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Disturbance Attenuation Control of A Class of Unceractuated Systems with Acceleration Measurement

Yuqing He
Robotics Laboratory, Shenyang, Institute of Automation, CAS
The Graduate School of CAS
Shenyang, 110016, China
heyuqing@sia.cn

Jianda Han
Robotics Laboratory
Shenyang Institute of Automation, CAS
Shenyang, 110016, China
Jdhan@sia.cn

***Abstract* - In this paper, a new kind of disturbance attenuation controller design method, based on the acceleration measurement, is proposed for a class of underactuated systems by combining with the backstepping and H∞ technique. Comparing with the traditional high gain acceleration feedback control method, where the acceleration feedback loop is important but difficult to design, the acceleration measurement in the new controller is only used to obtain the disturbance signals. Simultaneously, in order to overcome the heavy noise accompanied with the acceleration measurement and the algebraic loop problem, a linear pre-filter is introduced. The designing process concerns both nonlinear H∞ controller and linear filter theory, which make the disturbance be attenuated from the concepts of both linear filter in frequency domain and input-output finite gain L_2 stability in time domain. In the end, simulations are conducted with respect to the tracking control of unmanned model helicopter. The results are compared with those obtained by the tracking control without acceleration measurement to verify the feasibility of it.**

Index Terms - Disturbance attenuation, Acceleration feedback, Underactuated systems, Nonlinear H∞ control, Backstepping

I. Introduction

Precise control is an important technology for many mechatronics systems, especially unmanned vehicles which are widely used in the areas of satellite clusters, deep space exploration, air traffic control, and the UxVs in battlefield (Unmanned Ground/Surface/Air vehicles) [1]. However, there exist at least three difficulties or challenges for it:

a) The dynamics of most unmanned vehicles are highly nonlinear, time-varying and coupled, which might be too complicated to be used for control design;

b) Many unmanned systems are underactuated, i.e., a system possessing more degrees of freedom than independent inputs, which bring more difficulties for the controller design;

c) The working environments of unmanned vehicles are usually dynamic, complex and unstructured, which bring unpredictable disturbances to the control system, such as the air mass for UAV and the wave/wind for USV.

Thus, how to overcome the above difficulties and achieve good control performance has been one of the main tasks. Traditional robust or adaptive control methods for uncertainty and disturbance attenuation suffer from several problems, including conservativeness due to the inaccuracy in the pre-assumption of uncertainties, online divergence due to unknown external disturbances and the complication for real-time implementation.

It is a fact that many underactuated mechatronics systems can be modeled as cascade form that can be stabilized by using the backstepping technique [2]. Yet, the backstepping technique itself cannot attenuate the influence of uncertainties on either model errors or external disturbances well. Most recently, backstepping technique is combined with some normal robust control schemes [3][4]. Whereas the backstepping-type robust control still has the disadvantages of normal robust control mentioned above. And, most of these methods did not consider the disturbance information which can usually be measured in reality.

On the other hand, the acceleration based robust feedback control technique (AFC) has been successfully used for suppressing uncertainties and external disturbances of fullactuated nonlinear mechatronics systems by the measurement of acceleration signals [5][6][7]. This is partly because most of the disturbance and uncertainties of the mechatronics systems exist as a form of forces and moment, which has algebraic relation with the acceleration signals from the Newton's second law of motion. However, a great disadvantage of the AFC is that it still cannot be used in nonlinear and underactuated systems. And, it should be noted that in mechatronics systems, the disturbance information can be directly obtained by the acceleration measurement. Thus, the AFC design is actually a disturbance attenuation robust control problem with measurable disturbance signals.

In this paper, based on the acceleration measurement, a 'pre-filter' is designed combining with the H∞ control and backstepping technique to obtain a new robust controller with known disturbance information for a class of underactuated mechatronics systems, which we call acceleration enhanced H∞ (AE-H∞) controller. And, in the end, the simulation results of the new controller are used to an unmanned helicopter model to verify the feasibility of which.

II. Some Existed Disturbance Attenuation Methods

Disturbance attenuation is an important and difficult problem in the control of mechatronics systems. And disturbance decoupling [8][9] and H∞ [10][11][12] are two often-used methods for both linear and nonlinear systems.

For the disturbance decoupling problem, researchers seek to obtain a states or outputs feedback law to completely remove the effect of the external disturbance on the output. However, only small parts of nonlinear system can be disturbance decoupled in theory. Comparatively, the idea of

H∞ control is broadly used in recent years to try to reduce the effect of the external disturbance on the output. Next, we will briefly introduce the idea of H• control for the following input-affine nonlinear system

$$\begin{aligned} \dot{x} &= F(x,u) + G(x)\omega \\ y &= H(x) \end{aligned} \tag{1}$$

where $x \in IR^n$ are states, $u \in IR^m$ are control inputs, $\omega \in IR^r$ are external disturbances, and $y \in IR^p$ are outputs, $F(x)$, $G(x)$, and $H(x)$ are smooth functions of states with $F(0) = 0$ and $H(0) = 0$.

System (1) is said to have L_2-gain less than or equal to γ, if inequality

$$\int_0^t \|y(t)\|^2 d\tau \le \gamma^2 \int_0^t \|\omega(t)\|^2 d\tau \tag{2}$$

is satisfied for all $t \ge 0$ and all $u \in L_2[0,t]$. And the H∞ controller design is to find a feedback $u = K(x)$ such that Eq. (2) is satisfied along with the trajectory of the closed loop system.

From reference [11], inequality (2) is satisfied, if there exists a smooth $V(x) \ge 0$ to satisfy the following HJI inequality

$$\frac{\partial V}{\partial x}F(x,K(x)) + \frac{1}{4\gamma^2}\frac{\partial V}{\partial x}G(x)G^T(x)(\frac{\partial V}{\partial x})^T + H^T(x)H(x) \le 0 \tag{3}$$

As we have observed that both the H• technique and the disturbance decoupling technique refuse considering the disturbance signals which can often be measured in reality. And, there exist little work on how to use the disturbance information to improve the disturbance attenuation of both disturbance decoupling and H• technique. In this paper, we will try to solve this problem by introducing a pre-filter.

III. H∞ Control Enhanced by Measurable Disturbance

The dynamics of underactuated mechatronical systems can be expressed as following [2]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = F(q)\tau + \Delta \tag{4}$$

where q are the generalized position vector of the system, which is belonged to IR^n; $M(q)$ is the inertia matrix which is a positive definition symmetric $n \times n$ matrix; the term of $C(q,\dot{q})\dot{q}$ contains two types of terms involving $q_i \dot{q}_j$ including the so called Centrifugal terms and Coriolis terms; $G(q)$ is the gravity terms (or the frictions terms); τ is the control vector; and Δ denote the external disturbances or the uncertainties.

If $F(q) = [0, I_m]^T$ in Eq.(4), and we partition the vector of q as $[q_1, q_2] \in IR^{(n-m)} \times IR^m$, where q_1 and q_2 denote the actuated and unactuated configuration vectors, respectively, the Eq.(4) can be denoted as

$$\begin{bmatrix} m_{11}(q) & m_{12}(q) \\ m_{21}(q) & m_{22}(q) \end{bmatrix}\ddot{q} + \begin{bmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} + \Delta \tag{5}$$

Eq. (5) is a general expression of underactuated mechatronics systems. Controller design of (5) is difficult, and which is not the main work of this paper. And what we will mainly deal with, in this paper, is a special case of Eq. (5) that can be transformed as:

$$\begin{cases} \dot{z} = f_1(z) + g_1(z)\xi_1 + h_1(z)\Delta \\ \dot{\xi}_1 = f_2(z,\xi_1) + g_2(z,\xi_1)\xi_2 + h_2(z)\Delta \\ \vdots \\ \dot{\xi}_r = f_2(z,\xi_r) + g_2(z,\xi_r)\tau + h_r(z)\Delta \end{cases} \tag{6}$$

where $z \in IR^{n-mr}$ $\xi_1, \xi_2, \ldots, \xi_r \in IR^m$ are the states; Δ are disturbances and ignored or unknown items; $\tau \in IR^m$, $m = n$, is the control input of the system; $f_i(\ )$ and $g_i(\ )$ (i=1,2,…,r) are known smooth functions.

Eq. (6) is often called as 'strict-feedback' systems. And, it has been showed that many underactuated systems can be expressed as the form of Eq. (6) by coordinate transformation or ignoring some coupling terms [13],[14],[15],[16].

It is well known that system (6) can be stabilized by backstepping technique [2]. And in this section, we will give the corresponding version of H∞ controller with measurable disturbances by using the idea of 'pre-filter'.

For the purpose of simplification, we give the following theorem firstly.

Theorem I:

Considering the following disturbance perturbed nonlinear system

$$\dot{z} = f_1(z) + g_1(z)\xi + h_1(z)\Delta \tag{7-a}$$

$$\dot{\xi} = f_2(z,\xi) + g_2(z,\xi)u + h_2(z,\xi)\Delta \tag{7-b}$$

where $g_2(\ )$ is reversible and there exists a function $l(z)$: $IR^m \rightarrow IR$ such that $g_1(z)l(z) = h_1(z)$. Suppose that with ξ picked as the virtual control input to the subsystem dynamics (7-a), and with $q_1(z) = l^T(z)l(z)$ some arbitrary but fixed nonnegative-definite function, there exists a control law $a_1(z)$ such that the following inequality is satisfied by a nonnegative-definite value function $V_1(z)$:

$$\frac{\partial V_1}{\partial z}[f_1(z) + g_1(z)a_1(z)] + \frac{1}{4\gamma^2}\frac{\partial V_1}{\partial z}h_1(z)h_1^T(z)(\frac{\partial V_1}{\partial z})^T + q_1(z) \le 0 \tag{8}$$

Then, there exists a controller $a(z,\xi,v,\dot{v})$ such that the following system

$$\begin{aligned} \dot{z} &= f_1(z) + g_1(z)\xi - h_1(z)v + h_1(z)(\Delta + v) \\ \dot{\xi} &= f_2(z,\xi) + g_2(z,\xi)u - h_2(z,\xi)v + h_2(z,\xi)(\Delta + v) \\ y &= l(z) \end{aligned} \tag{9}$$

has L_2-gain less than or equal to γ, taking $(\Delta + v)$ as new disturbance signals.

Proof of Theorem I:

Firstly, we suppose a new variable

$$z_1 = \xi - a_1(z) - l(z)v \tag{10}$$

Compute the derivative of z_1, we have

$$\begin{aligned} \dot{z} &= f_1(z) + g_1(z)a_1(z) + g_1(z)z_1 + h_1(z)(\Delta + v) \\ \dot{z}_1 &= f_3(z,\xi,v,\dot{v}) + g_3(z,\xi)u - h_3(z,\xi,v)v + h_3(z,\xi,v)(\Delta + v) \end{aligned} \tag{11}$$

where

$$f_3(z,\xi,v,\dot{v}) = f_2(z,\xi) - [\frac{\partial a_1(z)}{\partial z} + \frac{\partial l(z)v}{\partial z}][f_1(z)+g_1(z)\xi] - l(z)\dot{v}; g_3(z,\xi) = g_2(z,\xi);$$

$$h_3(z,\xi,v) = \{h_2(z,\xi) - [\frac{\partial a_1(z)}{\partial z} + \frac{\partial l(z)v}{\partial z}]h_1(z)\}$$

Introduce a new value function candidate for the system (11),

$$V_2(z,z_1) = V_1(z) + \frac{1}{2}z_1^T z_1 \tag{12}$$

Then, we have

$$\frac{\partial V_2}{\partial z}[f_1(z)+g_1(z)\xi - h_1(z)v] + \frac{\partial V_2}{\partial z_1}[f_3(z,\xi,v,\dot{v}) + g_3(z,\xi)u + h_3(z,\xi,v)v] - q_1(z) + z_1^T Q_1 z_1 + \frac{1}{4\gamma^2}[\frac{\partial V_2}{\partial z} \quad \frac{\partial V_2}{\partial z_1}]\begin{bmatrix} h_1(z) \\ h_3(z)\end{bmatrix}\begin{bmatrix} h_1^T(z) & h_3^T(z)\end{bmatrix}\begin{bmatrix}(\frac{\partial V_2}{\partial z})^T \\ (\frac{\partial V_2}{\partial z_1})^T\end{bmatrix} \tag{13}$$

$$\le z_1^T\{f_3(z,\xi,v,\dot{v}) + g_3(z,\xi)u - h_3(z,\xi,v)v + Q_1 z_1 + g_1^T(z)(\frac{\partial V_1}{\partial z})^T + \frac{1}{4\gamma^2}[2h_3 h_1^T(\frac{\partial V_1}{\partial z})^T + h_3 h_3^T z_1]\}$$

where Q_1 is a positive definitely matrix.

Take

$$u = g_3^{-1}(z,\xi)\{-f_3(z,\xi,v,\dot{v}) + h_3(z,\xi,v)v - Q_1 z_1 - g_1^T(z)(\frac{\partial V_1}{\partial z})^T - \frac{1}{4\gamma^2}[2h_3 h_1^T(\frac{\partial V_1}{\partial z})^T + h_3 h_3^T z_1]\} \tag{14}$$

Eq.(13) can be made less than or equal to 0. Thus, we conclude that controller (14) make system (11), which is equivalent with system (9), has L_2-gain less than or equal to γ with $l(z)$ and $Q_1^{1/2}z1$ as outputs and $\Delta + v$ as external disturbances.

End of Theorem I.

Now, if the signals of Δ, $\dot{\Delta}$ are both measurable, we can wipe off the effect of the disturbance on z by letting $v = -\Delta$, simultaneously, the effect of the disturbance on ξ can be evaluated from Eq. (10). Unfortunately, in general cases, the derivative of Δ is difficult to obtain due to the heavy noise in reality. In order to avoid using $\dot{\Delta}$, we can design a linear pre-filter as following

$$\dot{v} = -kv - b\Delta \tag{15}$$

Eq. (15) is a linear filter, and from the linear system theory, which can be used to attenuation the influence of disturbance Δ because the signals $\Delta + v$ is the final external signals acted on the closed loop system.

With respect to more general cases of strict-feedback system (6), we can repeat using the process during the proof of Theorem I to obtain a similar controller:

$$u = a(z,\xi_1,\cdots,\xi_r,v,\cdots,v^{2r-2}) \tag{16}$$

and the following pre-filters:

$$v^{(r)} = -k_1 v - \cdots - k_r v^{(r-1)} - b\Delta \tag{17}$$

IV. Obtain Disturbance Signals By Accelerations Measurements

Measurement or estimation of external disturbance signals is necessary and important in the new disturbance attenuation robust controller of section III.

From section III, Most of mechatronics systems can be modeled as Eq. (4) and be rewritten as following:

$$\ddot{q} = f(q,\dot{q}) + g(q)u + \Delta \tag{18}$$

where $f(\ )$ and $g(\ )$ are some pre-defined function of the generalized position and velocity vectors of q and $\dot{q}$; u is the control input; Δ is the summation of external disturbance and the uncertainty of the systems, which will be called disturbance in the following sections for the purpose of simplification.

From Eq. (18), the disturbance signals Δ can be obtained immediately by simple algebraic computation,

$$\Delta = \ddot{q} - f(q,\dot{q}) - g(q)u \tag{19}$$

It is clear that the acceleration signals are needed in order to obtain the disturbance from Eq. (19). Fortunately, it is well known that acceleration signals can be easily measured in most of mechatronics systems, such as unmanned helicopter, robotic system, satellite, etc. Nevertheless, there still exist some special cases where the acceleration signals cannot be measured directly, such as the angular acceleration measurement, the heavily coupled acceleration measurement, and the relative acceleration signals measurement. For these special situations, the clean and little-delayed acceleration signals can be estimated using our early designed estimating algorithms as [17].

Remark: It should be noted that, although the disturbance can be directly obtained by the acceleration measurement, the pre-filter designing of Eq. (15) or Eq. (17) is still necessary due to two reasons: On the one hand, the high-order of the disturbance signals need the high-order of acceleration signals which will introduce heavily magnified noises; On the other hand, the direct use of disturbance from Eq. (19) in controller (14) will lead to the algebraic loop problem, i.e., the directly feeding of acceleration measurement, which includes algebraically the control input, into the input of the system being controlled. And this can deteriorate the closed loop performance and is usually unallowed in real applications.

V. Disturbance Attenuation Control of A 6-DOF Helicopter Model

In this section, we will apply the proposed acceleration enhanced H∞ controller design method to control an underactuated model helicopter.

A. *Dynamics of Model Helicopter*

The complete dynamics of a helicopter, including actuator dynamics, aerodynamics, and body dynamics, is usual too complicated to be used for the purpose of controller design [18]. Thus, the helicopter dynamics are often considered as a rigid body incorporating with simplified aero- and actuator dynamics. The motion equation of a model helicopter can be written with respect to the body frame with x-axis pointing to its head, y-axis going to the right of the body, and z is defined by the right-handed rule, i.e.

$$\begin{bmatrix} \dot{p} \\ \dot{v}^p \\ \dot{\Theta} \\ \dot{\omega}^b \end{bmatrix} = \begin{bmatrix} v^p \\ \frac{1}{m} R f^b \\ \Psi \omega^b \\ J^{-1}(\tau^b - \omega^b \times J\omega^b) \end{bmatrix} \tag{20}$$

where $p \in IR^3$ and $v^p \in IR^3$ are the position and velocity vector in inertia frame; $R \in SO(3)$ is the rotation matrix of the body frame relative to the inertia frame; ω^b is angular velocity vector. $\Theta = [\phi \quad \theta \quad \psi]^T$ is Euler angle vector; m and J are respectively, the mass and inertia of the helicopter; Ψ is the transformation matrix from angular velocity to angular position; f^b and τ^b are force and moment of the helicopter presented in body frame, including disturbance force and moment.

The aerodynamics of a helicopter, which can be considered as a lumped model consisting of main rotor, tail rotor, horizontal stabilizer, vertical stabilizer and fuselage, is the main origin of the uncertainty. And for the purpose of simplification, most researchers design the controllers by considering only the aerodynamics of main rotor and tail rotor as following equations:

$$f^b = \begin{bmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{bmatrix} + R^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + \Delta_1$$

$$\tau^b = \begin{bmatrix} L_M \\ M_M + M_T \\ N_M \end{bmatrix} + \begin{bmatrix} Y_M h_M + Z_M y_M + Y_T h_T \\ -X_M h_M + Z_M l_M \\ -Y_M l_M - Y_T l_T \end{bmatrix} + \Delta_2 \tag{21}$$

where X, Y, Z and L, M, N are, respectively, the force and torque about the x, y, z axis in body frame; the subscript $_M$ and $_T$ denotes the main and tail rotor; h_M, y_M, h_T, l_M, l_T are some distance constants; Δ_1 and Δ_2 denote the unmodeled aerodynamic uncertainties (force and torque) including the ignored horizontal stabilizer, vertical stabilizer and fuselage, and exogenous disturbances such as the force and torque induced by air mass and wind. And Eq. (21) can be extended by the following equations.

$$\begin{cases} X = X_M = -T_M \sin a_{1s}; Y = Y_M = T_M \sin b_{1s}; \\ Z = Z_M = -T_M \cos a_{1s} \cos b_{1s}; L = L_M = S_{L1} b_{1s} + S_{L2} Q_M \\ M = M_M + M_T = S_{M1} a_{1s} + S_{M2} T_M + S_{M3} Q_T; \\ N = N_M = S_{N1} Q_M + S_{N2} T_T; \\ T_M = S_{T_M 1} \theta_M + S_{T_M 2}, T_T = S_{T_T} \theta_T + S_{T_T 2}; \\ Q_M = S_{Q_M 1} \theta_M + S_{Q_M 2}, Q_T = S_{Q_T} \theta_T + S_{Q_T 2} \end{cases} \tag{22}$$

where, a_{1s} and b_{1s}, are respectively, the longitudinal and lateral flapping angle of main rotor with respect to shaft; T_M and T_T are force of main and tail rotor respectively.

B. Controller Design

The dynamics equation denoted by Eq. (20), (21)can be denoted as Eq. (23) after ignoring the couplings between rolling moments and lateral acceleration [19],

$$\begin{aligned} \ddot{p} &= \frac{1}{M} R \begin{bmatrix} 0 \\ 0 \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \Delta_1 \\ \dot{\Theta} &= \Psi \omega^b \\ \dot{\omega}^b &= J^{-1}(\tau^b - \omega^b \times J\omega^b + \Delta_2) \end{aligned} \tag{23}$$

where Z and τ^b is inputs.

Eq. (23) can be rewritten as following

$$\ddot{p}_1 = p_2 + \Delta_1$$

$$\ddot{p}_2 = f_2(p_1, \dot{p}_1, p_2, \dot{p}_2) + g_2(p_1, \dot{p}_1, p_2, \dot{p}_2)u + h_2(p_1, \dot{p}_1, p_2, \dot{p}_2)\Delta_2 \tag{24-a}$$

$$\ddot{\psi} = f_3(p_2, \dot{p}_2, \psi, \dot{\psi}) + g_3(p_2, \dot{p}_2, \psi, \dot{\psi})u + h_3(p_2, \dot{p}_2, \psi, \dot{\psi})\Delta_2 \tag{24-b}$$

where

$$f_2 = \frac{2}{M} \dot{Z} R(\omega^b \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}) + \frac{1}{M} Z R[\omega^b \times (\omega^b \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})] - \frac{1}{M} Z R\{[J^{-1}(\omega^b \times J\omega^b)] \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\} \tag{25}$$

$$g_2 = \frac{1}{M}[ZR\bar{J} \quad R\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}]; h_2 = \frac{1}{M} ZR\bar{J}; \bar{J}\Delta_2 = J(\Delta_2 \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}) \tag{26}$$

$$\begin{aligned} f_3 &= -[0 \quad \sin\phi/\cos\theta \quad \cos\phi/\cos\theta] J^{-1}(\omega^b \times J\omega^b) + \\ &\quad q\dot{\phi}\cos\phi/\cos\theta + q\dot{\theta}\sin\theta\sin\phi/\cos^2\theta - \\ &\quad r\dot{\phi}\sin\phi/\cos\theta + r\dot{\theta}\sin\theta\cos\phi/\cos^2\theta; \\ g_3 &= [0 \quad \sin\phi/\cos\theta \quad \cos\phi/\cos\theta] J^{-1}; \\ h_3 &= [0 \quad \sin\phi/\cos\theta \quad \cos\phi/\cos\theta] J^{-1}; u = [(\tau^b)^T \quad \ddot{Z}]^T \end{aligned} \tag{27}$$

Eq. (24) have the standard form of Eq. (6), thus, we can design the AE-H∞ controller of them. And the controller design of the trajectory tracking control of helicopter can be divided into the following 4 steps:

Step I:

Design a nonlinear AE-H∞ controller with measurable disturbances of (24-a) as section III:

$$g_2(p_1, \dot{p}_1, p_2, \dot{p}_2)u = u_1 = a(p_1, \dot{p}_1, p_2, \dot{p}_2, v_1, \dot{v}_1, \ddot{v}_1, v_2) \tag{28}$$

Step II:

Design a nonlinear AE-H∞ controller with measurable disturbances of (24-a) as section III:

$$g_3(p_2, \dot{p}_2, \psi, \dot{\psi})u = u_2 = k(p, \dot{p}) + h(p, \dot{p})v_2 \tag{29}$$

Step III:

Design linear pre-filters as Eq. (17).

$$\dddot{v}_1 = -k_1 v_1 - k_2 \dot{v}_1 - k_3 \ddot{v}_1 - b_1 \Delta_1$$
$$\dot{v}_2 = -k_4 v_2 - b_2 \Delta_2 \tag{30}$$

Step IV:

Obtain the real controller:

$$u = \begin{bmatrix} g_2(p_1, \dot{p}_1, p_2, \dot{p}_2) \\ g_3(p_2, \dot{p}_2, \psi, \dot{\psi}) \end{bmatrix}^{-1} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{31}$$

C. Simulation Results

In order to verify the performance of the proposed new AE-H∞ controller design method, we use the high fidelity model presented in [19] as the target being controlled. The controller is designed according to the simplified model denoted by Eq. (20), (21) and (22) with the parameters of

$$\begin{aligned} &S_{L1} = -65.0398, S_{L2} = -0.062, S_{M1} = 65.0398, S_{M3} = -1, \\ &S_{M2} = -0.01, S_{N1} = -1, S_{N2} = 0.898, S_{T_M 1} = 1777, \\ &S_{T_M 2} = 39.8, S_{T_T 1} = 106.2, S_{T_T 2} = 6.9, S_{Q_M 1} = 95.6, \\ &S_{Q_M 2} = -1.8, S_{Q_T 1} = -3.9, S_{Q_T 2} = -0.03, h_M = 0.2340, \\ &y_M = 0, h_T = 0.062, l_M = 0.01, l_T = 0.8980, M = 9.502 \end{aligned} \tag{32}$$

The controller parameters in Eq. (28), Eq. (29) and Eq. (30) are respectively selected as:

$$\gamma = 50, \quad Q_1 = 5I, \quad Q_2 = 50I$$
$$k(p_1, \dot{p}_1) = -4p_1 - 2.8\dot{p}_1, \quad q(p_1, \dot{p}_1) = -5p_1^T p_1 - 6\dot{p}_1^T \dot{p}_1 \tag{33}$$
$$V(p_1, \dot{p}_1) = 3.3125 p_1^T p_1 + 1.25 p_1^T \dot{p}_1 + 0.3905 \dot{p}_1^T \dot{p}_1$$

$$\begin{aligned} &\gamma = 50, k(p_1, \dot{p}_1) = -4p_1 - 2.8\dot{p}_1, \\ &q(p_1, \dot{p}_1) = -5p_1^T p_1 - 6\dot{p}_1^T \dot{p}_1 \\ &V(p_1, \dot{p}_1) = 3.3125 p_1^T p_1 + 1.25 p_1^T \dot{p}_1 + 0.3905 \dot{p}_1^T \dot{p}_1 \end{aligned} \tag{34}$$

$$k_1 = -125, k_2 = -75, k_3 = -15, k_4 = -125, k_5 = -10, k_6 = -10 \tag{35}$$

The simulations are conducted with respect to a step-response issue, i.e., the helicopter is controlled to maneuver a step change from the initial states of $x_0=y_0=z_0=1.0$ *m*, $\phi_0=\theta_0=\Psi_0=0.1$ rad to $x=y=z=0.0$ *m*, $\Psi=0.0$ rad.

In order to demonstrate the good performance of the new controller, we compare the simulation results with the linearized controller in [16] with the designed poles are -1.4±1.4283j, -5, -5, -5, and the direct H∞ controller, i.e., controller without the signals of v_1 and v_2. Fig.1 demonstrates the situation while there is only the uncertainty due to model simplification but no external force or torque disturbances. From Fig.1 we can see that under the control of linearized controller and the direct H∞ controller, the model uncertainty causes a stable tracking error, which has been attenuated successfully by the control with acceleration measurement.

Fig.2 shows the case that external force disturbances of 50*N* are abruptly occurring to the first equation of (24) at every 20*s*, i.e.,

$$\ddot{p}_1 = p_2 + g e_3 + \Delta_{1m} + \Delta_{1d} \tag{36}$$

where Δ_{1m} is the uncertainty due to model simplification and Δ_{1d} is external disturbance and

$$\Delta_{1d} = \begin{cases} 0, & t < 20s \\ [50, 0, 0]^T, & 20 \le t < 40s \\ [50, 50, 0]^T, & 40 \le t < 60s \\ [50, 50, 50]^T, & t \ge 60s \end{cases} \tag{37}$$

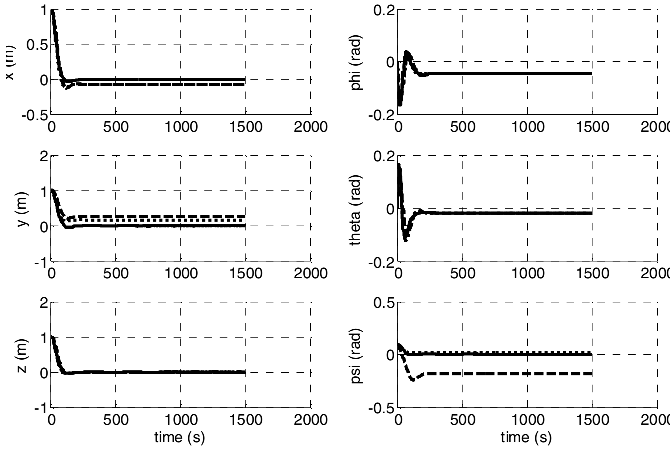

Fig.1 model uncertainty rejection, where the solid line is under the control with AE-H∞ controller, the dotted line is the H∞ controller , and the dashed line is the one with linearized controller.

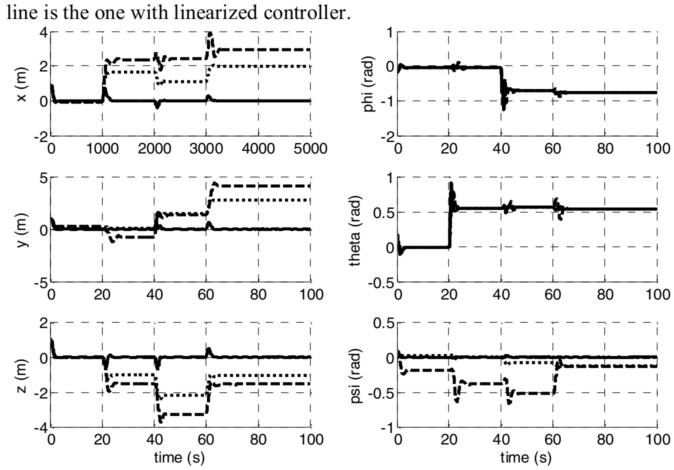

Fig.2 model uncertainty and external step-changed force disturbance rejection, where the solid line is under the control with AE-H∞ controller, the dotted line is the H∞ controller , and the dashed line is the one with linearized controller.

In Fig.2 we can see that both the linearized controller and the direct H∞ controller cannot overcome the disturbance forces and there exist stable position errors. On the other hand, the solid lines indicate that the stable position errors are rejected by the proposed AE-H∞ controller. It should be noted that the offsets of ϕ and θ in Fig.2 after disturbance occuring are necessary for the helicopter to resist the disturbances.

Besides the step disturbance like (37), sine-changed torque disturbance Δ_2 is also tested, i.e.

$$\Delta_2 = \begin{cases} 0, & t < 20s \\ [A\sin(\omega t), A\sin(\omega t), A\sin(\omega t)] & t \ge 20s \end{cases} \tag{38}$$

where $A=5$, $\omega=0.5$. And the results are demonstrated in Fig.3 and Fig.4, from which we can clearly see the improvement by the proposed AE-H∞ controller.

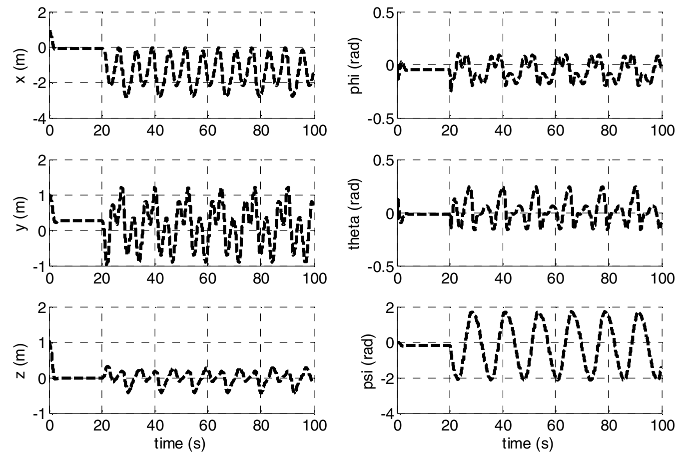

Fig.3 linearized controller for both model uncertainty and external sine-changed torque disturbance rejection

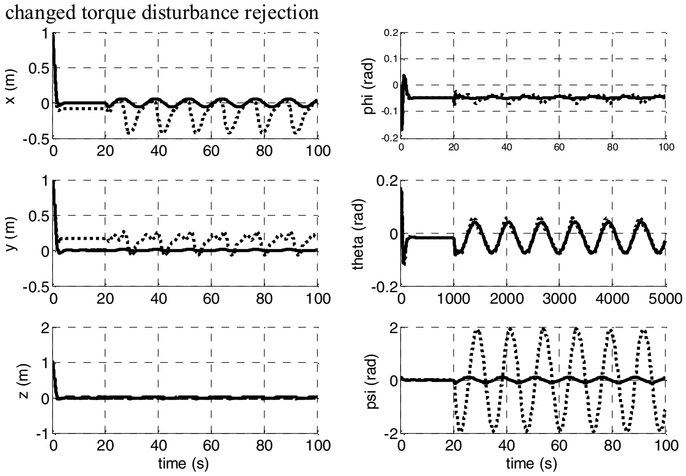

Fig.4 model uncertainty and external sine-changed torque disturbance rejection where the solid line is under the control with AE-H∞ controller, the dashed line is the H∞ controller.

VI. Conclusion

In this paper, we have presented a new acceleration enhanced H• controller design approach for the uncertainty and disturbance attenuation of a class of underactuated mechatronics systems. And the measured or estimated acceleration signals are used to obtain the disturbance signals. Simultaneously, In order to overcome the influence of noise and the algebraic loop in traditional AFC, a concept of pre-filter is introduced. And the proposed controller successfully improves the robustness on the external disturbance and model uncertainty, which is verified by the simulation results with respect to the underactuated dynamics of a model helicopter.

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