Adaptive Robust Control Techniques Applied to the Yaw Control of a Small-scale Helicopter

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Abstract —This paper presents a new robust controller design approach to the yaw control of a small-scale helicopter mounted on an experimental platform. The yaw dynamic system is linearized into a linear system, which is modelled by an affine uncertainty model. We proposed a novel robust H∞ feedback controller with adaptive mechanisms for the linear system with guaranteed control performances. The feedback gains are obtained by the solutions of a series of linear matrix inequalities (LMIs). The designed approach reduces conservatism inherent in robust control with a fixed gain controller and improves performances in time-response. Numerical simulations illustrate the theoretical results.

Index Terms — Helicopter; Robust control; Adaptive control; Uncertainty; H∞ control

I. INTRODUCTION

Rotorcraft-based unmanned aerial vehicles (RUAVs), are indispensable for many places where human intervention is considered difficult or dangerous. A helicopter can operate in different flight modes, such as vertical take-off/landing, hovering, longitudinal/lateral flight, pirouette, and bank-to-turn. Due to their versatility in maneuverability, helicopters are capable to fly in restricted areas and to hover efficiently for long periods of time. These characteristics make helicopters invaluable for terrain surveying, surveillance and clean-up of hazardous waste sites.

The design for flight control system of helicopter has been dominated by linear control techniques. The last decade has witnessed remarkable progress in RUAV research including modelling [2] and control theory [3-5], which are based on an accurate and fixed model. However, the complicated dynamics of helicopter lead to both parametric and dynamic uncertainty. Unmeasurable states, sensor and actuator noise, saturation, bandwidth limitations, friction and delays, all of these may make the resulting closed-loop system out of the region of stability, so the controller should be designed to be robust for those effects.

Robust control theory allows the design of control systems based on a simple low-order plant model with well-defined uncertainty bounds that account for model simplifications, non-linearity, and variations in operating conditions. Furthermore, approximate low-order plant models are more easily identified from flight test data and results in less complexity.

Recently, a considerable amount of work has been done to design robust controllers for linear system with parameter uncertainty. Since an adequate level of performance is required in practice, recent literatures have focused on quadratic stabilizing control with some performance such as LQR, H∞ or H2 [6-10] disturbance attenuation and closed-loop pole location based on LMI or other methods. A design technique well suited to the control of helicopters is the technique of H∞ control. Being an inherently multivariable technique and also being able to provide robust stability for systems subjected to uncertainty make H∞ control an ideal candidate. A number of simulation studies and flight test [6-9] have investigated robust control methods on rotorcraft using H∞ techniques. The effective method which can deal with H∞ problem is the linear matrix inequality (LMI) method [11-12].

While a single controller with a fixed gain is considered, the resulting controllers designed by any method above inherently become conservative. On the other hand, adaptive control [13] theories have been greatly developed as controller design methodologies for system with uncertainties. The typical adaptive control scheme is the parameter adaptive control, in which unknown parameters are estimated explicitly, and control parameters are determined based on these estimates. However, even in the so-called “ideal case”, a stable adaptive controller can’t guarantee good transient response.

It is worthwhile considering incorporate some kind of adaptation mechanism with robust control methods. In this paper, we deal with the yaw control problem of a small-scale helicopter mounted on an experimental platform. The helicopter yaw dynamic can be modelled by an affine uncertainty model. For the general affine uncertainty linear model, a simpler robust controller based on LMI method and indirect adaptive method is given, which aims to reduce conservatism inherent in fixed gain and to improve transient behaviour even in existence of disturbance. Then we apply the controller to the yaw control of small-scale helicopter. Both linear and nonlinear simulations of yaw dynamic model are performed, and the effectiveness of the proposed method is demonstrated.

II. YAW DYNAMIC OF SMALL-SCALE HELICOPTER

A. Modelling yaw dynamics
where blade speed

by 

respectively;

where, the yaw dynamics has the form:

\[
\begin{align*}
\dot{\phi} &= r \\
I_{zz} \dot{r} &= N_{mr} + N_{tr} + N_{fs} + N_{hs} + N_{vf} + N_{trf}
\end{align*}
\]  
(1)

where \(\phi\) and \(r\) are the yaw angle and angular velocity of the helicopter respectively; \(I_{zz}\) is the inertia around z-axis; \(N\) denotes the torque acted on the helicopter; the subscripts of \(mr, tr, fis, hs\) and \(vf\) denote respectively, main rotor, tail rotor, fuselage, horizontal and vertical fin.

In hovering and low-velocity flight, the torque generated by main rotor and force generated by tail rotor are dominant. By simplifying the fuselage and vertical fin damping, the yaw dynamics can be rewritten as:

\[
\begin{align*}
\dot{\phi} &= r \\
I_{zz} \dot{r} &= -Q_{mr} + T_{pr} + b_{r} r + b_{\phi} \phi
\end{align*}
\]  
(2)

where \(Q_{mr}\) is the torque of main rotor, \(T_{pr}\) is the thrust of tail rotor, \(b_{r}\) is the distance between the tail rotor and z-axis, \(b_{\phi}\) and \(b_{r}\) are damping constants.

\[
T_{pr} = \frac{1}{2} \rho a_{mr} b_{mr} c_{mr} \Omega_{mr}^2 \int_{r_{mr}}^{r} \left( \Omega_{mr} r_{mr}^2 - \frac{v_{mr}^2}{\Omega_{mr}^2} \right) dr_{mr}
\]  
(3)

\[
v_{mr} = \sqrt{\frac{T_{pr}}{2 \rho A_{mr}}}
\]  
(4)

where \(\rho, a_{mr}, b_{mr}, c_{mr}, \Omega_{mr}, \theta_{mr}, r_{mr}, v_{mr}, A_{mr}\) are respectively, density of air, slope of the lift curve, number of rotor, chord of the blade, speed of the tail rotor, pitch angle, radial distance, induced speed of the tail rotor and area of the tail rotor disc.

Combining (3) with (4), we have

\[
T_{pr} = \frac{1}{2} \rho a_{mr} b_{mr} c_{mr} \Omega_{mr}^2 \int_{r_{mr}}^{r} \left( \Omega_{mr} r_{mr}^2 - \frac{v_{mr}^2}{\Omega_{mr}^2} \right) dr_{mr}
\]

\[
= C_1 \theta_{mr} + \frac{1}{2} C_2 (C_2 + \sqrt{C_2^2 + 4C_1^2} \theta_{mr})
\]

with

\[
C_1 = \frac{1}{\rho} \frac{r_{mr}^2}{2 \pi} \pi R_{mr}^2 (R_{mr}^2 - R_{mr0}^2)
\]

\[
C_2 = \frac{1}{8} \rho a_{mr} b_{mr} c_{mr} \Omega_{mr}^2 \sqrt{2/\rho \pi R_{mr}^2 (R_{mr}^2 - R_{mr0}^2)}
\]

Similarly, the force of the main rotor is:

\[
T_{mr} = C_3 \theta_{mr} + \frac{1}{2} C_4 (C_4 + \sqrt{C_4^2 + 4C_3^2} \theta_{mr})
\]

where,

\[
C_3 = \frac{1}{8} \rho a_{mr} c_{mr} \Omega_{mr}^2 (R^2 - R_{mr0}^2)
\]

\[
C_4 = \frac{1}{8} \rho a_{mr} c_{mr} \sqrt{2/\rho \pi R^2 (R^2 - R_{mr0}^2)}
\]

The torque generated by main rotor is:

\[
Q_{mr} = \int_{r_{mr}}^{r} \left( \rho \Omega_{mr}^2 C_3 c_{mr} \phi + \frac{\rho \Omega_{mr}^2 C_4 c_{mr}}{2} \right) dr_{mr}
\]

where \(\phi = v_{r}/(\Omega_{mr})\), \(C_1 = a_{mr} c_{mr} \theta_{mr} + a_{mr} \theta_{mr} + a_{mr} c_{mr} \theta_{mr}^2\),

where \(a, \alpha, c, \phi, v_{r}, \Omega\) are respectively slope of the lift curve, the angle of attack of the blade element, speed radial distance, chord of the blade, inflow angle, induced speed and rotor speed of the main rotor.

After integral manipulation with the help of Maple, we obtain

\[
Q_{mr} = C_2 \rho c_{mr} \Omega_{mr}^3 (R^2 - R_{mr0}^2) \theta_{mr} + \frac{\rho \Omega_{mr}^2 C_4 c_{mr}}{2}
\]

\[
+ \left[ 8C_{mr} + \frac{\pi R^2}{2} (2C_3 \theta_{mr} + C_2 + C_2 \theta_{mr} + 4C_1 \theta_{mr}) (R_{mr0}^2 - R^2) \right] C_4 \rho c_{mr} \Omega_{mr}^2
\]

\[
- 4a_{mr} \sqrt{2/\rho \pi R^2 (R_{mr0}^2 - R^2)} + 6C_2 \rho c_{mr} \Omega_{mr}^2 (R_{mr0}^2 - R^2)
\]

\[
+ 6C_2 \rho c_{mr} \Omega_{mr}^2 (R_{mr0}^2 - R^2) + 3a_{mr} C_2 (R_{mr0}^2 - R^2)
\]

\[
+ 6C_2 \rho c_{mr} \Omega_{mr}^2 (R_{mr0}^2 - R^2) + 3a_{mr} C_2 (R_{mr0}^2 - R^2)
\]

where \(R, \theta_{mr} \) are respectively, radial and pitch angle of main rotor.

**B. Simplified model**

From (2) we can see that there exist couplings between main rotor torque \(Q_{mr}\) and tail rotor thrust \(T_{pr}\). And (5) and (6) further demonstrate that the models are highly nonlinear and too complex to be used for control design. Instead of the dynamics described by (5) and (6), a simplified model is proposed for control design.

By plotting the torque vs pitch angle, we can find that relation between \(Q_{mr}\) and \(\theta_{mr}\) approximated with quadratic polynomial

\[
Q_{mr} = k_{\phi} \theta_{mr}^2 + k_{\theta} \theta_{mr} + k_{\psi}
\]

where \(k_{\phi}, k_{\theta}, k_{\psi}\) depend on the shape of the blades and the speed of main rotor \(\Omega\), while \(\theta_{mr}\) are constant. So, \(k_{\phi}, k_{\theta}, k_{\psi}\) are constants.

Similarly, the lift of tail rotor, \(T_{pr}\), can be written:

\[
T_{pr} = k_{\theta} \theta_{mr}^2 + k_{\theta} \theta_{mr} + k_{\psi}
\]

Then we can obtain the following nonlinear model:

\[
\begin{align*}
\dot{\phi} &= r \\
I_{zz} \dot{r} &= -k_{\phi} \theta_{mr}^2 + k_{\theta} \theta_{mr} + k_{\psi} \quad (7)
\end{align*}
\]

The nonlinear dynamic can be presented by a state space description:
\[
\dot{x} = f(x,u) + \zeta
\]

with \( \zeta = \eta(Q_{\omega'}) + \omega \),

where, \( \eta(Q_{\omega'}) \) is the disturbance due to main rotor, \( \omega \) is the uncertainty due to wind and so on, \( x = [\Phi \quad r]^{T}, u = \theta_{pr} \).

The nonlinear system can be linearized in a trim point \( (x_{0}, u_{0}) \) by calculating the Jacobian matrix which presents the full derivative matrix of \( F \) at \( x \) and \( u \):

\[
\dot{x} = Ax + Bu + \zeta
\]

with

\[
A = \frac{\partial f}{\partial x} \bigg|_{x_{0},u_{0}} = \begin{bmatrix} 0 & 1 \\ a_{1} & a_{2} \end{bmatrix}, \quad B = \frac{\partial f}{\partial u} \bigg|_{x_{0},u_{0}} = \begin{bmatrix} 0 \end{bmatrix}
\]

where, \( a_{1} = b_{2}I_{2}^{-1}, a_{2} = b_{1}I_{2}^{-1} \) and

\( a_{3} = 2k_{2}I_{2}^{-1}t_{pr} + k_{1}I_{2}^{-1} + b_{2}\Omega \).

### III ADAPTIVE ROBUST CONTROL DESIGN

In this section, we propose the control method which is general for a linear time-invariant affine uncertainty model described by

\[
\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + B_{e}(\theta)\omega(t)
\]

\[
y(t) = C_{i}x(t)
\]

where \( x(t) \in R^{n} \) is the state, \( u(t) \in R^{m} \) is the control input, \( y(t) \in R^{p} \) is the measured output and \( \omega(t) \in R^{l} \) is an exogenous disturbance which belongs to \( L_{2}[0,\infty) \), respectively. Here \( \theta = (\theta_{1}, \ldots, \theta_{N}) \in R^{N} \) is the vector of uncertain parameters, and

\[
A(\theta) = A_{0} + \sum_{i=1}^{N} \theta_{i}A_{i}, \quad B(\theta) = B_{0} + \sum_{i=1}^{N} \theta_{i}B_{i}
\]

\[
B_{e}(\theta) = B_{e0} + \sum_{i=1}^{N} \theta_{i}B_{e,i}
\]

where \( A_{0}, A_{1}, \ldots, A_{N}, B_{0}, B_{1}, \ldots, B_{N}, B_{e0}, B_{e1}, \ldots, B_{eN} \) are known constant matrices. We also assume that lower and upper bounds are known for the parameters, i.e. \( \theta_{i} \in [\theta_{i}, \bar{\theta}_{i}] \).

Considering the lower and upper bounds \( \{\theta_{i}, \bar{\theta}_{i}\} \), the following set can be defined

\[
\Omega = \{\theta = (\theta_{1}, \ldots, \theta_{N}) : \theta_{i} \in [\theta_{i}, \bar{\theta}_{i}]\}
\]

The control design considered in this paper is to find an algorithm such that:

1. The closed-loop system is stable for all \( \theta_{i} \in [\theta_{i}, \bar{\theta}_{i}] \).
2. The output \( y(t) \) tracks the reference signal \( r_{d}(t) \) with zero steady-state error, that is \( \lim_{t \to \infty} e(t) = 0 \)

where \( e(t) = r_{d}(t) - y(t) \), and with a known performance bound for the resulting closed-loop system which will be defined later.

It is well known that an integral control can effectively eliminate the steady tracking error. In order to obtain a robust tracking controller with state feedback plus tracking error integral, we introduce the following augmented state-space description

\[
\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t) + \bar{B}_{e}(\theta)\omega(t)
\]

\[
y(t) = C_{i}x(t)
\]

where

\[
\bar{A}(\theta) = \begin{bmatrix} 0 & -C_{i} \\ 0 & A(\theta) \end{bmatrix}, \quad \bar{B}(\theta) = \begin{bmatrix} 0 \\ B(\theta) \end{bmatrix}, \quad \bar{B}_{e}(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & B_{e}(\theta) \end{bmatrix}
\]

Choose the controlled output \( z(t) \in R^{q} \), defined by

\[
z(t) = Cx(t) + Du(t)
\]

where \( C \) and \( D \) are constant weighting matrices which can be adjusted to achieve satisfactory response. For the flight tracking controller design problem, the \( L_{2} \) gain will be used to characterize the closed-loop performance. Thus the design problem can be reduce to the following: Find a state feedback controller \( u(\theta) \) such that

1. The augmented closed-loop system (10) is robust stable for all \( \theta_{i} \in [\theta_{i}, \bar{\theta}_{i}] \).
2. The augmented closed-loop system (10) admits a given \( L_{2} \) gain bound \( \gamma \) from the signal \( \omega_{2}(t) \) to the regulated output \( z(t) \), that is

\[
\int_{0}^{T} z^{T}(t)z(t)dt \leq \gamma^{2} \int_{0}^{T} \omega^{T}_{2}(t)\omega(t)dt \quad \text{for all } T \geq 0
\]

and for all \( \theta_{i} \in [\theta_{i}, \bar{\theta}_{i}] \).

Here, the controller is designed as:

\[
u(t) = K(\hat{\theta})\bar{x}(t) = (K_{0} + K^{'}(\hat{\theta}))\bar{x}(t)
\]

where \( \hat{\theta}(t) = (\hat{\theta}_{1}(t), \hat{\theta}_{2}(t), \ldots, \hat{\theta}_{N}(t)), \hat{\theta}_{i}(t) \) is the estimate of \( \theta_{i} \), \( K_{0} \) is a fixed gain while \( K^{'}(\hat{\theta}(t)) = \sum_{i=1}^{N} K_{i}^{'}(\hat{\theta}_{i}(t)) \) is the parameter-dependent gain. In the following, we will analyze that such a control may bring more robustness and less sensitivity to disturbance.
Denote \( \Omega_2 = \{ \hat{\theta} = (\hat{\theta}_0, \ldots, \hat{\theta}_N) : \hat{\theta}_i \in [\theta_i, \bar{\theta}_i] \} \)

The closed-loop dynamics

\[
\begin{bmatrix}
\dot{x}(t) = A(\hat{\theta})x(t) + B(\hat{\theta})K(\hat{\theta})x(t) + \overline{B}_w(\hat{\theta})w(t) \\
z(t) = Cx(t) + DK(\hat{\theta})x(t)
\end{bmatrix}
\]

(13)

stability can be demonstrated, and the following result is stated.

**Theorem:** The augmented closed-loop system (13) is robust stable and \( H_\infty \) disturbance attenuation is less or equal to \( \gamma \), if there exist matrices \( X, Y_0, Y_i, i=1, \ldots, N \), such that for any \( \Theta \in \Omega_1, \hat{\Theta} \in \Omega_2 \), the following linear matrix inequalities hold:

\[
\begin{bmatrix}
M_i + M_i^T - \overline{B}_w^T(\hat{\Theta}) (CX + DY(\hat{\Theta}))^T \\
\gamma^2 I & 0 \\
0 & -I
\end{bmatrix} < 0
\]

(14)

where, * denotes the symmetric part,

\[
Y(\Theta) = Y_0 + \sum_{i=1}^{N} Y_i \hat{\theta}_i
\]

\[
M_i = \overline{A}(\Theta)X + \overline{B}(\Theta)Y_0 + \overline{B}_w \sum_{i=1}^{N} Y_i \hat{\theta}_i
\]

and also \( \hat{\Theta}_i \) is determined according to the adaptive law

\[
\hat{\Theta}_i(t) = \text{Proj}_{[\theta_i, \bar{\theta}_i]}\{L_i\}
\]

(16)

\[
= \begin{cases}
0 & \text{if } \hat{\theta}_i = \theta_i \text{ and } L_i \leq 0 \\
\text{or } \hat{\theta}_i = \bar{\theta}_i \text{ and } L_i \geq 0 \\
L_i & \text{otherwise}
\end{cases}
\]

where, \( L_i = -l_i \overline{X}(t)P \overline{B}_w K_i \overline{x}(t), \quad l_i \geq 0, i=1, \ldots, N \).

\text{Proj}_{[\cdot, \cdot]}\{13\} \text{ denote the projection operator whose role is to project the estimates } \hat{\Theta}_i(t) \text{ to the interval } [\theta_i, \bar{\theta}_i]. Then the controller

\[
u(t) = (Y_0X^{-1} + \sum_{i=1}^{N} Y_iX^{-1}\hat{\theta}_i)\overline{x}(t)
\]

**Proof:**
We choose the following candidate Lyapunov function

\[
V = \overline{x}(t)P \overline{x}(t) + \sum_{i=1}^{N} \hat{\Theta}_i(t) / l_i
\]

(17)

where \( \hat{\Theta}_i(t) = \hat{\Theta}_i(t) - \theta_i \). Then from the derivative of \( V \) along the closed-loop system (13), we can get

\[
\dot{V}(t) + z^T(t)z(t) - \gamma^2 \omega^2(t)\omega(t)
\]

\[
\leq \overline{x}^T(t)(P[A(\Theta) + \overline{B}(\Theta)K_0 + \sum_{i=1}^{N} K_i \hat{\theta}_i + \sum_{i=1}^{N} \overline{B}_w^T(\hat{\Theta})^T] \overline{x}(t) + (C + DK(\hat{\Theta}))^T(C + DK(\hat{\Theta})) + \frac{1}{\gamma^2} \overline{P} \overline{B}_w^T(\Theta) \overline{B}_w^T(\Theta) \overline{x}(t)
\]

(18)

Thus

\[
\dot{V}(t) + z^T(t)z(t) - \gamma^2 \omega^2(t)\omega(t) \leq \overline{x}^T(t)W\overline{x}(t)
\]

(19)

where

\[
W = M + M^T + \frac{1}{\gamma^2} \overline{P} \overline{B}_w^T(\Theta) \overline{B}_w^T(\Theta) P
\]

(20)

with

\[
M = P\overline{A}(\Theta) + \overline{B}(\Theta)K_0 + \overline{B}_w \sum_{i=1}^{N} K_i \hat{\theta}_i + \sum_{i=1}^{N} \overline{B}_w \sum_{i=1}^{N} K_i \hat{\theta}_i
\]

(21)

Now the design condition that \( \dot{V}(t) + z^T(t)z(t) - \gamma^2 \omega^2(t)\omega(t) \leq 0 \) is reduced to \( W < 0 \).

Let \( X = P^{-1}Y_0 = K_0X, Y_i = K_iX \). By pre-and post-multiplying inequality \( W < 0 \) by \( X \), the resulting inequality is

\[
W_i = M_i + M_i^T + (CX + DY(\hat{\Theta}))^T(CX + DY(\hat{\Theta}))
\]

(22)

\[
+ \frac{1}{\gamma^2} \overline{B}_w^T(\Theta) \overline{B}_w^T(\Theta) < 0
\]

By Schur complement, we know \( W_i < 0 \) is equivalent to (14) and (15) for any \( \Theta, \hat{\Theta} \in [\theta_i, \bar{\theta}_i] \). Due to the fact that (14) depends affinely on \( \Theta, \hat{\Theta} \) (\( \Theta \) and \( \hat{\Theta} \) are independent) and based on classical LMI theory [12], it is well known that if for any \( \hat{\Theta} \in \Omega_2 \) and \( \Theta \in \Omega_1 \), (14) and (15) holds, then for any \( \hat{\Theta}, \hat{\Theta} \in [\theta_i, \bar{\theta}_i] \), (14) and (15) is satisfied. Using Matlab LMI toolbox, we can solve (14) and (15) for any \( \hat{\Theta} \in \Omega_2 \) and \( \Theta \in \Omega_1 \), which are a finite number of LMIs, and get the corresponding \( X, Y_0 \), and \( Y_i \).

By Schur complement if (14) and (15) holds, we have

\[
M_i + M_i^T < 0 \quad \text{which implies } \dot{V}(t) \leq 0
\]

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Furthermore, we have
\[
\dot{V}(t) + z^T(t)z(t) - \gamma^2 \omega_n^T(t) \omega_n(t) \leq 0
\]  
(23)

Integrating the above-mentioned inequality (23) from 0 to T on both sides, we obtain
\[
V(T) - V(0) + \int_0^T z^T(t)z(t)dt \leq \gamma^2 \int_0^T \omega_n^T(t) \omega_n(t) dt
\]
Due to \(V(T) \geq 0\) for any \(T \geq 0\), we have
\[
\int_0^T z^T(t)z(t)dt \leq \gamma^2 \int_0^T \omega_n^T(t) \omega_n(t) dt + \sum_{i=1}^N \frac{\hat{\theta}_i^2(0)}{l_i}
\]
where, \(\hat{\theta}_i(0) = \hat{\theta}_i(0) - \theta_i(0)\), \(i = 1\cdots N\) are the initial values of estimate errors. Due to \(\theta_i\), \(\hat{\theta}_i(0) \in [\theta_i - \hat{\theta}_i, \theta_i + \hat{\theta}_i]\) is bounded, we have \(\hat{\theta}_i(0) \in [\theta_i - \hat{\theta}_i, \theta_i + \hat{\theta}_i]\).

We can choose \(l_i\) relatively large so that \(\sum_{i=1}^N \frac{\hat{\theta}_i^2(0)}{l_i}\) is sufficiently small and approximately equal to 0. Thus with zero initial condition, i.e. \(\xi(0) = 0\), we have
\[
\int_0^T z^T(t)z(t)dt \leq \gamma^2 \int_0^T \omega_n^T(t) \omega_n(t) dt \quad \text{for any } T \geq 0
\]
where \(\gamma\) is the \(H_{\infty}\) disturbance attenuation index. \(\square\)

The following is an problem to optimize the \(H_{\infty}\) performance.

Algorithm 1: \(\gamma\) is minimized if the following optimization problem is solvable

\[
\min \eta \quad \text{s.t.} \quad (14)(15)
\]
where \(\eta = \gamma^2\). We can obtain the minimized \(\gamma\) by solve the problem above.

### IV. SIMULATIONS

The proposed control method is verified by the simulation model obtained from the helicopter-on-arm platform, shown as Fig.1. A small-scale electrical helicopter is mounted at the end of a two-DOF arm, while the weight of the helicopter is perfectly balanced at the other side of the arm. First, the parameters of the nonlinear yaw dynamic model are identified by least square method, and followings are the result:

\[
\begin{aligned}
\dot{\phi} &= r + \eta_1 \\
\dot{r} &= k_2 r + k_4 \phi^2 + k_5 \Omega \theta + k_6 \phi + \eta_2
\end{aligned}
\]  
(24)

with \(k_1 = -1.3828\), \(k_2 = 63.0923\), \(k_3 = 11.6514\), \(k_4 = -0.1380\) \(k_5 = -3.3286\), \(\Omega = 1200\). \(\eta_1\) and \(\eta_2\) are disturbances. Fig.2 demonstrates the fitness of identified model of (24) with respect to the measure performance, from which we can see that the simulation model match with system very well. System (24) can be linearized, and system matrices are as follow:

\[
A_0 = \begin{bmatrix} 0 & 1 \\ -3.3286 & -1.3828 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ -3.3286 & 0 \end{bmatrix}
\]
\[
A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -1.3828 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]
\[
B_0 = \begin{bmatrix} 0 \\ 72.2633 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 72.2633 \end{bmatrix}
\]
\[
\theta_i \in [-0.2 \quad 0.2], \quad i = 1, 2, 3.
\]

In the following simulations, the initial conditions are: \(\varphi(0) = 0\), \(r(0) = 0\). The tracking command of \(\varphi\) is \(\varphi_d = -50\), \(0 \leq t \leq \text{t}_{\text{off}}\) and we use the following disturbance:

\[
\eta_1 = \begin{bmatrix} 30 & 1 \leq t \leq 2 \\ 30 & 10 \leq t \leq 11 \\ 0 & \text{else} \end{bmatrix}, \quad \eta_2 = \begin{bmatrix} 30 + 30 \sin 2t & 1 \leq t \leq 2 \\ 30 + 30 \sin 2t & 10 \leq t \leq 11 \\ 30 \sin 2t & \text{else} \end{bmatrix}
\]

An adaptive robust tracking controller is designed to control yaw model of the helicopter using the proposed approach in Section III. Using the Matlab, we can get the gains of adaptive robust controller:

\[
K_0 = [3.7717 \ -2.4740 \ -0.4695] \quad \text{with } K_0 = [3.7717 \ -2.4740 \ -0.4695]
\]
\[
K_1 = [0 \quad 0.0039 \quad 0]
\]
\[
K_2 = [0 \quad 0 \quad 0.0010]
\]
\[
K_3 = [-0.2746 \quad 0.1801 \quad 0.0342]
\]

To verify the robustness and response of our method, simulation results using the nonlinear model with our method are carried out and the results are given in Figs. 3. It takes 7 seconds for \(\varphi\) to track the desired value –50 degree in presence of disturbance at 1 second. After the second pulse disturbance takes place, \(\varphi\) converges to the tracking command quickly. The response curve of \(r\) is given in Fig.3b. It is easy to see that using our adaptive robust controller the closed-loop nonlinear system is stable and has zero tracking error even in presence of disturbance.

We have also implemented the fixed gain robust control [14] with \(K_0 = [3.7717 \ -2.4740 \ -0.4695]\) and PID control method on the nonlinear model. The parameters of PID control are:

\[
K_p = 0.01, \quad K_i = 0.000025, \quad K_d = 0.025
\]

which are real parameters for our experiments. Tracking results are shown in Fig. 4. From Fig.4, we can see that PID control can’t track the desired command, and the best tracking result of fixed gain controller is not as good as under our proposed method.

Summarizing these simulations, it is noted that the proposed adaptive robust controller design method can improve the system performance in the presence of uncertainty and disturbance.
V. CONCLUSIONS

In this paper, we introduce a new robust $H_{\infty}$ feedback controller with adaptive mechanisms for the linearized yaw dynamic with guaranteed control performance using LMI method. The proposed control design method is general for a linear time-invariant affine uncertainty model. The proposed controller reduces conservatism inherent in a robust control with a fixed gain and improves performances in time-response. Simulations of yaw dynamic model of small-scale helicopter are performed, and numerical results illustrate the theoretical developments.

REFERENCES


Fig. 1 Helicopter-on-arm Platform

Fig. 2 Comparison between the flight data and computed from the identified model

Fig. 3a $\psi$ behaviour of nonlinear model

Fig. 3b $\tau$ behaviour of nonlinear model

Fig. 4 $\psi$ behaviour of nonlinear model with fixed robust control, PID control and adaptive robust control