Dynamic Feedback Tracking Control of Tracked Mobile Robots with Estimated Slipping Parameters

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Abstract—The trajectory tracking control problem of a tracked vehicle with slipping is considered in this paper. The slipping effects are analyzed and modeled as three time-varying parameters, which can be estimated simultaneously with robot’s pose using nonlinear estimators such as unscented Kalman filter. Dynamic feedback linearization integrated with a globally exponential stabilizing state feedback is applied to achieve the tracking control objective. Simulation results are provided to demonstrate the effectiveness of proposed method.

I. INTRODUCTION

With the wide developments of mobile robots in many planetary explorations, urban reconnaissance and rescue mission scenarios, and so on, autonomous navigation tasks in outdoor environments has currently received considerable attention during these years [1]-[3]. Compared to the applications in structured environment, these tasks are much more challenging because the interactive effectiveness between robots and dynamic environments is time-varying and unpredictable. Among them the slipping of robots on certain terrain is obviously a crucial factor for the health and activities of the mobile robot. However, not so many attentions have been given to this problem [4]-[6].

In order to improve the accuracy of the localization and control the robot more efficiently, it is necessary to establish the model with slipping as time-varying parameters. On the other hand, slip parameters should be estimated with the pose of the vehicle simultaneously. To solve this nonlinear joint estimation problem, some real time nonlinear filter such as the Unscented Kalman filter (UKF) can be employed [7]-[8]. Compared to EKF [9], the UKF does not need to calculate Jacobians and can achieve more accurate estimations without linearization. Therefore, UKF is well suited for online application in systems with fast dynamics.

The navigation control problem is basically divided into two questions: point stabilization and tracking reference trajectory. The point stabilization problem has been investigated for nonholonomic mobile robot control systems by many authors. According to Brockett’s necessary conditions for stability, the nonholonomic systems cannot be stabilized to a point with smooth static-state feedback [10]. To deal with this problem, some researchers have proposed controllers that utilize discontinuous control laws, piecewise continuous control laws, smooth time-varying control laws, or hybrid controllers to achieve point stabilization [11]-[13].

As indicated by Z. P. Jiang, it is not clear that the stabilization methodologies available now may be extended directly to tracking problems for nonholonomic systems [14]. Y. Kanayama proposed a linearization based tracking control scheme using Lyapunov analysis [15]. The scheme was extended to a simplified dynamic model of the mobile robot [16]. Other pioneer studies on Lyapunov stable time-varying state tracking control law can be found in [17]-[18]. Many variations and improvements followed in latter researches [19]-[21]. Jiang proposed a backstepping based tracking control method for a class of nonholonomic robot system that can be transformed into chained system [14]. Tracking controller obtained with input-output linearization is used in [22], a saturation feedback controller is proposed in [23] and dynamic feedback linearization technique is used in [24].

In this paper, the longitudinal and lateral slipping are considered and processed as three time-varying parameters. The Unscented Kalman filter is used to estimate the state and slipping parameters simultaneously. With the estimated slipping parameters, a dynamic feedback linearization method combining with an exponential stabilizing feedback algorithm is used to achieve the trajectory tracking control objective.

Our paper is organized as follows. In Section II a kinematic model of tracked vehicles is established, where slipping is modeled as three time-varying parameters. In Section III a nonlinear estimator, unscented Kalman filter, is introduced in details for joint estimation of robot states and slipping parameters. The dynamic feedback linearization combining with an exponential stabilizing feedback for tracking control problem is presented in Section IV. Section V presents some simulation results. Finally, some concluding remarks are offered in Section VI.

II. SLIPPING MODELING OF TRACKED VEHICLES

In this section a general kinematic model with slipping of tracked vehicles will be developed. The slipping is described by three time-varying parameters. Fig. 1 shows the platform of the vehicle undergoing general plane motion.

In order to describe the motion of the tracked vehicle, a fixed reference frame \( F_1(x_m, y_m) \) and a moving frame \( F_2(x_n, y_n) \) attached to the vehicle body with origin at the geometric center \( O_n \) of the vehicle are defined. The motion of the vehicle is composed of the translation velocity \( \mathbf{v} = [v_x, v_y]^T \) and the rotational velocity \( \omega_r = \frac{d\psi}{dt} \), where \( v_x \) and \( v_y \) stand for the projection of the \( \mathbf{v} \) to axes of the frame \( F_2 \), and \( \psi \) is the yaw
angle. $v_i$ is termed with the side-slipping velocity. $I$ is the instantaneous centre of rotation. Because of the sideslip, it often shifts ahead $O_a$ by a mount $d$. An angle $\alpha$ exists between the line connecting $I$ to $O_a$ and the perpendicular of $I$ to $x$ axis of frame $F_2$.

\[ i_L = \frac{r_0 \omega_L - v_i}{r_0 \omega_L} \quad (1) \]
\[ i_K = \frac{r_0 \omega_K - v_i}{r_0 \omega_K} \quad (2) \]
where $r$ is the radius of the wheels which drive the tracks; $\omega_L, \omega_K$ are the angular velocities of the left and right wheels.

The sideslip ratio can be described by the mount $d$, or equivalently defined as
\[ \sigma = \tan \alpha \quad (3) \]
In this paper, the mount $d$ is chosen to stand for sideslip.

In the frame $F_2$, a suitable model with slipping can be written as
\[ \dot{x} = \frac{r_0 \omega_L (1-i_L) + r_0 \omega_K (1-i_K)}{2} \quad (4) \]
\[ \dot{y} = -\frac{r_0 \omega_L (1-i_L) + r_0 \omega_K (1-i_K)}{b} \quad (5) \]
\[ \dot{\psi} = -\frac{r_0 \omega_L (1-i_L) + r_0 \omega_K (1-i_K)}{b} \quad (6) \]
where $b$ is the distance between two tracks.

Upon the transformation from $F_2$ to $F_1$, the kinematic model is modeled as
\[ \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi + \sigma \sin \psi & 0 & 0 \\ \sin \psi - \sigma \cos \psi & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (7) \]
Or simply re-writing (7) as
\[ \eta = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} r (1-i_L) \omega_L + r (1-i_K) \omega_K \\ -r (1-i_L) \omega_L + r (1-i_K) \omega_K \end{bmatrix} = \frac{2}{b} \begin{bmatrix} r (1-i_L) \omega_L + r (1-i_K) \omega_K \\ -r (1-i_L) \omega_L + r (1-i_K) \omega_K \end{bmatrix} \quad (9) \]
and $u = (\omega_L, \omega_K)^T$ is regarded as the real control input which can be used to control $\eta$ according to the relationship
\[ \eta = Tu \quad (10) \]
with the transformation matrix $T$ as
\[ T = \begin{bmatrix} (1-i_L)/2 & (1-i_K)/2 \\ -(1-i_L)/b & (1-i_K)/b \end{bmatrix} \quad (11) \]

Thus we have $u = T^{-1} \eta$:
\[ \begin{bmatrix} \omega_L \\ \omega_K \end{bmatrix} = T^{-1} \begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 1/(1-i_L) & -b/2(1-i_L) \\ 1/(1-i_K) & -b/2(1-i_K) \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (12) \]
Notice that the lateral velocity $v_i = -d \omega$, which determines velocity of sideslip, is not integrable and hence describes the nonholonomic constraint as
\[ (\sin \psi - \cos \psi) d \quad \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = A \dot{q} = 0 \quad (13) \]

Our object is to solve the tracking control problem of the tracked vehicle under the kinematic model (7) in which the slipping is considered as three time-varying parameters $i_L, i_K$ and $\sigma$. Two steps should be taken: first, the three slip parameters should be estimated online, as well as the pose state $q$; second, a robust tracking control law should be found and proved.

### III. UNSCENTED KALMAN FILTER

To estimate the states and slip parameters, joint estimation technique can be used, that is, states and parameters are estimated simultaneously using a same filter [25]. It is often used to solve the state feedback control with uncertain parameters, or the modeling of the parameters with noise and states that can’t be measured directly. Because of the incorporation of the states and the parameters, more accurate results may be made using this approach. In the localization of the tracked vehicle with slip, the pose and the slip parameters should be estimated at the same time. A new state vector is defined as a combination of the old state and parameter vector, $x_s = (X \ Y \ \psi \ i_L \ i_K \ \sigma)^T$. In this augmented state, the dynamic of the slip parameters are often unknown. They can be rewritten as
\[ \begin{bmatrix} p_{k+1} \\ \eta_k \end{bmatrix} = \begin{bmatrix} p_k \\ w_{s,k} \end{bmatrix} \quad (14) \]
where $p_k \in \mathbb{R}^r$ is the parameter vector; $w_{s,k} \in \mathbb{R}^r$ is the additive process noise which drives the model.

The unscented Kalman filter is introduced to joint estimate the state and slip parameters. Unlike EKF [9], the UKF does not approximate the nonlinear process and observation models. Instead, it uses the true nonlinear models and approximates the distribution of the state random variable.
The UKF, which does not need to compute the Jacobian, uses the so-called unscented transform (UT) and sigma points to propagate all of them through models. As a result, the UKF often leads to more accurate estimations than EKF.

Given the following general nonlinear system

\[ \begin{align*}
    x(k+1) &= f(x(k), u(k)) + w_k \\
    y(k+1) &= h(x(k)) + v_{k+1}
\end{align*} \]  

where \( w_k \) and \( v_{k+1} \) are process and measurement noise with covariance \( Q_k \) and \( R_{k+1} \), respectively. The initial state vector is defined as \( x_0 \). The UKF algorithm is given below [7]

1. Initialize with
   \[ \mathbf{x}_0 = E[x_0] \]  

2. For \( k = 0, 1, \ldots \)
   (a) Calculate sigma points
      \[ X_k = \left\{ \begin{array}{c} \hat{x}_k^a \\
                      \hat{x}_k^b \\
                      \gamma \sqrt{P_k} \hat{x}_k^b - \gamma \sqrt{P_k} \hat{x}_k^a \end{array} \right\} \]  
      with the weights
      \[ W_k^{(m)} = A / (L + \lambda), W_k^{(c)} = A / (L + \lambda) + \left( 1 - \alpha^2 + \beta \right) \]
      \[ W_k^{(c)} = 1 / 2(L + \lambda), I = 1, 2, \ldots, L \]
      where \( L \) is the dimension of the augmented states; \( \alpha \) controls the size of the sigma point distribution and should be ideally a small number to avoid sampling non-local effects when the nonlinearities are strong; and \( \beta \) is a non-negative weighting term, which can be used to acknowledge the information of the higher order moments of the distribution. For a Gaussian prior the optimal choice is \( \beta = 2 \), and to guarantee positive semi-definiteness of the covariance matrix \( \kappa \geq 0 \) is chosen. The rest parameters are defined as
      \[ \lambda = \alpha^2 / (L + \kappa) - \kappa, \gamma = \lambda / (L + \lambda) \]  
   (b) The prediction step
      \[ \mathbf{X}_{k+1,k} = f(\mathbf{X}_k, \mathbf{u}_k) \]
      \[ \tilde{\mathbf{x}}_{k+1,k}^a = \frac{2L}{i=0} \sum W_k^{(m)} \mathbf{X}_{k+1,k}^a(i) \]
      \[ \mathbf{P}_{k+1,k}^a = \sum W_k^{(c)} \left[ \mathbf{X}_{k+1,k}^a(i) - \tilde{\mathbf{x}}_{k+1,k}^a \right] \left[ \mathbf{X}_{k+1,k}^a(i) - \tilde{\mathbf{x}}_{k+1,k}^a \right]^T \]
      \[ \mathbf{X}_{k+1,k} = \left[ \begin{array}{c} \mathbf{X}_{k+1,k}^a \\
                      \hat{\mathbf{x}}_{k+1,k}^b \\
                      \gamma \sqrt{\mathbf{P}_{k+1,k}^a} \hat{\mathbf{x}}_{k+1,k}^b - \gamma \sqrt{\mathbf{P}_{k+1,k}^a} \hat{\mathbf{x}}_{k+1,k}^a \end{array} \right] \]
      \[ \mathbf{Y}_{k+1,k} = h(\mathbf{X}_{k+1,k}) \]
      \[ \tilde{\mathbf{y}}_{k+1,k} = \mathbf{H}_{k} \mathbf{X}_{k+1,k} \]
   (c) The update step
      \[ \mathbf{P}_{k+1,k} = \sum W_k^{(c)} \left[ \mathbf{Y}_{k+1,k} - \tilde{\mathbf{y}}_{k+1,k} \right] \left[ \mathbf{Y}_{k+1,k} - \tilde{\mathbf{y}}_{k+1,k} \right]^T \]
      \[ \mathbf{P}_{k+1}^u = \mathbf{P}_{k+1,k} - K_k \mathbf{P}_{k+1,k} \mathbf{K}_k^T \]

Note that the above equations are for the most general form of the UKF. For some special cases, such as the model is linear, the algorithm can be simplified significantly.

IV. TRACKING CONTROL AND LYAPUNOV ANALYSIS

A. Control Objective

In this chapter the tracking control problem will be considered. As defined in previous work (e.g., see [15] and [16]), the reference trajectory is generated via a reference robot that moves according to the following dynamic trajectory:

\[ \mathbf{q}_r = \mathbf{S}(\mathbf{q}_r) \mathbf{n}_r \]  

where \( \mathbf{q}_r(t) = \begin{bmatrix} x_r(t) & y_r(t) & \psi_r(t) \end{bmatrix}^T \in \mathbb{R}^3 \) is the desired time-varying position and orientation trajectory, and \( \mathbf{n}_r(t) = \begin{bmatrix} v_r(t) & \omega_r(t) \end{bmatrix}^T \in \mathbb{R}^2 \) is the reference time-varying linear and angular trajectory. With regard to (36), it is assumed that the signal \( \mathbf{n}_r(t) \) is constructed to produce the desired motion and that \( \mathbf{n}_r(t) \cdot \mathbf{n}_r(t) \cdot \mathbf{q}_r(t) \cdot \mathbf{q}_r(t) \) are bounded for all time. Here \( \psi_r(t) \) is assumed to be in \( (-\pi, \pi) \). Another assumption should be made is that \( v_r \) does not go to zero as \( t \to \infty \). The control objective is described as following:

Control Objective: Given the current configuration \( \mathbf{q}(t) \) in (12) and the reference trajectory \( \mathbf{q}_r(t) \) in (36), design the auxiliary tracking control law \( \mathbf{u}(t) \) such that

\[ \lim_{t \to \infty} (\mathbf{q}(t) - \mathbf{q}_r(t)) = 0 \]  

then the real control can be obtained through (10).

B. Dynamic Feedback Linearization

In order to design the system be fully linearized and input-output decouple by means of a dynamic state feedback, first, define the linearizing outputs as

\[ \mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\
                     \mathbf{z}_2 \end{bmatrix} = \begin{bmatrix} x + d \cos \psi \\
                     y + d \sin \psi \end{bmatrix} \]

Thus we can get the dynamic model of the linearizing outputs:

\[ \dot{\mathbf{z}} = \begin{bmatrix} \cos \psi \\
                     \sin \psi \end{bmatrix} \mathbf{v} \]

Because the input \( \omega \) is not contained in (39), another differential process should be done as

\[ \mathbf{z} = \begin{bmatrix} \cos \psi \\
                     -\sin \psi \\
                     \sin \psi \end{bmatrix} \begin{bmatrix} \mathbf{v} \\
                     \mathbf{w} \end{bmatrix} \]

An integrator is added to on the input
where \( a \) stands for the longitudinal acceleration of the robot. Now (40) can be rewritten as
\[
\ddot{\xi} = \begin{pmatrix} \cos \psi & -\ddot{\xi} \sin \psi \\ \sin \psi & \ddot{\xi} \cos \psi \end{pmatrix} \begin{pmatrix} a \\ \omega \end{pmatrix} = \alpha(q, \xi) \begin{pmatrix} a \\ \omega \end{pmatrix}
\] (42)

When the decoupling matrix \( \alpha(q, \xi) \) is designed, which means the longitudinal velocity \( v \) does not degrade to zero. Under this assumption, the control law
\[
\begin{pmatrix} a \\ \omega \end{pmatrix} = \begin{pmatrix} \cos \psi & -\ddot{\xi} \sin \psi \\ \sin \psi & \ddot{\xi} \cos \psi \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}
\]
(44)
where \( s = (s_1, s_2)^T \) is the trajectory input-output jerk reference. So we can obtain the fully linearized system
\[ \ddot{\xi} = s \] (45)
The resulting dynamic compensator is obtained by combing (40), (41) and (44)
\[
\ddot{\xi} = s_1 \cos \psi + s_2 \sin \psi
\]
(46)
\[
\omega = \left(s_1 \cos \psi - s_1 \sin \psi\right)/\ddot{\xi}
\]
Note that the above dynamic feedback linearizing controller has a potential singularity at \( \ddot{\xi} = v = 0 \), which should be taken into account carefully. When the reference velocity \( v \) is not identically zero, we can choose nonzero \( \ddot{\xi} \) temporarily when \( \ddot{\xi} = 0 \). After that moment \( \ddot{\xi} \) will be no longer zero because the persistence of trajectory.

### C. Exponential Stabilizing State Feedback

A globally exponential stabilizing state feedback controller can be easily chosen to complete the control design:
\[
\begin{align*}
s_1 &= \dot{z}_r + k_{d1}(\dot{z}_r - z_1) + k_{p1}(\dot{z}_r - z_1) \\
s_2 &= \dot{z}_r + k_{d2}(\dot{z}_r - z_2) + k_{p2}(\dot{z}_r - z_2)
\end{align*}
\] (47)

with the PD gains chosen such that
\[ \lambda^2 + k_p \lambda + k_p < 0 \]
(48)

is Hurwitz. Obviously we can choose
\[
k_{p1} > 0, k_{d1} > 0, i = 1, 2
\]
(49)

After the auxiliary control input \( \eta = [v, \omega]^T \) is designed, the real control input can be obtained from (10) \( u = T^{-1}\eta \).

### V. Simulations

In this section, we performed some simulations on the kinematic model of the tracked vehicle with slipping, using the methods described in previous section. The experimental platform used in study is a tracked vehicle with two arms as Fig. 2.

The sensors equipped on the vehicle include 2 accelerometers, a 2-DOF electrolytic tilt sensor, a pair of cameras, a gyroscope and a GPS. The vehicle has the ability of striding over some obstacles, and can localize itself in both 2D and 3D environments, although only planar motion is considered in the study.

In the simulation, the angular velocities of the two wheels that drive the two main tracks are considered as input variables. The physical parameters in the model are chosen as follows: \( b = 0.65m \), \( r = 0.35m \). The three states about the vehicle’s pose are measurements. The total time of the simulation is chosen as \( t = 60s \), and the time-step is 0.1s. The noises used in the model, including the process noise and the measurement noise, are assumed to be the Gaussian noise with zero mean value and 5% variance of the nominal values. The reference trajectories are used. One is the line trajectory with the input \( v = 0.1m/s \) and \( \omega = 0 \) rad/s. The other is the circle trajectory with \( v = 0.1m/s \) and \( \omega = 0.1 \) rad/s. In order to demonstrate the tracking performance, abrupt changes are simulated to occur in the three slipping parameters at time \( t = 30s \), that is, \( i_q = 0.1 \), \( i_k = -0.1 \), \( \sigma = 0.1 \). In both situation, the states and the time-varying slipping parameters are jointly estimated using UKF, and then the tracking control method mention in Section IV are implemented to achieve the tracking control objective.

In order to achieve satisfactory result, proper tuning of the filters is essential. The values in \( Q_i \) and \( R_{i+1} \) for Kalman based filters should be determined carefully. For UKF, the three filter parameters are chosen as \( \alpha = 1 \), \( \beta = 2 \), \( \gamma = 0 \). The reference input \( v_r, \omega_r \) are chosen as following [26]:

- \( 0 \leq t < 5: v_r = 0.25\left(1 - \cos \frac{\pi t}{5}\right), \omega_r = 0 \)
- \( 5 \leq t < 20: v_r = 0.5, \omega_r = 0 \)
- \( 20 \leq t < 25: v_r = 0.25\left(1 + \cos \frac{\pi t}{5}\right), \omega_r = 0 \)
- \( 25 \leq t < 30: v_r = 0.15\left(1 - \cos \frac{2\pi t}{5}\right), \omega_r = -v_r/1.5 \)
- \( 30 \leq t < 35: v_r = 0.15\left(1 - \cos \frac{2\pi t}{5}\right), \omega_r = v_r/1.5 \)
- \( 35 \leq t < 40: v_r = 0.25\left(1 + \cos \frac{\pi t}{5}\right), \omega_r = 0 \)
- \( 40 \leq t: v_r = 0.5, \omega_r = 0 \)
Fig. 3 shows the tracking performance of three slipping parameters using UKF filter, Fig. 4 shows tracking errors, Fig. 5 shows the trajectory and Fig. 6 shows the input $u = (\omega_L, \omega_R)^T$. In Fig. 3 and Fig. 5, the red dashed line stands for the reference value, while the blue real line is the results of our methods.

In order to test the performance and robustness of the controller, an abrupt change is added at time $t = 30\text{ s}$. Fig. 3 shows that UKF can track the time-varying slipping parameters quickly, even when abrupt changes happen. Fig. 4 – Fig. 6 shows our proposed control law can achieve the tracking objective very well, through an adaptation process when abrupt change happens.

VI. CONCLUDING REMARKS

The longitudinal and lateral slipping of a tracked mobile robot is considered and modeled as three time-varying parameters. The kinematic model containing the slipping parameters is proposed and UKF is used to joint estimate the pose and slipping parameters. A dynamic feedback linearization integrated an exponential stabilizing feedback control law is proposed to achieve the trajectory-tracking objective, using the estimated slipping parameters. A combination trajectory is used to test our algorithms and the simulation results are given. More works should be done to investigate the physical mechanism of slipping and the dynamic tracking control will be studied in future.

REFERENCES
