Robust Operation strategy design for an Electronic Market Enabled Supply Chain with Uncertain Variable Costs

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Abstract—An operation model is proposed in this paper for an electronic market enabled supply chain with uncertain variable costs. The supply chain consists of multi-supplier and multi-customer, suppliers can provide many kinds of product to different customers directly or through electronic market. Uncertain variable costs are described as symmetrical and limited intervals. A robust operation model for this supply chain is constructed by using the robust optimization method based on interval analysis. The operation model is a multi-objective programming problem satisfying several conflict objectives, such as meeting the demands of all customers, minimizing the total system cost, the availabilities of suppliers’ capacities not below a certain level, and robustness of decision to uncertain variable costs. The result of a numerical example proved that the solution of the model is the most conservative. However, it can ensure the robustness of the operation of the supply chain effectively.

Keywords—Electronic markets; Supply chain management; Variable cost; Uncertainty

I. INTRODUCTION

Recently, research on electronic markets has generally focused on the benefits of electronic markets from transaction cost economic and strategic perspectives. For example, economic-based research has examined the role of intermediaries [1], and compared neutral to biased e-markets [2]. However, very little academic research has been conducted on how actually operate an electronic market. Yet, clearly in order to maintain the continued vitality of electronic markets, it will be necessary to operate in a cost-effective manner. Recently, Keskinocak and Tayur (2001) modeled a supply chain in an electronic market environment consisting of suppliers and customers and an intermediary who matches customer orders to availability as a large-scale linear integer program [3]. Carlton and Judy (2004) extended this model, it provided a model of an electronic market linking customers and suppliers either directly or via an intermediary [4]. However, uncertain environments were not considered in all above studies.

In fact, customer requirement (demand), cost, quality, due time, priority and so on are uncertain in actual supply chain. Therefore, problems how to express these uncertainties and how these uncertainties impact the strategies of supply chain operation must be considered in the studies of supply chain operations management.

As for the expression of uncertainties, three distinct methods are frequently mentioned [5]: first, the distribution-based approach, where the normal distribution with specified mean and standard deviation is widely invoked for modeling uncertain demands and/or parameters; second, the fuzzy-based approach, wherein the forecast parameters are considered as fuzzy numbers with accompanied membership functions; third, the scenario-based approach, in which several discrete scenarios with associated probability levels are used to describe expected occurrence of particular outcomes. Moreover, Dimitris and Melvyn (2004) describe the uncertainty as symmetrical and limited intervals, and propose a robust formulation that is linear, is able to withstand parameter uncertainty without excessively affecting the objective function [13].

In this paper, we considered the operation of a supply chain consisting of multi-suppliers and multi-customers with uncertain variable costs under electronic market. Based on Ref. [13], we describe uncertain variable costs as symmetrical and limited intervals, and proposed an operation model for this supply chain by using the robust optimization method based on interval analysis approach.

II. THE OPERATION MODEL

A. Notations

There are many products, suppliers and customers in the supply chain. We denote them with index \(i (i = 1, 2, \cdots, I)\), \(j (j = 1, 2, \cdots, J)\) and \(k (k = 1, 2, \cdots, K)\) respectively. Other parameters, decision variables and their connotations are listed blow.

Parameters: \(w_{ik}\) - variable cost to supply one unit of product \(i\) directly to customer \(k\) from supplier \(j\); \(wms_{ij}\) - variable cost per unit to supply product \(i\) from supplier \(j\) to electronic market; \(wmc_{ik}\) - variable cost per unit to supply product \(i\) from electronic market to customer \(k\); \(s_i\) - setup cost for product \(i\) at supplier \(j\); \(f_{jk}\) - fixed cost of satisfying (part of) customer \(k\) ’s demand from supplier \(j\); \(fms_{ij}\) - fixed cost of supplier \(j\) dealing with the electronic market; \(fmc_{ik}\) - fixed cost of electronic market dealing with customer \(k\); \(C_{ij}\) - capacity (number of units) for

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product \( i \) at supplier \( j \); \( T_j \) - total capacity available at supplier \( j \); \( c_j \) - unit capacity used to make one unit of product \( i \) at supplier \( j \); \( d_i \) - demand for product \( i \) at customer \( k \); \( P_{ik} \) - penalty per unit of unsatisfied demand for product \( i \) at customer \( k \); \( EMC \) - capacity of the electronic market (EM).

Decision variables: \( x_{ijk} \) - quantity of product \( i \) supplied directly by supplier \( j \) to customer \( k \); \( x_{msk} \) - quantity of product \( i \) sent from the electronic market to customer \( k \); \( x_{msj} \) - quantity of product \( i \) sent from supplier \( j \) to the electronic market; \( u_{ik} \) - quantity of unsatisfied demand for product \( i \) at customer \( k \).

\[
\begin{align*}
  z_{jk} &= \begin{cases} 
    1, & \text{if part of customer } k \text{'s demand is satisfied from supplier } j \text{ directly} \\
    0, & \text{otherwise.}
  \end{cases} \\
  z_{mc_k} &= \begin{cases} 
    1, & \text{if the electronic market sends any product to customer } k \\
    0, & \text{otherwise}
  \end{cases} \\
  y_{ij} &= \begin{cases} 
    1, & \text{if supplier } j \text{ produces product } i \\
    0, & \text{otherwise}
  \end{cases} \\
  z_{ms_j} &= \begin{cases} 
    1, & \text{if supplier } j \text{ sends any product to the electronic market; } \\
    0, & \text{otherwise}
  \end{cases}
\end{align*}
\]

**B. Model**

In the process of this supply chain operation, we consider 3 objectives described as follows.

Objective 1: to meet demands of all customers completely.

\[
\begin{align*}
  \min \ & P_f \times (d^-_i + d^+_i) \\
  \text{s.t.} \ & (TD - \sum_k u_{ik}) / TD + d^-_i - d^+_i = 100\% 
\end{align*}
\] (1)

Where, \( TD \) is the total demand of all customers, \( P_f \) is a preferential factor, it is a large enough constant; \( d^-_i \) and \( d^+_i \) are slack and surplus of the objective.

Objective 2: to minimize the total system cost.

\[
\begin{align*}
  \min \ & P_C \times d^+_i \\
  \text{s.t.} \ & C + d^-_i - d^+_i = TC \\
  & \sum_i \sum_k w_{ijk} x_{ijk} + \sum_j \sum_k wms_{kj} x_{msj} \\
  & + \sum_j \sum_k wms_{kj} x_{msj} + \sum_j s_{ij} y_{ij} \\
  & + \sum_k f_{jk} z_{jk} + \sum_j fms_{ij} z_{msj} \\
  & + \sum_k fms_{ij} z_{msj} + \sum_k p_{ik} u_{ik} - C \leq 0
\end{align*}
\] (2)

Where, \( P_C \) is a preferential factor, it is a large enough constant; \( TC \) is the expected cost of the system, it is a constant also; \( d^-_i \) and \( d^+_i \) are slack and surplus of the objective.

In practice, variable costs \( w_{ijk} \), \( wms_{kj} \) and \( wmc_{ik} \) are uncertainty. Here, we suppose variable costs’ uncertainties can be denoted as symmetrical and limited intervals \([w_{ijk} - \hat{w}_{ijk}, w_{ijk} + \hat{w}_{ijk}]\) , \([wms_{kj} - \hat{wms}_{kj}, wms_{kj} + \hat{wms}_{kj}]\) and \([wmc_{ik} - \hat{wmc}_{ik}, wmc_{ik} + \hat{wmc}_{ik}]\) . Where, \( \hat{w}_{ijk} \), \( \hat{wms}_{kj} \) and \( \hat{wmc}_{ik} \) are maximum fluctuations of standard variable cost \( w_{ijk} \), \( wms_{kj} \) and \( wmc_{ik} \) respectively.

Because of \( w_{ijk} \)’s uncertainty, (3) would be disturbed by \( I \times J \times K \) parameters. This uncertain disturbance can be expressed as:

\[
\begin{align*}
  \max \ & \sum_i \sum_j \sum_k \hat{w}_{ijk} x_{ijk} \\
  & - x_{ijk} \leq x_{ijk} \leq x_{ijk}' \\
  & x_{ijk}' \geq 0, \ \forall i, j, k
\end{align*}
\]

For the uncertainty of \( wms_{kj} \), (3) would be disturbed by \( I \times J \) parameters. We describe this uncertain disturbance as follows.

\[
\begin{align*}
  \max \ & \sum_i \sum_j \sum_k \hat{wms}_{kj} y_{ij} \\
  & - x_{msj} \leq x_{msj} \leq x_{msj}' \\
  & x_{msj}' \geq 0, \ \forall i, j
\end{align*}
\]

Similarly, for the uncertainty of \( wmc_{ik} \), (3) would be disturbed by \( I \times K \) parameters. This uncertain disturbance can be described as:

\[
\begin{align*}
  \max \ & \sum_i \sum_k \sum_k \hat{wmc}_{ik} z_{jk} \\
  & - x_{msj} \leq x_{msj} \leq x_{msj}' \\
  & x_{msj}' \geq 0, \ \forall i, k
\end{align*}
\]

Through linearizing these non-linear disturbances by using the robust optimization method based on interval analysis, (3) is equivalent to constraints as following.

\[
\begin{align*}
  \sum_i \sum_j \sum_k w_{ijk} x_{ijk} + (I \times J \times K) x + \sum_i \sum_j \sum_k \xi_{ijk} \\
  & + \sum_i \sum_j \sum_k wms_{kj} y_{ij} + (I \times J) y + \sum_i \sum_j \sum_k \xi_{ij} \\
  & + \sum_i \sum_j \sum_k wmc_{ik} z_{jk} + (I \times K) z + \sum_i \sum_j \sum_k \xi_{ik} \\
  & + \sum_i \sum_j \sum_j s_{ij} y_{ij} + \sum_i \sum_k \sum_j f_{jk} z_{jk} + \sum_i \sum_j \sum_j fms_{ij} z_{msj} \\
  & + \sum_k \sum_j \sum_k fms_{ij} z_{msj} + \sum_i \sum_k \sum_k p_{ik} u_{ik} - C \leq 0 \\
  \end{align*}
\] (4)

\[
\begin{align*}
  & x + \xi_{ijk} \geq \hat{w}_{ijk} x_{ijk}', \ \forall i, j, k \\
  & y + \xi_{ij} \geq \hat{wms}_{kj} y_{ij}', \ \forall i, j
\end{align*}
\] (5)

\[
\begin{align*}
  & x + \xi_{ik} \geq \hat{wmc}_{ik} z_{jk}', \ \forall i, k
\end{align*}
\] (6)
\[ z + \zeta_{ik} \geq c_{ik}x_{mc_{ij}}, \quad \forall i, k \]  
(7)

\[-x_{ij} \leq x_{ij} \leq \epsilon, \quad \forall i, j, k \]  
(8)

\[-xms_{ij} \leq xms_{ij} \leq xms_{ij}, \quad \forall i, j \]  
(9)

\[-xmc_{ij} \leq xmc_{ij} \leq xmc_{ij}, \quad \forall i, k \]  
(10)

Where, variables \( x, y, z, \epsilon, \zeta_{ij}, \zeta_{ik} \) are all nonnegative.

Objective 3: to ensure the availabilities of suppliers’ capacities not below a certain level \( \alpha, 0 \leq \alpha \leq 1 \).

\[
\min P_p \times \sum d_j
\]
(11)

Where, \( P_p \) is a preferential factor, it is a large enough constant; \( d_j^+ \) and \( d_j^- \) are slack and surplus of the objective.

To integrate all the 3 objectives above, we can rewrite the operation model for the supply chain as follows:

Objective function:

\[
\min P_p \times (d_j^+ + d_j^-) + P_p \times d_j^+ + P_p \times \sum d_j
\]

Constraint conditions:

Besides constraints (1), (2), (4) to (11), matching suppliers to customers are constrained by available supplier and electronic market capacity, as well as customer demand for each product. The specific constraints are listed below.

\[
\sum x_{ij} + xms_{ij} - y_jC_j \leq 0, \quad \forall i, j
\]  
(12)

\[
\sum \sum c_{ij}x_{ij} + \sum c_{ij}xms_{ij} \leq T_j, \quad \forall j
\]  
(13)

\[
x_{ij} - \zeta_{ij}d_{ik} \leq 0, \quad \forall i, j, k
\]  
(14)

\[
xms_{ij} \leq \sum d_j, \quad \forall i, j
\]  
(15)

\[
xmc_{ik} \leq \sum d_{ik}, \quad \forall i, k
\]  
(16)

\[
\sum x_{ik} + xmc_{ik} + u_k = d_{ik}, \quad \forall i, k
\]  
(17)

\[
\sum \sum xms_{ij} = \sum xmc_{ik}, \quad \forall i
\]  
(18)

\[
\sum \sum xms_{ij} \leq EMC
\]  
(19)

Nonnegative constraints:

\[ x_{ij}, xms_{ij}, xmc_{ik}, u_k, x, y, \epsilon, \zeta_{ij}, \zeta_{ik}, \zeta_{ik} \geq 0 \]  
(20)

\[ y_j, z_{ij}, \epsilon, \zeta_{ij}, \zeta_{ik} \geq 0 \]  
(21)

Now, we design a numerical example to verify the robustness of the model.

### III. A NUMERICAL EXAMPLE AND RESULT ANALYSIS

The supply chain we considered consists of 3 suppliers and 4 customers, each supplier produces 2 products, the standard variable costs of \( w_{ij}, wms_{ij}, wmc_{ik} \) are listed in Table I to Table III. The capacity of electronic market is 500, and demands of all products are shown in Table IV, other parameters come from Ref. [14].

<table>
<thead>
<tr>
<th>Table I. Standard Variable Cost per Unit: ( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Supplier</strong></td>
</tr>
<tr>
<td><strong>Product 1</strong></td>
</tr>
<tr>
<td><strong>Product 2</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II. Standard Variable Cost per Unit: ( wms )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Supplier 1</strong></td>
</tr>
<tr>
<td><strong>Supplier 2</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table III. Standard Variable Cost per Unit: ( wmc )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Customer</strong></td>
</tr>
<tr>
<td><strong>Product 1</strong></td>
</tr>
<tr>
<td><strong>Product 2</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table IV. Demands of Products: ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Customer</strong></td>
</tr>
<tr>
<td><strong>Product 1</strong></td>
</tr>
<tr>
<td><strong>Product 2</strong></td>
</tr>
</tbody>
</table>

Let the expected total cost of the system is 0, and the availabilities of suppliers’ capacities are all 80%. We used successive multi-objective programming algorithm and the software LINGO9.0 for solving our mixed-integer programming model on a PC with Pentium IV 2.20GHZ.

Case I: All variable costs are certain. the total system cost is 29906. The operation strategy of the supply chain is shown in Figure 1.

![Figure 1. The optimal operation strategy as all variable costs are certain](image)

The results of calculation show, \( d_j^+ = d_j^- = 0 \). From (1), we can find that all customers demand are satisfied, namely, the objective 1 can be realized.

\( C=29906 \) and \( d_j^+ = 29906 \), from (2), we can find that the total system cost is 29906, so the objective 2 cannot be realized. The total system cost exceeds the expected cost (zero in this example) by 29906. The reason is that we suppose the expected system cost is zero (i.e., \( TC=0 \)) in our numerical example for calculation convenience, which cause \( d_j^+ = 29906 \).

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\( d_{ij}^* = 1500, d_{ik}^* = 3460 \) and \( d_{ik}^* = 1440 \), from (4), we can find that supplier 3 can realize the 3rd operation objective, but the consumed capacity of supplier 1 and 2 is lower than the expected goal. The margin is 1500 and 3460, respectively.

Case II: All variable costs maximum fluctuation ratio are equal to 5% of standard variable cost. That is, \( w_i = 5\% w_{pi} \), \( s_i = 5\% wms \) and \( c_{a} = 5\% wmc_a \). The total system cost is 31339.75. The operation strategy is same as that of Case I.

Case III: All variable costs maximum fluctuation ratio are equal to 10% of standard variable cost. The total system cost is 32773.5, all the optimal values of decision variables are same as that of Case I.

When maximum variable costs fluctuation ratio are equal to 1%, 2%, ..., 10%, We resolved the model, the total system costs and the differences to certain case are list in Table V. The optimal operation strategies are all unchangeable when the maximum fluctuation ratio of variable costs doesn’t exceed 10%.

**TABLE V. OPTIMAL TOTAL SYSTEM COSTS AND THE DIFFERENCE TO CERTAIN CASE**

<table>
<thead>
<tr>
<th>Maximum fluctuation ratios</th>
<th>Optimal total system costs</th>
<th>Difference to certain case (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>30192.75</td>
<td>0.96</td>
</tr>
<tr>
<td>2%</td>
<td>30479.5</td>
<td>1.92</td>
</tr>
<tr>
<td>3%</td>
<td>30766.25</td>
<td>2.88</td>
</tr>
<tr>
<td>4%</td>
<td>31053</td>
<td>3.84</td>
</tr>
<tr>
<td>5%</td>
<td>31339.75</td>
<td>4.79</td>
</tr>
<tr>
<td>6%</td>
<td>31626.5</td>
<td>5.75</td>
</tr>
<tr>
<td>7%</td>
<td>31913.25</td>
<td>6.71</td>
</tr>
<tr>
<td>8%</td>
<td>32200</td>
<td>7.67</td>
</tr>
<tr>
<td>9%</td>
<td>32486.75</td>
<td>8.63</td>
</tr>
<tr>
<td>10%</td>
<td>32773.5</td>
<td>9.59</td>
</tr>
</tbody>
</table>

From the above, we can conclude that, when the fluctuation of variable costs doesn’t exceed a certain range, the operational strategies of supply chain are all unchangeable, the uncertainties of variable costs have no impacts on the operational performance of supply chain. Namely, under variable costs uncertainty, our model is robust.

IV. CONCLUSION

In this paper, we considered the operation of an multi-suppliers and multi-customers supply chain with uncertain variable costs under electronic market, and proposed a multi-objective operation model for this supply chain. The result of a numerical example shows that the model we provided is robust to uncertain variable costs. The modeling method we suggested is meaningful for modeling the operation of electronic market enabled supply chain with uncertainty.

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