Robust Optimization Model for Remanufacturing System in Uncertain Reverse Logistics Environment

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Abstract - The remanufacturing process of reusable parts in uncertain reverse logistics environment is considered. An existing framework for remanufacturing system is adopted. In this framework, the manufacturer has two alternatives for supplying parts: either ordering the required parts to external suppliers or overhauling returned products and bringing them back to ‘as new’ conditions. The numbers of returned products are uncertain and can be described as a scenario set with certain probability. By using the approach of robust optimization based on scenario analysis, a robust optimization model is constructed to maximize the total cost savings by optimally deciding the quantity of parts to be processed at each remanufacturing facilities, the number of purchased parts from subcontractor. The results of a numerical example show that the model we proposed is both solution-robust and model-robust.

Index Terms - Reverse logistics, Remanufacturing, Uncertainty, Robustness

I. INTRODUCTION

Reverse logistics can be defined as the logistics activities all the way from used products no longer required by the customer to products again usable in the market. Reverse logistics can be categorized various types according to the product recovery option. Ref. [1] suggested various product recovery options as direct reuse, resale, repair, refurbishing, remanufacturing, cannibalization, and recycling. Also, these options are to be reclassified into three broad categories such as reuse, recycling, and remanufacturing. In reuse, the returned product can be used more than once in the same form after cleaning or reprocessing, such as container, pallet, and bottle. On the other hand, recycling denotes material recovery without conserving any product structure, for example, metal, glass, paper, and plastic. Finally, remanufacturing, the subject of this study, is an industrial process in which worn-out products are restored to like-new condition, such as electronic machine, toner cartridge, and automobile part. An example is Hewlett-Packard, which collects an empty laser-printer cartridge from customers for using again (refer to Ref. [2]).

There are numerous researches on remanufacturing system which address many various topics from definition to practical cases in real industry. Many analytic and quantitative approaches are also found in various problems such as forecasting, production planning/control, inventory control/management, and location. For example, the impact of remanufacturing in economy is studied by Ref. [3], and more fundamentally, Ref. [4] provides arguments for why used products should be remanufactured. A good overview on quantitative models for recovery production planning and inventory control is given by Ref. [5]. Ref. [6] proposes an hybrid manufacturing/remanufacturing system with stocking points for serviceable and remanufacturable products. Ref. [7] proposes a general mixed-integer programming model and solution procedure for a reverse distribution problem focused on the strategic level. Moreover, Ref. [8] deals with remanufacturing execution at operational level. They propose a general framework for remanufacturing system in reverse logistics environment and a mathematical model to maximize the total cost savings by optimally deciding the quantity of parts to be processed at each remanufacturing facilities, the number of purchased parts from subcontractor.

In practice, the operation environment of remanufacturing system is uncertain. Uncertainties come from final products manufacturing, distributing, raw material supplying, used products collecting, reusing, recycling, remanufacturing, and so on. However, only few researches are found for development of a general framework and robust optimization model about remanufacturing system under uncertain environments. Therefore, in addition to the previous researches on the various specific areas of remanufacturing, our study focuses on developing a robust optimization model for remanufacturing system in uncertain reverse logistics environment.

Based on the framework of the remanufacturing system in Ref. [8], we propose a robust optimization model for this remanufacturing system with uncertain returned products, and design a numerical example to verify the validity of the model.

II. ROBUST OPERATING MODEL FOR REMANUFACTURING

A. Framework for Remanufacturing System

We consider the remanufacturing system in Ref. [8], it starts with returned products including end-of-life product from customers. Then, they are collected to the collection facilities, the returned products are disassembled to remanufacturing and the rests, beyond the remanufacturing...
capacity, are sent to the remanufacturing subcontractor. The cleaned products from the collection site are disassembled in the disassembly site. Disassembled parts are classified into the reusable parts and non-reusable parts. The former go to refurbishing site for repairing and cleaning, the latter dealing with wastes go to the disposal site to landfill or incinerate. After refurbishing process, ‘as new’ parts are stocked as part inventory together with new parts from the external supplier and remanufacturing subcontractor. Finally parts in inventory are supplied to the manufacturing plants according to the company’s own production plan. The framework for this remanufacturing system is shown in the shade region of Fig. 1.

Different from the remanufacturing system in Ref. [8], the number of returned products we considered here are uncertain and can be described as a scenario set with certain probability. In the framework as shown in Fig. 1, manufacturing company has two alternatives for supplying parts: either ordering the required parts to external suppliers or overhauling the returned products and bringing those back to ‘as new’ conditions. The company is interested in minimizing total remanufacturing cost so that eventually it can maximize total profit. To achieve the goal, while meeting part demands from manufacturing plants, the company should determine how many returned products should be thrown into the remanufacturing process such as refurbishing and disassembling for ‘as new’ condition, and how many new parts need to be purchased from the external supplier. The decision under uncertain reverse logistics environment can be optimally made using the robust optimization model in the following section.

![Fig. 1 Conceptual framework for remanufacturing system – the shaded region](image)

### B. Notations

#### Indices

- \( p \) Product index, \( p = \{1, \ldots, P\} \)
- \( i \) Part index, \( i = \{1, \ldots, I\} \)
- \( t \) Time period, \( t = \{1, \ldots, T\}, (T : \text{planning horizon}) \)

#### Parameters

- \( \mathbf{R}_i \) The required quantity of part \( i \) at time \( t \)
- \( \mathbf{CP}_p \) The collected quantity of product \( p \) at time \( t \)
- \( \mathbf{BOM}_p \) The number of part \( i \) from disassembling one unit of product \( p \)
- \( \mathbf{CSC}_p \) The capacity of process in the collection site
- \( \mathbf{RC}_p \) The capacity of the refurbishing site
- \( \mathbf{DC}_p \) The capacity of the disassembly site
- \( \mathbf{VP}_i \) The volume occupied by one unit of part \( i \)
- \( \mathbf{PIC}_p \) The inventory capacity of the part inventory
- \( \lambda_i \) The upper bound of disposal rate for disassembled part \( i \)
- \( \mathbf{CPIC}_p \) The unit inventory holding cost of collected product \( p \) in the collection site
- \( \mathbf{DspC}_i \) The disposal cost of disassembled part \( i \)
- \( \mathbf{DSUC}_p \) The set-up cost for disassembling collected product \( p \)
- \( \mathbf{DSVC}_p \) The unit operation cost for disassembling collected product \( p \)
- \( \mathbf{OUTC}_p \) The subcontract cost for product \( p \)
- \( \mathbf{RSC}_i \) The set-up cost for refurbishing disassembled part \( i \)
- \( \mathbf{RVC}_i \) The unit operation cost of refurbishing disassembled part \( i \)
- \( \mathbf{PIC}_i \) The unit inventory holding cost of part \( i \) in the part inventory
- \( \mathbf{PPC}_i \) The unit purchasing cost of part \( i \) from supplier at time \( t \)
- \( \mathbf{DIC}_t \) The idle cost of the disassembly facility
- \( \mathbf{RIDC}_t \) The idle cost of the refurbishing facility

#### Decision variables

- \( \mathbf{DP}_p \) The number of disassembled product \( p \) at time \( t \)
- \( \mathbf{RPart}_i \) The number of refurbished part \( i \) at time \( t \)
- \( \mathbf{WPart}_i \) The number of disposed part \( i \) at time \( t \), \( \mathbf{RPart}_i = \mathbf{WPart}_i - \mathbf{DP}_p \)
- \( \mathbf{CPI}_p \) The inventory level of product \( p \) at time \( t \)
- \( \mathbf{DPart}_i \) The number of disassembled part \( i \) at time \( t \), \( \mathbf{DPart}_i = \sum_{p=1}^{P} \mathbf{BOM}_p \cdot \mathbf{DP}_p \)
- \( \mathbf{PI}_i \) The inventory level of part \( i \) at time \( t \)
- \( \mathbf{PPart}_i \) The number of purchased part \( i \) at time \( t \)
- \( \mathbf{RSU}_i \) The binary variable for set-up of refurbishing part \( i \) at time \( t \)
- \( \mathbf{DSU}_p \) The binary variable for set-up of disassembly product \( p \) at time \( t \)
- \( \mathbf{OUT}_p \) The number of outsourcing product \( p \) at time \( t \)
such that 1

The number of part $i$ from subcontractor,

$$OPart_i = \sum_{p=1}^{P} BOM_{pi} \ast OUT_{pi}$$

C. The Model

In the remanufacturing system shown in Fig.1, the numbers of returned products from customers are uncertain. Let $PS = \{1, 2, \cdots, S\}$ be a set of future possible scenarios for these uncertain returned products, each with a probability of occurrence $p_s$ such that $\sum_{s=1}^{S} p_s = 1$. Let $CP_s^t$ denote the number of collected product $p$ at time $t$ in scenario $s \in PS$.

Let $OUT_{pi}^t$ be the control variables that can be adjusted once the uncertain number of returned products are observed, and other decision variables are design variables whose optimal values are independent of any realizations of the uncertain number of returned products. Thus, for any scenario $s \in PS$, the number of outsourcing product $p$ at time $t$ is dependent on different scenario $s \in PS$, denoted by $OUT_{ps}^t$. Moreover, let $\epsilon_{ps}^t$ be deficient outsourcing product $p$ at time $t$ in scenario $s \in PS$. In addition, we introduce a parameter $\omega_{ps}$, it denotes cost saving loss with unit outsourcing product $p$ decrease.

According to above indices and parameters, the mathematical formulation for this problem can be stated as follows.

Objective

The objective is to maximize the cost saving from remanufacturing process. The objective function of operating model can be expressed as follows.

$$\max Z = \sum_{i=1}^{I} \sum_{t=1}^{T} PPI_{pi} \ast R_{it}$$

$$- \sum_{s=1}^{S} p_s \left[ \sum_{t=1}^{T} \left( OUT_{pi}^t \ast OUT_{pi}^t + \omega_{ps} \ast \epsilon_{ps}^t \right) \right]$$

$$- \sum_{s=1}^{S} p_s \left( \sum_{i=1}^{I} \left( CPI_{pi} \ast NI_{pi}^t + DSUC_{pi} \ast DSU_{pi}^t + DSVC_{pi} \ast DP_{pi}^t \right) \right)$$

$$- \sum_{i=1}^{I} \left( DspC_{i} \ast WP_{ia} + RSC_{i} \ast RSU_{ia}^t + RVC_{i} \ast RP_{ia}^t \right)$$

$$- \sum_{i=1}^{I} \left( IPCI_{i} \ast PI_{ia} + PPIC_{i} \ast PP_{ia} \right)$$

$$- \sum_{i=1}^{I} \left( DSU_{pi} \ast DI\DCl - \sum_{i=1}^{I} \left( (1 - RSU_{ia}^t) \ast RICD \right) \right)$$

It is measured by the gap between the purchasing cost for all parts from the external supplier and the remanufacturing process cost for returned products and their parts. The remanufacturing process cost includes not only set-up and operation cost but also idle cost at each remanufacturing facilities. This means that our model tries to maximize both the cost saving from remanufacturing process and the utilization of remanufacturing facilities at the same time. Restricted conditions

The number of parts by disassembling products at the collection site.

$$DPart_{ia} = \sum_{p=1}^{P} BOM_{pi} \ast DP_{pi}, \forall i, t$$

The number of parts by disassembling products at the remanufacturing subcontractor.

$$OPart_{ia} = \sum_{p=1}^{P} BOM_{pi} \ast OUT_{pi}^t, \forall i, t, s$$

The balance equations for the product and part inventory.

$$CP_{ps}^t + CPI_{ps}^t = OUT_{ps}^t + DP_{ps} + CPI_{ps}^t + \epsilon_{ps}^t, \forall p, t, s$$

$$RPart_{ia} + WP_{ia} = DPart_{ia}, \forall i, t$$

$$RPart_{ia} + OPart_{ia} + PI_{i,t-1} = R_f + PI_i, \forall i, t$$

The inventory quantities of part cannot exceed the predetermined capacity of the part inventory ($PICP$).

$$\sum_{i=1}^{I} WP_{ia} \ast PI_i \leq PICP, \forall t$$

The number of products at the collection site cannot exceed its capacity ($CSP_{pf}$).

$$OUT_{pf}^t + DP_{pf} \leq CSP_{pf}, \forall p, t, s$$

The disassembly quantity cannot exceed the capacity of the disassembly site ($DCP_p$).

$$DP_{pt} \leq DCP_p, \forall p, t$$

The refurbishing quantity cannot exceed the capacity of the refurbishing site ($RPC_p$).

$$RPart_{pt} \leq RPC_p, \forall i, t$$

Set-up constraints for setup at the refurbishing site and the disassembly site.

$$RPart_{ia} \leq M \ast RSU_{ia}, \forall i, t$$

$$DP_{pt} \leq M \ast DSU_{pt}, \forall p, t$$

Where, $M$ is a very large predetermined value.

The manufacturing companies cannot exceed $\lambda_i$ of disassembled parts to dispose.

$$\sum_{i=1}^{I} WP_{ia} \leq \lambda_i \ast \sum_{i=1}^{I} DPart_{ia}, \forall i$$

Where, $\lambda_i$ is a predetermined value.

Nonnegative conditions

$$OUT_{ps}^t, \epsilon_{ps}^t, DP_{ps}^t, CPI_{ps}^t \geq 0, \forall p, t, s$$

$$RPart_{ia}, OPart_{ia}, WP_{ia}, PI_{ia} \geq 0, \forall i, t$$

$$RSU_{ia} \in \{0, 1\}, \forall i, t, DSU_{pt} \in \{0, 1\}, \forall p, t$$

III. SIMULATION ANALYSES OF NUMERICAL EXAMPLES

Using a numerical example, we will illustrate how the model works in the proposed framework and the robustness of the proposed model. Some data come from Ref. [8]. We assume that there are three types of products ($P = 3$), five types of parts from those products ($I = 5$), 10 time-periods for planning horizon ($T = 10$), and four scenarios ($S = 4$).
Table I shows the amount of used products that are collected from customers to the collection site at each period in scenario \( s \in PS \) (\( CP_{ps} \)) and the occurrence probability of each scenario.

Table II shows the part requirements of manufacturing plants according to their own production plan during ten time periods (\( R_{ps} \)). The quantity of parts requirement is generated as many as 1.5 times of the quantity of returned products so that external supplier may supply the shortage of remanufacturing quantity.

Table III indicates a ‘bill-of-material (BOM)’ of each product by which parts are assembled to a product (\( BOM_{pi} \)).
parts at refurbished site. It is easily noticed that the refurbishing capacity is nearly fully utilized for parts 2, 3, and 5. This is because the corresponding demands are relatively higher than that of other parts, as shown in Table II. The shortage of part requirements is purchased from the external supplier as seen in Table VI. Note that there is no purchase for parts 1 and 4 at period 1. The numbers of parts from subcontractor are shown in Table VII. The results of other decision variables are not shown here.

According to the data listed in Table I to Table III and others we given above, the cost saving and the difference to uncertain case for different collect products scenario are calculated. The results are shown in Table VIII.

As the results shown, the cost saving of the remanufacturing system with uncertain returned products is the least one among different cases. Compared with the cost saving in scenario 1, 2, 3 and 4, the cost saving of the remanufacturing system with uncertain returned products only decreased 5.79%, 7.73%, 6.11% and 7.47% respectively. That is, the optimal cost saving of the model under uncertain returned products remains ‘close’ to optimal for any realization of the scenario set. Moreover, the optimal solution of robust model is solved according to all returned products scenarios, so this optimal solution is conservative for all the scenarios. When one of the scenarios is realized, the optimal decision variables solved by the robust model do not necessarily equal to the case when the numbers of collect products are certain and may possibly inferior to this optimal solution. However, this optimal solution of robust model is one of the feasible solutions when this returned products scenario is realized. Therefore, we can conclude that the model we proposed in this paper is both solution-robust and model-robust.

### Table IV
**The Number of Products to Go to the Subcontractor for Remanufacturing Outsourcing**

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<th>t=1</th>
<th>t=2</th>
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<td>0</td>
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<td>Product 2</td>
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<td>122</td>
<td>52</td>
<td>62</td>
<td>62</td>
<td>132</td>
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<td>72</td>
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<td>Product 3</td>
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<td>50</td>
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<td>30</td>
<td>20</td>
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### Table V
**The Number of Refurbished Parts from Remanufacturing Operations**

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<td>Part 3</td>
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<td>1000</td>
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<tr>
<td>Part 4</td>
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<td>840</td>
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<tr>
<td>Part 5</td>
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### Table VI
**The Number of Parts Purchased from External Supplier**

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### Table VII
**The Number of Parts from Subcontractor**

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<td>100</td>
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<td>760</td>
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<td>400</td>
<td>720</td>
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<td>360</td>
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<td>100</td>
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### Table VIII
**The Cost Saving and the Difference to Uncertain Case for Different Cases**

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<tr>
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<th>Scenario 1</th>
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<th>Scenario 3</th>
<th>Scenario 4</th>
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<td>1534940</td>
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<tr>
<td>Difference to uncertain case (%)</td>
<td>5.79</td>
<td>7.73</td>
<td>6.11</td>
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### IV. CONCLUSIONS

This paper discussed a notion of remanufacturing system in uncertain reverse logistics environment. Based on the general framework in view of supply planning in Ref. [8], we developed a robust optimization model to optimize the supply planning function. The model determines the quantity of products/parts processed in the remanufacturing facilities/subcontractors and the amount of parts purchased from the external suppliers while the number of returned products are uncertain, the goal of the model is to maximize the total remanufacturing cost saving. We presented a
numerical example, the results show that the model we proposed is both solution-robust and model-robust.

Our research results can be guidelines on the relevant research. The proposed model can be applicable to the various industries after customizing for specific industries in uncertain reverse logistics environment. However, as the proposed model is introduced in terms of a general framework, many future works are accordingly needed. Above all, the framework with remanufacturing can be effectively enhanced by adopting more industry practices and so the robust optimization model does.

REFERENCES


