An Adaptive UKF Algorithm and Its Application in Mobile Robot Control

Qi Song and Juntong Qi
Shenyang Institute of Automation
Graduate School of the Chinese Academy of Sciences
Shenyang, Liaoning Province, China
{songqim & qijt}@sia.cn

Jianda Han
Shenyang Institute of Automation
Chinese Academy of Sciences
Shenyang, Liaoning Province, China
jdhan@sia.cn

Abstract - In order to improve the performance of the UKF a novel adaptive filter method is proposed. The error between the covariance matrixes of innovation measurements and their corresponding estimations/predictions is utilized as the cost function. Based on the MIT rule, an adaptive algorithm is designed to online update the covariance of the process uncertainties by minimizing the cost function. The updated covariance is further fed back into the normal UKF. Such an adaptive mechanism is intended to compensate the lack on the priori knowledge of process uncertainty distribution and improve the performance of UKF for the applications such as active state and parameter estimations. Simulations are conducted with respect to the dynamics of an omni-directional mobile robot, and the results obtained by the proposed AUKF are compared with those by normal UKF to demonstrate the effectiveness and improvements.

Index Terms – Adaptive UKF Innovation MIT Process Covariance

I. INTRODUCTION

Autonomous control is a key technology for autonomous system [1]. How to overcome uncertainties and achieve high performance control is one of the main issues with the autonomous control.

In recent years, the encouraging achievement from sequential estimation makes it becoming an important direction for online modeling and model-reference control [2]. The most popular state estimator for nonlinear system is the extended Kalman Filter (EKF) [3]. Although widely used, EKF’s have some deficiencies including the requirement of the sufficient differentiability of the state dynamics and a susceptibility to bias and divergence in the state estimates. Unscented Kalman Filter (UKF), on the other hand, uses the nonlinear model directly instead of linearizing it [4]. The UKF has the same computational complexity with the EKF. Since the nonlinear models are used without linearization, the UKF does not need to calculate Jacobians and can achieve the second-order accuracy (the accuracy of EKF is first-order).

However, since the UKF is in the framework of the Kalman filter, it can only achieve good performance under certain assumptions about the system’s mathematical model. These assumptions include: 1) sufficient accuracy in modeling the dynamic and observation models; 2) complete information of the noise statistics; 3) proper initial conditions.

But in practice this information is usually not totally known and the performance of the filter may be seriously degraded from the theoretical performance. In order to avoid the problems an adaptive filter can be applied. Innovation represents additional information available from the new measurement zk and is considered as the most relevant source of information to the filter adaptation. There have been many investigations about adaptive filtering, which focus on utilizing new statistical information obtained from the innovation to online tune the noise covariance matrix. Maybeck [5] used a maximum-likelihood estimator for designing an adaptive filter that can estimate the system errors covariance matrix. Lee [6] modified Maybeck methods by introducing a window scale factor. The new automated adaptive algorithms are integrated into the UKF and DDF which can be applied to the nonlinear system. One disadvantage of the algorithm is that is not very robust numerically. Loebis et al [7] present an adaptive EKF method, which adjusts the measurement noise covariance matrix employing the principles of fuzzy logic. However in practice it is always difficult to determine the values of the increment of the covariance at each instant of time.

In this paper we propose an on-line innovation-based adaptive scheme of UKF to adjust the noise covariance. The filter parameter is tuned by using a MIT adaptation rule to minimize the cost function of innovation sequence. Extensive simulations are conducted with respect to the dynamics of an omni-directional mobile robot. It demonstrates that the estimate accuracy with the adaptive approaches is significantly better than the conventional UKF.

II. STANDARD UKF

In this section the principle of classical UKF is introduced. Consider the general discrete nonlinear system:

\[
\begin{align*}
  x_{k+1} &= f(x_k, u_k) + w_k \\
  y_k &= h(x_k) + v_k
\end{align*}
\]

where \(x_k \in \mathbb{R}^n\) is the state vector, \(u_k \in \mathbb{R}^r\) is the known input vector, \(y_k \in \mathbb{R}^m\) is the output vector at time \(k\). \(w_k\) and \(v_k\) are, respectively, the disturbance and sensor noise vector, which are assumed to Gaussian white noise with zero mean.

The UKF estimation can be expressed as:

Initialization
\[ \begin{align*}
\bar{x}_0 &= E[x_0] \\
P_0 &= E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T]
\end{align*} \]

**Sigma Points Calculation and Time Update**

\[ \begin{align*}
\chi_{k-1} &= [\bar{x}_{k-1}, \bar{x}_{k-1} + \sqrt{(n + \lambda)}P_{k-1}, \\
\chi_{k-1} &= f(\chi_{k-1}) \\
\bar{x}_{k-1} &= \sum_{i=0}^{2n} w^m_i \chi_{i,k-1}^* \\
P_{k-1} &= \sum_{i=0}^{2n} w^m_i (\chi_{i,k-1}^* - \bar{x}_{k-1}) \\
Z_{k-1} &= \left[ \bar{x}_{k-1}, \bar{x}_{k-1} + \sqrt{(n + \lambda)}P_{k-1}, \\
\gamma_{k-1} &= h(\chi_{k-1}) \\
\bar{y}_{k-1} &= \sum_{i=0}^{2n} w^m_i \gamma_{i,k-1}
\end{align*} \]

where

\[ \begin{align*}
w^m_0 &= \frac{\lambda}{n + \lambda} \\
w^c_0 &= \frac{n - \lambda}{n + \lambda} + \frac{\alpha}{\lambda^2} (\lambda - 1) + \beta \lambda \\
w^p_0 &= \frac{1}{\lambda} + \frac{\lambda}{(n + \lambda)} \\
\lambda &= n(\alpha^2 + 1)
\end{align*} \]

**Measurement Update**

\[ \begin{align*}
P_{\bar{y}_k} &= \sum_{i=0}^{2n} w^m_i (\gamma_{i,k-1}^* - \bar{y}_{k-1}) \\
\gamma_{k-1} &= h(\chi_{k-1}) \\
P_{\bar{y}_k} &= \sum_{i=0}^{2n} w^m_i (\gamma_{i,k-1}^* - \bar{y}_{k-1}) \\
K_k &= P_{\bar{y}_k} P_{\bar{y}_k}^{-1} \\
P_k &= P_{\bar{y}_k} - K_k P_{\bar{y}_k} K_k^T \\
\bar{x}_k &= \bar{x}_{k-1} + K_k (\bar{y}_k - \bar{y}_{k-1})
\end{align*} \]

where the variable definitions are given as follows: \( \{w_i\} \) is a set of scalar weights, \( n \) is the state dimension. The parameter \( \alpha \) determines the spread of the sigma points around \( \bar{x} \) and usually set to \( 10^{-4} \leq \alpha \leq 1 \). The constant \( \beta \) is used to incorporate part of the prior knowledge of the distribution of \( x \) and for Gaussian distributions \( \beta = 2 \) is optimal. \( Q \) and \( R \) are the disturbance and sensor noise covariance respectively.

### III. Adaptive UKF

**A. Adaptive Parameter**

In many adaptive filtering algorithms the covariance matrices \( R \) and \( Q \) are the main parameters to tune online. In principle, an adaptive filter can estimate both \( R \) and \( Q \). However, adaptive filtering algorithms that try to update both the observational noise and the system noise are not robust [8]. The measurement noise statistics are relatively well known compared to the system model error. In this paper, the adaptive estimation of the process noise covariance \( Q \) is considered. Usually, the process noise covariance \( Q \) is a diagonal matrix. So the estimation of \( Q \) can be simplified as the estimation of its diagonal elements.

**B. Cost Function**

Most innovation-based adaptive filter methods are to minimize the time-averaged innovation covariance. However, with this criterion we may obtain a minimum “true” innovation covariance, but this covariance could be completely different from the one computed by the filter [9]. In this paper, a recursive algorithm to minimize difference between the filter-computed and the actual innovation covariance is formulated. The time-averaged innovation covariance is used as an approximation to the actual one:

\[ S_k = \frac{1}{N} \sum_{n=k-N}^{k} v_n v_n^T \]

where \( N \) is the size of the estimation window. The \( v_k \) is innovation and can be written as:

\[ v_k = y_k - \bar{y}_{k-1} \]

where \( y_k \) and \( \bar{y}_{k-1} \) are the real measurement received by the filter and its estimated (predicted) value respectively.

From the measurement update equation (5) of the standard UKF, we can obtain the filter-computed innovation covariance:

\[ \hat{S}_k = \sum_{i=0}^{2n} w^m_i (\gamma_{i,k-1}^* - \bar{y}_{k-1}) (\gamma_{i,k-1}^* - \bar{y}_{k-1})^T + R \]

Then the criterion function for adaptive UKF is to minimize

\[ V_k = tr(\Delta S_k^2) = tr \left[ (S_k - \hat{S}_k)^T \right] \]

**C. Adaptive Law**

With the MIT rule, the parameter can be adjusted in the negative gradient direction of the criterion function, i.e.

\[ \delta^m_k = -\eta \frac{\partial V_k}{\partial \delta^m_k} \]

where \( \eta > 0 \) is the tuning rate to determine the convergence speed and \( \delta^m_k \) is the \( m \)th diagonal element of the process noise matrices at time \( k \).

**D. Adaptive UKF**

Equation (10) leads to the following recursive scheme:
\[ q^w_k = q^w_{k-1} - \eta \frac{\partial V_k}{\partial q_k} dt \]  

where \( dt \) is the sampling interval. This scheme can be incorporated into the UKF equations to update \( Q \).

In order to calculate (11), we need to take the derivative of \( V_k \). From (9), we will have

\[ \frac{\partial V_k}{\partial q_k} = \frac{\partial}{\partial q_k} \left[ tr(\Delta S_k) \right] = tr \left( \frac{\partial \Delta S_k}{\partial q_k} \Delta S_k + \Delta S_k \frac{\partial \Delta S_k}{\partial q_k} \right) \]  

where

\[ \frac{\partial \Delta S_k}{\partial q_k} = \frac{\partial}{\partial q_k} \left( S_k - \tilde{S}_k \right) = \frac{\partial S_k}{\partial q_k} - \frac{\partial \tilde{S}_k}{\partial q_k} \]  

From (6) and (7) we can get the equation we need for the first term:

\[ \frac{\partial S_k}{\partial q_k} = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\partial v^T_k y + v}{\partial q_k} \right) \]  

\[ = \frac{1}{N} \sum_{i=1}^{N} \left( -\frac{\partial \tilde{Y}_k}{\partial q_k} \left( y_k - \tilde{y}_k \right) \right) \]  

and the second term can be obtained from (8):

\[ \frac{\partial \tilde{S}_k}{\partial q_k} = \sum_{i=0}^{2n} \left[ \frac{\partial \tilde{Y}_k}{\partial q_k} \left( y_k - \tilde{y}_k \right) \right] \]  

\[ \text{Gradient of Prediction} \]

\[ \frac{\partial \tilde{Y}_k}{\partial q_k} = \sum_{i=0}^{2n} \frac{\partial \tilde{Y}_k}{\partial q_k} \]  

\[ \text{Derivative Updates} \]

\[ \frac{\partial P_{i,j,k-1}}{\partial q_k} = \sum_{i=0}^{2n} \left[ \frac{\partial \tilde{Y}_k}{\partial q_k} \right] \]  

IV. SIMULATION

The simulations are carried out with respect to the dynamics of the 3-DOF omni-directional mobile robot developed in SIA.

A. Simulation Model
The dynamic model of the mobile robot is [10]:
\[
(2Mr_0^2 + 3nI_0)\ddot{x}_w + 3nI_0 \dot{y}_w \dot{\phi}_w + 3n^2c_2 \dot{c}_{x_w} = m (\beta \mu_1 + 2u_2 \cos \phi_w + \beta_1 \mu_1)
\]
\[
(2Mr_0^2 + 3nI_0)\ddot{y}_w + 3nI_0 \dot{x}_w \dot{\phi}_w + 3n^2c_2 \dot{c}_{y_w} = m (\beta \mu_1 + 2u_2 \sin \phi_w + \beta_2 \mu_1)
\]
\[
(3n^2I = \text{diag} \{10^{-12}, 10^{-12}, 10^{-12} \} \quad t < 10s
\]
\[
\begin{align*}
Q_T &= \text{diag} \{10^{-10}, 10^{-10}, 10^{-10}, 10^{-10} \} & t \geq 10s
\end{align*}
\]

The tracking performance of the adaptive UKF with respect to changes of the process noise statistics is tested. The change of the true process noise intensity is assumed as:
\[
\begin{align*}
Q_T &= \text{diag} \{10^{-12}, 10^{-12}, 10^{-12}, 10^{-8} \} \quad t < 10s
\end{align*}
\]

In UKF, the prior knowledge of the process noise covariance is selected as \(Q = Q_T\).

The velocity estimation errors of the classical UKF and the adaptive UKF under the same condition of the process noise intensity change are illustrated in Fig.1. As can be seen, under the incorrect noise information the classical UKF can not produce optimal estimates due to the violation of the optimality conditions. On the other hand, the estimation errors in adaptive case are quickly overcome and almost the same as its previous size.

C. The Changes of Parameters

In this section the performance of the adaptive UKF for parameter estimations is validate.

The true values of the friction coefficient and the motor axis inertia are designed to change according to:
\[
\begin{align*}
\dot{c} &= c_0, \quad I_w = I_w^0 & t < 10s \\
\dot{c} &= c_0 + c_s, I_w = I_w^0 + I_w^s & t \geq 10s
\end{align*}
\]

where the constant change value in 10s is \(c_s = 0.0031 \text{ kgm}^2/\text{s}\) and \(I_w^s = 0.0164 \text{ kgm}^2\). The state vector is subject to zero mean additive white noise with covariance:
\[
\begin{align*}
Q_T &= \text{diag} \{10^{-14}, 10^{-14}, 10^{-14}, 10^{-10}, 10^{-10} \}
\end{align*}
\]

Other conditions of the system are the same as that in the section 3.1.

In order to estimate the parameters, the joint estimation, which treats the parameter vector as a dynamical variable, is used. Since the dynamics of parameter is unknown, the parameter can be assumed as a non-correlated random drift vector and modeled by:
\[
\theta_k = \theta_{k-1} + w_{k-1}
\]

where \(w_{k-1}\) is the Gaussian white noise with zero mean, called the pseudonoise. The pseudonoise is very important to the time-varying parameter estimation [11]. Properly adjusting the pseudonoise covariance will lead to better tracking of the time-varying parameter.

In the simulation, the UKF parameters are designed as:
\[
\begin{align*}
\dot{c}_0 &= c_0, \quad \dot{I}_w^0 = I_w^0 \\
\dot{P}_0 &= \text{diag} \{10^{-8}, 10^{-8}, 10^{-8}, 10^{-8} \} \\
\dot{Q} &= \text{diag} \{10^{-18}, 10^{-17} \}
\end{align*}
\]

Other parameters of the UKF are the same as that mentioned in the section 3.1.
Fig. 1 State Estimation Errors with the Time-Varying Process Noise
The parameter estimate results are shown in Fig. 2, from which we can see that the standard UKF with fixed value noise covariance can’t track the parameters change due to the lower pseudonoise intensity, which will not provide enough drive power to the parameter estimate. As for the adaptive UKF, the intensity of the pseudonoise will increase during the parameter change by the adaptive parameter update. This can accelerate the convergence of the parameter estimates and make the UKF react quickly and track the abrupt change successfully after a short period (~3 seconds) of adaptation. Fig. 3 illustrates the performance comparison of the standard UKF and the adaptive UKF with respect to velocity estimate errors due to the parameter change. As can be seen, the tracking errors of standard UKF are much more significant than those by the adaptive UKF.

V. CONCLUSION

In this paper, the estimation errors of the UKF with the unknown noise statistics are analyzed. A novel adaptive UKF, which is based on the innovation covariance matrix and the MIT adaptive law, is proposed. Also, a recursive algorithm has been formulated. Simulations on the dynamics of omnidirectional mobile robot are conducted to verify the proposed scheme. Results show that the adaptive UKF outperforms the conventional UKF in terms of the fast convergence and estimation accuracy by tuning the process noise covariance matrix $Q$.

REFERENCES