximum of \( \Pi_i \) for fixed \( w'_j \), and vice versa. If \( w_i > w^*_i \), then \( Q^*_i \) decreases and \( Q^*_2 \) increases, \( A'_i \) increases, \( A'_j \) increases, and \( A'_j \) decreases, implying that \( \frac{\partial^2 Q'_i}{\partial w'_i^2} \) remains negative. Hence, \( \Pi_i \) is concave in \( w_i \) for \( w_i > w^*_i \) for fixed \( w^*_j \), therefore, there can be no other local maximum greater than \( w^*_i \).

V. COMPARISON OF THE EQUILIBRIUM UNDER WHOLESALE PRICE AND BUYBACK CONTRACTS

This section presents numerical examples to compare the equilibrium solution when the manufacturers offer wholesale price and buyback contracts under symmetric products’ condition. We start with the following example: \( \theta = 20, \beta = 2, \gamma = 1, c = 2, c_c = 1, c_u = 1 \), demand is uniformly distributed between 0 and 10, and \( b_1 = b_2 = 3 \) when buyback contracts are offered. Table 1 provides the equilibrium results.

Recall that a manufacturer prefers to offer a buyback contract for any fixed contract offer by the other manufacturer. However, this preference does not carry over to the equilibrium analysis. Competition between the manufacturers is different when they both offer buyback contract than when they both offer wholesale-price contracts. Apparently, it can be a more aggressive type of competition. Note, the supply chain is better off with buyback contract relative to wholesale-price contracts even though the supply chain can be better off. In other words, although in a serial supply chain a retailer may have just cause to fear buyback contract, she may actually prefer the buyback contracts when offered by competing manufacturers.

While there are many research chances to study the problem of multiple competing manufacturers and a common retailer, for example under products with different substitutability level or asymmetric products condition. These interesting problems would be explored in the future research.

REFERENCES


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<td>38.23</td>
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</table>

TABLE 1. RESULTS FOR THE EQUILIBRIUM UNDER WHOLESALE AND BUYBACK CONTRACT

VI. CONCLUSION

Different from market structures researched before, contracts serve as a tool for competition tools rather than coordinating the whole supply chain in our paper. This paper is attempt to consider buyback contracts other than wholesale-price contracts in systems with multiple competing manufacturers and a single common retailer. We demonstrate that the manufacturers may be worse off with buyback contracts relative to wholesale-price contracts even though the supply chain can be better off. In other words, although in a serial supply chain a retailer may have just cause to fear buyback contract, she may actually prefer the buyback contracts when offered by competing manufacturers.

While there are many research chances to study the problem of multiple competing manufacturers and a common retailer, for example under products with different substitutability level or asymmetric products condition. These interesting problems would be explored in the future research.