Model Identification and Active Modeling Control for Small-Size Unmanned Helicopters: Theory and Experiment

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In this paper, a semi-decoupled state space model of a small-size helicopter is developed for hovering conditions in order to simplify the model identification process. An enhanced identification algorithm in frequency domain is proposed and implemented to estimate the parameters in the state space model using real flight data collected from a SERVOHELI-40 small-size unmanned helicopter. Based on adaptive set-membership filter and the built model, this paper also presents a strategy of model-error estimation and elimination to solve the impact of model mismatch in real flight. The accuracy of the identified model is verified by simulation in time domain, using a different set of hovering flight data. Using the identified model for hovering as the nominal model, the developed strategy is tested on SERVOHELI-40 UAV platform through real flight experimentation. The results have shown the accuracy of the developed semi-decoupled model and the effectiveness of the proposed optimal estimation algorithm in frequency domain, and that proposed strategy for the estimation and elimination of the model-error is feasible and practical for the flight control of unmanned helicopters in full flight envelope.

I. Introduction

Unmanned helicopters are increasingly popular platforms for unmanned aerial vehicles (UAVs). With abilities to hover, to take off and to land vertically, unmanned helicopters extend the potential applications of UAVs. Compared with traditional full-size helicopters, small-size helicopters tend to be naturally more maneuverable and more responsive. However, only a modest part of the helicopter's inherent qualities are exploited in the relevant literature, and there are few suitable state-space models available for advanced flight control design.

Helicopter system modeling and identification are highly versatile procedures for extracting dynamic model of a helicopter from the measured response to specific control inputs. Modeling of full-size helicopters based on first principles has already been reported in the literature, these include tilt-rotor aircraft XV-15 ¹, helicopter BO-105 ², UH-60 ³ and SH-2G ⁴. However, there are only several reported applications of system identification techniques to modeling of small-size helicopters, including the model identification of YAMAHA R-50 ⁵ and X-Cell ⁶ for flight control, and a six-DoF dynamic modeling of Raptor 50 V2 for simulations ⁷. The difficulties in modeling small-size helicopters lie in high frequency dynamics and noise from sensors. The environment also needs to be considered due to their structural characteristics, and a large amount of flight data is necessary for system identification.

A reliable dynamic state-space model for small-size unmanned helicopters was first presented in reference ⁸. The model explicitly accounts for the stabilizer bar and rotor system, based on first principles, which has a strong influence on the flight dynamics characteristics of small-size unmanned helicopters. Optimal estimation of the model parameters is done in frequency domain for difference flight modes, using sweeping inputs. However, some parameters in the model cannot be measured from the input or output data because of too many coupling parameters in the system matrix, and the strong correlation among parameters makes the initial estimation difficult to select. Some constraints must be added during identification and the initial estimation of parameters must be done based on

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a large number of flight experiments, which is costly. On the other hand, although the linear model structure is easy
for flight controller design, due to the linearization, the variable working point and dynamics may cause the nominal
parameters to change in transient state in real flight, which cannot be completely simulated by computers. Hence,
the model errors and immeasurable states must be estimated and submitted to the nominal flight controller, which is
designed based on nominal model, to keep the helicopter system stable and robust.

In recent years, the encouraging achievement in sequential estimation makes it an important direction for online
modeling and model-reference control. Among stochastic estimations, the most popular one is the Kalman-type
filters (KFs). Although widely used, the KFs suffer from sensitivity to bias and divergence in the estimates,
relying on assumptions on statistic distribution such as white noise and known mean or covariance for optimal
estimation. In many cases, it is more practical to assume that the noises or uncertainties are unknown but bounded
(UBB). In view of this, the set-membership filter (SMF), which computes a compact feasible set in which the true
state or parameter lies only under the UBB noise assumption, provides an attractive alternative. This supply an
effective method to estimate the model errors and immeasurable states in real flight, and compensation for the
nominal controller can be implemented based on the estimate to keep the helicopter system stable and robust.

In this paper, based on the current research, we do the following works for flight model building and the online
compensation of the model errors for nominal controllers in real flight. First, for effective model identification, the
original model in reference is decomposed into three groups (longitudinal, lateral, and yaw-heave coupling), and a
semi-decoupled model is obtained. Each group has a decoupled system matrix, and the coupling characteristics are
presented only in the control matrix. Thus, the number of unknown parameters and control inputs are reduced and
the control loops are semi-decoupled. Considering the changing dynamics in full flight envelope, the
semi-decoupled model is enhanced through an additional model error expression.

To identify the unknown parameters in the MIMO semi-decoupled model, a new cost function is proposed to
make the traditional method of SISO system frequency estimation applicable to the MIMO state-space models. The
proposed cost function is presented in the additional form of the frequency error of every input-output pair for
transfer matrix, and the parameters are identified by minimizing the cost function. An enhanced frequency
identification process is proposed to make the traditional identification process more efficient for small-size
helicopter identification. The proposed identification process first uses linear regression in time domain to get the
crude state estimation, and then runs the frequency response method to obtain the free parameters. Based on the
value of the free parameters, a norm convergence criterion, which is about the parameter matrix error between
adjacent iterations, is defined to avoid excessive iteration. The simplified model and proposed identification method
free the selection of initial estimation and constraint is not required.

To eliminate the model mismatch’s influence to nominal controllers, based on the adaptive set-membership filter, a
joint estimator is applied for the on-line estimation of the above model’s uncertainty since the unknown statistic
characteristic of noise. Through extending the system states, this proposed estimator can obtain the states, model
errors and their boundaries at the same time. Base on the value and boundary of the model error, which are
estimated by the adaptive set-membership filter, a novel optimal strategy for on-line compensation is designed.
Through minimizing a cost function of the model uncertainty and the indication of the health of set-membership
filter, this strategy not only deals with the tracking error, but also keeps the estimator-controller system stable at the
same time.

At last, the improved model and method of frequency identification are applied to the SERVOHELI-40, a
small-size helicopter, in hovering and cruising flight modes. The data was collected from the experimental platform
by applying frequency sweeping input, and the nominal model’s accuracy is verified by simulations in time-domain,
using the data from another flight experiment. At the same time, model errors and their boundaries are estimated by
the proposed estimator in transient state and compensated by the developed strategy. The conclusion, obtained from
the simulation and flight experiment, is that the developed semi-decoupled model is feasible in practice and the
proposed optimal estimation algorithm in frequency domain is effective. Successful identification of model
parameters has been achieved using the data of a one-minute hovering and cruising flight experiment, and the model
errors and their boundaries, which can be used by flight controllers, can be accurately obtained and completely
eliminated to improve the robustness and tracking performance.

II. Experiment Platform Setup

The entire experiment was implemented on the SERVOHELI-40 small-size helicopter platform (Fig. 1).
ServoHeli-40 aerial vehicle is a high quality helicopter, which is changed by us using a RC technical grade helicopter operating with a remote controller. The modified system allows the payload of more than 10 kilograms, which is sufficient to take the whole airborne avionics box and the communication units for flight control. The fuselage of the helicopter is constructed with sturdy duralumin, and composite body and the main rotor blades are replaced with heavy-duty fiber glass reinforced ones to accommodate extra payloads. The vehicle is powered by a ZENOAH engine which generates 9hp at about 10000 rpm, a displacement of 80cc and practical angular rate ranging 2,000 to 16,000 rpm. The full length of the fuselage is 2120mm, as well as the full width of it is 320mm. The total height of the helicopter is 730mm, the main rotor is 2150mm and the tail rotor is 600mm.

Designing the avionics box and packing the box appropriately under the fuselage of the helicopter are two main tasks to implement of the UAV helicopter system. In the actual flight environment, the weight and the size of the avionics box are strict limited. Our airborne control box, which is shown in Fig. 2, is a compact aluminum alloy package mounted on the landing gear. The center of gravity of the box lies on the IMU device, where is not the geometry center of the system that keeps the navigation data from IMU accurate. The compass and the IMU, are taken as the horizontal center of the gravity of the avionics system and the other components are installed on the same line.

The platform, which is fixed with a 3-axis gyro, a three-axis acceleration sensor, a compass and a GPS, can save the data of velocities, angular rates, Euler angle accelerations and positions into a SD card through an ARM processor. An Extended Kalman Filter (EKF) is used to estimate the values of sensors. There is a CPLD used for sampling control inputs from the remote control of the pilot. The rotor speed is controlled by a Governor, an electronic unit for engine control. Table 1 shows the physical characteristics of SERVOHELI-40 small-size helicopter.
Table 1 Physical characteristics of SERVOHELI-40 small-size helicopter

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>2.12m</td>
</tr>
<tr>
<td>Height</td>
<td>0.73m</td>
</tr>
<tr>
<td>Main rotor diameter</td>
<td>2.15m</td>
</tr>
<tr>
<td>Stabilizer bar diameter</td>
<td>0.75m</td>
</tr>
<tr>
<td>Rotor speed</td>
<td>1450rpm</td>
</tr>
<tr>
<td>Dry weight</td>
<td>20kg</td>
</tr>
<tr>
<td>Engine</td>
<td>2-stroke, air cooled</td>
</tr>
<tr>
<td>Flight time</td>
<td>45 min</td>
</tr>
</tbody>
</table>

III. Dynamics Analysis

In reference 8, a parameterized state-space model is presented that explicitly accounts for the stabilizer bar and rotor system based on first principles. The model can be expressed in the following form

\[
\begin{bmatrix}
\Delta \dot{u} \\
\Delta \dot{v} \\
\Delta \dot{\phi} \\
\Delta \dot{\phi} \\
\Delta \dot{\theta} \\
\Delta \dot{\alpha} \\
\Delta \dot{\beta} \\
\Delta \dot{\gamma} \\
\Delta \dot{d}
\end{bmatrix} = A \Delta \dot{X} + B \Delta U
\]

where \( u, v, w \) are longitudinal, lateral, and vertical velocity, \( p, q, r \) are roll, pitch, and yaw angle rates, \( \phi \) and \( \theta \) are the angles of roll and pitch, respectively, \( a \) and \( b \) are the first harmonic flapping angle of main rotor, \( c \) and \( d \) are the first harmonic flapping angle of stabilizer bar, \( r_{fb} \) is the feedback control value of the angular rate gyro, \( \delta_{lat} \) is the
lateral control input, $\delta_{lon}$ is the longitudinal control input, $\delta_{ped}$ is the yawing control input, and $\delta_{col}$ is the vertical control input. $X$ is a $13 \times 1$ vector of states, $A$ is the $13 \times 13$ system matrix, $B$ is the $13 \times 4$ control matrix, and $U$ is the $4 \times 1$ control inputs. All the symbols except gravity acceleration $g$ in $A$ and $B$ are unknown. Thus, all of the states and control inputs in (1) are physically meaningful and defined in body-axis, which is described in Fig. 3.

![Fig. 3 Helicopter with its body-fixed reference frame](image)

The measurement equation for the state space model can be determined according to the type of sensors on the helicopter platform. For our SERVOHELI-40 small-size unmanned helicopter platform, which will be described in further detail in the following section, all velocities, angular rates, Euler angles and accelerations are available for identification and control, so we select the measurement equation as

$$
I_{66} \begin{bmatrix}
\delta_{lon} \\
\delta_{lat} \\
\delta_{a} \\
\delta_{c}
\end{bmatrix}
= \begin{bmatrix}
I_{66} & 0_{62} & 0_{62} & 0_{63} \\
0_{66} & I_{22} & 0_{23} & 0_{33} \\
0_{66} & 0_{22} & I_{33}
\end{bmatrix}
\begin{bmatrix}
\delta_{lon} \\
\delta_{lat} \\
\delta_{a} \\
\delta_{c}
\end{bmatrix}
+ \begin{bmatrix}
0_{60} & 0_{60} \\
0_{30} & 0_{30} \\
0_{30} & I_{33}
\end{bmatrix}
\begin{bmatrix}
X_{lon} \\
X_{lat}
\end{bmatrix}
$$

where $I_{m \times m}$ is the $m \times m$ unit matrix and $0_{m \times n}$ is the $m \times n$ zero matrix.

Equation (1) has thirteen states and four coupling control channels. There are forty-four unknown parameters in the system matrix and control matrix. Thus, the matrix computation is complex and some parameters cannot be directly identified from the input-output data because they are unmeasured. This is known as over-parameterized and therefore constraints must be used during identification, for example, in reference, the author suppose $N_{r fb} = -N_{ped}$ and $K_{r fb} = 2N_r$. At the same time, the strong correlation among parameters also makes the initial estimation difficult to select because of the coupling control loops and the large number of control inputs. The identification process using equation (1) directly will be complex and costly.

To simplify the identification process, we consider the fact that lateral and longitudinal dynamics are approximately decoupled when hovering, and decompose the model into three parts: lateral dynamics, longitudinal dynamics and coupling dynamics of altitude with yaw. Each part is coupled with others through the control matrix. We set lateral and longitudinal coupling parameters in the matrix $A$ to zero, and add some free control coefficients in matrix $B$ to compensate the coupling dynamics. Then, each part involves only two control inputs and five states at most, and their parameters are simple to identify. The semi-decoupled state equations for the three parts are

$$
\begin{bmatrix}
\Delta u \\
\Delta q \\
\Delta \theta \\
\Delta a \\
\Delta c
\end{bmatrix}
= A_{lon} \begin{bmatrix}
X_{lon} \\
X_{lat}
\end{bmatrix}
+ B_{lon} \begin{bmatrix}
\delta_{lon} \\
\delta_{lat}
\end{bmatrix}
= A_{lon} \Delta X_{lon} + B_{lon} \Delta \delta_{lon}
$$
\[
\begin{bmatrix}
\Delta v \\
\Delta \dot{p} \\
\Delta \phi \\
\dot{b} \\
\dot{d}
\end{bmatrix}
= \Delta \dot{X}_{lat} =
\begin{bmatrix}
Y_y & 0 & g & Y_y & 0 \\
L_y & 0 & 0 & L_y & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 / \tau_f & B_\phi / \tau_f \\
0 & -1 & 0 & 0 & -1 / \tau_f
\end{bmatrix}
\begin{bmatrix}
\Delta v \\
\Delta \dot{p} \\
\Delta \phi \\
\dot{b} \\
\dot{d}
\end{bmatrix}
+ \begin{bmatrix}
Y_{lon} & Y_{lat} \\
L_{lon} & L_{lat} \\
0 & 0 \\
B_{lon} & B_{lat} \\
D_{lon} & D_{lat}
\end{bmatrix}
\begin{bmatrix}
\Delta X_{lon} \\
\Delta X_{lat} \\
\delta_{lon} \\
\delta_{lat}
\end{bmatrix}
= A_{lat} \Delta X_{lat} + B_{lat} \Delta u_{lat}
\] (4)

\[
y_{lat} = (I_{3 \times 3} 0_{3 \times 2}) \Delta X_{lat} = C_{lat} \Delta X_{lat}
\]

\[
\begin{bmatrix}
\Delta \dot{v} \\
\Delta \dot{\phi} \\
\Delta \dot{b}
\end{bmatrix}
= \Delta \dot{X}_{lat-hane} =
\begin{bmatrix}
Z_w & Z_r & 0 \\
N_w & N_r & -N_{ped} \\
0 & K_r & -K_{r fb}
\end{bmatrix}
\begin{bmatrix}
\Delta v \\
\Delta \phi \\
\Delta \dot{b}
\end{bmatrix}
+ \begin{bmatrix}
Z_{ped} & Z_{col} \\
N_{ped} & N_{col} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{ped} \\
\delta_{col}
\end{bmatrix}

= A_{lat-hane} \Delta X_{lat-hane} + B_{lat-hane} \Delta u_{lat-hane}
\] (5)

For normal missions of unmanned helicopters, the flight modes often include hovering (velocity under 5 m/s), cruising (velocity above 5 m/s), taking off and landing (distance to the ground is under 3 m, and there exists significant ground effect) and the transitions among these modes. Hence, the nominal model can be identified more easily in hovering than other flight modes, because the range of the working point is large and pumping signal can be inputted by a remote controller within the hovering flight mode. However, if we only use model (3)-(5) to describe helicopter dynamics in full flight envelope, the dynamics of the helicopter changes with flight mode, leading to model error of the nominal model, which is typically obtained by linearizing the nonlinear model at nominal working points. The model errors from linearization, external disturbance and unmodeled dynamics can be considered as additional process noise, which has non-linear relationship with system states \(X\), time \(t\) and process noise \(W\). Hence, we use non-linear function \(f(X, X, W, t) \in \mathbb{R}^{13}\), where \(X\) is the state vector and \(W \in \mathbb{R}^{13}\) is the process noise, to represent the varying model error in full flight envelope. Thus, a parameterized state-space model helicopter dynamics in the full flight envelope can be formulated as

\[
\begin{align*}
\dot{X} &= A_0 X + B_0 U + B_f f(X, X, W, t) \\
Y &= CX
\end{align*}
\] (6)

where \(A_0 = \text{diag}(A_{lon}, A_{lat}, A_{yaw-heave})\), \(B_0 = \begin{bmatrix} B_{lon} & B_{lat} & B_{yaw-heave} \end{bmatrix}^T\), \(U = \begin{bmatrix} \delta_{lon} & \delta_{lat} & \delta_{ped} & \delta_{col} \end{bmatrix}^T\), \(C = \text{diag}(C_{lon}, C_{lat}, C_{yaw-heave})\) and \(B_f = I_{13 \times 13}\). \(A_0\) and \(B_0\) can be identified in different flight modes, with different parameters.

### IV. Parameter Identification and Online Model-error Elimination

#### A. Preliminary Work for Parameter Identification in Frequency Domain

In the frequency domain, the input and output of a system are related through the frequency-response function:

\[
Y(j \omega) = H(j \omega)U(j \omega)
\] (7)

where \(U(j \omega)\) and \(Y(j \omega)\) are the Fourier transforms of the input and output signals \(u(t)\) and \(y(t)\), respectively, and \(H(j \omega)\) is the Fourier transform of the impulse-response function. If \(u(t)\) and \(y(t)\) are finite-length discrete samples, the Fourier transform of the input and output signals \(u(t)\) and \(y(t)\) can be obtained through a discrete Fourier transform (DFT) as follows:

\[
Y(j \omega_k) = \sum_{n=0}^{N-1} y(t_n) e^{-j \omega_k t_n}, k = 0, 1, 2, \ldots N - 1
\] (8)

\[
U(j \omega_k) = \sum_{n=0}^{N-1} u(t_n) e^{-j \omega_k t_n}, k = 0, 1, 2, \ldots N - 1
\] (9)

where \(w_k = k \Omega_s\) is the discrete frequency point, \(\Omega_s = 2 \pi / (NT_s)\) is the frequency sampling interval, \(T_s\) is the time sampling interval, \(N = T_d / T_s\) is the number of sample points and \(T_d\) is the length of a data segment. Then,
for \( N_d \) experiments, the input auto-spectral density function \( G_{uu}(j\omega) \) and input-output cross-spectral density function \( G_{uy}(j\omega) \) can be estimated as follows:\(^{12}\):

\[
\hat{G}_{uu}(j\omega_k) = \frac{2}{N_dT_d} \sum_{n_d=1}^{N_d} U_{n_d}(j\omega_k)U_{n_d}^*(j\omega_k)
\]

\[
\hat{G}_{uy}(j\omega_k) = \frac{2}{N_dT_d} \sum_{n_d=1}^{N_d} Y_{n_d}(j\omega_k)Y_{n_d}^*(j\omega_k)
\]

Finally, the estimates of the frequency response at the discrete frequency points \( \omega_k \) are computed from

\[
\hat{H}(j\omega_k) = \frac{\hat{G}_{uy}(j\omega_k)}{\hat{G}_{uu}(j\omega_k)}
\]

A magnitude squared coherence function \( \gamma_{uy}^2 \) is defined here to represent the correlation metric between \( u(t) \) and \( y(t) \):

\[
\gamma_{uy}^2 = \frac{|G_{uy}|^2}{|G_{uu}|^2 |G_{yy}|}
\]

where \( G_{yy}(j\omega) \) is output auto-spectral density function.

In reference\(^{2}\), a cost function is defined as

\[
J = \sum_{i=1}^{n_d} \epsilon(\omega_i, \Theta)^T W(\omega_i) \epsilon(\omega_i, \Theta)
\]

where \( \omega_i \) is the frequency point, \( \epsilon(\omega_i, \Theta) \) is the vector of magnitude and phase error between predicted response based on parameters \( \Theta \) and the sampling data in frequency domain, and \( W(\omega_i) \) is a \( 2 \times 2 \) weight matrix related with the magnitude squared coherence \( \gamma_{uy}^2 \) (uses \( W_\gamma = 2.25(1 - \gamma_{uy}^2)^2 \) to emphasize the most reliable data\(^2\)). So, \( \Theta \) can be obtained by minimizing the cost function (13) for SISO systems. Thus, the process of identification for rotorcraft\(^{2}\) is illustrated in Fig. 2. The cost function \( J \) in (14) is a complex non-linear function of unknown parameters \( \Theta \), and we can minimize it through secant method\(^{10}\) to obtain \( \Theta \).

![Fig. 4 Flowchart of the frequency-domain identification process for SISO system](image)

**B. Enhanced Identification Process for MIMO System in Frequency Domain**

For a MIMO linear parameterized state-space model

\[
\begin{align*}
\dot{X} &= A(\Theta)X + B(\Theta)U \\
Y &= C(\Theta)X
\end{align*}
\]
where $\Theta$ is the unknown parameters, $X$ is the $n \times 1$ vector of states, $U$ is the $r \times 1$ control inputs, $A$ is the $n \times n$ system matrix, $B$ is the $n \times r$ control matrix, and $C$ is the $p \times n$ measurement matrix. However, the SISO cost function (14), in part $A$, is not applicable here because (15) is a MIMO state-space model. So, we have to consider a new cost function for all of the input-output pairs.

For the MIMO linear parameterized state-space model (15), the impulse-response matrix can be obtained as:

$$T(j\omega, \Theta) = C(\Theta)(j\omega - A(\Theta))^{-1}B(\Theta) + D(\Theta)$$  \hspace{1cm} (16)

If frequency points are selected as $(\omega_1, \omega_2, \ldots, \omega_n)$, based on the structure of cost function (14), we can define a new cost function as follows:

$$J_M = \sum_{i=1}^{n_T} \left\{ \sum_{k=1}^{n} w_k \left[ 2 \Delta(\Theta_k) + W_p(\Delta\omega_k)^2 \right] \right\}$$  \hspace{1cm} (17)

where $\Delta | \cdot |_l$ is the magnitude error between prediction and real frequency of $i$th impulse-response function at $\omega_k$, $\Delta\omega_k$ is the phase (deg) error between predictive and real frequency of $i$th impulse-response function at $\omega_k$, $W_\gamma$ is a weighting function dependent on the value of magnitude squared coherence function $\gamma_{uy}$, $W_g$ and $W_p$ are the relative weights for magnitude and phase squared errors, $n_T$ is the number of elements in $T(j\omega, \Theta)$. Thus, the process for SISO systems, illustrated in Fig. 4, is applied to an MIMO state-space model.

For practice use, the above frequency response identification process (Fig. 4) has the following two problems: first, the initial estimation of parameters must be adjusted for secant search method by hand based on a large number of flight experiments, which is costly for rotorcraft; second, no parameters convergence criterion exists to stop the iteration process, which may make numerical value unstable.

To get the initial estimation of the unknown parameters, we define the following one-step prediction error criterion

$$\sum_{i=1}^{N} \left\| \hat{X}_{t+1} - X_{t+1} \right\|^2$$  \hspace{1cm} (18)

where $\hat{X}_{t+1} = X_i + dt(A(\Theta)X_i + B(\Theta)U_i)$ is one-step prediction at time $t$, $dt$ is the sampling time. Let column vector $\hat{\theta}_i$ stand for the unknown parameters in the $i$th row of matrix $[A(\Theta) \ B(\Theta)]$, then we have

$$\hat{X}_{i,t+1} = \varphi_{i,t} \hat{\theta}_i + F_{i,t}$$  \hspace{1cm} (19)

where $\varphi_{i,t}$ is the mixed state and input row vector at time $t$ and its coefficient in the $i$th row of matrix $[A(\Theta) \ B(\Theta)]$ is unknown, and $F_{i,t}$ is the known term for calculating the $i$th element of the one-step prediction vector $\hat{X}_{t+1}$. Let $\hat{Q}_{i,t} = \hat{X}_{i,t+1} - F_{i,t}$, then we have

$$\hat{Q}_i = \varphi_i \hat{\theta}_i \Rightarrow \left\| \hat{X}_i - X_i \right\| = \left\| (\hat{X}_i - F_i) - (X_i - F_i) \right\| = \left\| \hat{Q}_i - \varphi_i \hat{\theta}_i \right\|$$  \hspace{1cm} (20)

$$\Rightarrow \sum_{i=1}^{N} \left\| \hat{X}_{t+1} - X_{t+1} \right\|^2 = \sum_{i=1}^{n} \left\| \hat{X}_i - X_i \right\|^2 = \sum_{i=1}^{n} \left\| \hat{Q}_i - \varphi_i \hat{\theta}_i \right\|^2 = \sum_{i=1}^{N} \sum_{i=1}^{n} J_i$$

where $\hat{Q}_i = (\hat{Q}_{1,t} \hat{Q}_{2,t} \ldots \hat{Q}_{N,t})$, $\hat{X}_i = (\hat{X}_{1,i} \hat{X}_{2,i} \ldots \hat{X}_{N,i})$, and $F_i = (F_{1,i} F_{2,i} \ldots F_{N,i})$. To minimize $\sum_{i=1}^{N} \left\| \hat{X}_{t+1} - X_{t+1} \right\|^2$, $\hat{\theta}_i$ must set $\frac{\partial J}{\partial \theta_i} = 0$ for every $i$, thus we can obtain $\hat{\theta}_i$ through linear regression as

$$\hat{\theta}_i = \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T \hat{Q}_i, \quad i = 1, 2, \ldots, n$$  \hspace{1cm} (21)

Thus, using linear regression, the initial estimation can be obtained in time domain by minimizing cost function (18).

To diagnose the parameters convergence, we propose the following criterion

$$\left\| A^{(t+1)} - A^{(i)} \right\| + \left\| B^{(t+1)} - B^{(i)} \right\| < \varepsilon$$  \hspace{1cm} (22)
where $A^{(i)}$ is the $i$th estimation of $A(\theta)$, $B^{(i)}$ is the $i$th estimation of $B(\theta)$, and $\varepsilon \in R$ is the threshold value set by hand. To maintain the nonlinear optimal search algorithm stable, we must avoid parameters’ sudden change between two iterations. We use the following equations to achieve this

$$
A^{(i+1)} = (1-\alpha)A^{(i)} + \alpha \bar{A}
$$

$$
B^{(i+1)} = (1-\alpha)B^{(i)} + \alpha \bar{B}
$$

where $\alpha \in (0,1)$ and $\bar{A}$ and $\bar{B}$ are the current parameter estimation by minimizing cost function (16).

Thus, an enhanced frequency identification process is proposed as the following steps:

1. Use Equation (21) to minimize cost function (18) to calculate the initial estimation of parameters $(0)^{A}$ and $(0)^{B}$, and set $i=1$.
2. Calculate the frequency response based on equations (7-11), and select the interesting frequency points $(\omega_1, \omega_2, ..., \omega_n)$.
3. Run the frequency identification process in Fig. 2 with the proposed cost function (17), and let

$$
(\hat{A}, \hat{B}) = \min_{\theta} J_M(\theta)
$$

4. Set

$$
A^{(i+1)} = (1-\alpha)A^{(i)} + \alpha \bar{A} \quad \text{and} \quad B^{(i+1)} = (1-\alpha)B^{(i)} + \alpha \bar{B}.
$$

5. If $\|A^{(i+1)} - A^{(i)}\| + \|B^{(i+1)} - B^{(i)}\| > \varepsilon$ then $i = i+1$ and go back to step 2), otherwise, let $\hat{A} = A^{(i+1)}$ and $\hat{B} = B^{(i+1)}$.

**Theorem**: the parameter estimation vector $\hat{\theta}$, obtained by minimizing cost function (18), is unbiased and consistent estimate for the real parameter vector $\theta$, and the enhanced frequency identification process has

$$
\lim_{i \to \infty} \begin{bmatrix} A^{(i)} \\ B^{(i)} \end{bmatrix} = [A \ B].
$$

Proof:

$$
\hat{\theta}_i = \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T Q_i \Rightarrow E\left\{ \hat{\theta}_i \right\} = E\left\{ \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T Q_i \right\}
$$

$$
= E\left\{ \left( \phi_i^T \phi_i \right)^{-1} \phi_i^T (\theta_i - \hat{\theta}_i)(\theta_i - \hat{\theta}_i)^T \phi_i \left( \phi_i^T \phi_i \right)^{-1} \right\}
$$

$$
= \sigma^2 \left( \phi_i^T \phi_i \right)^{-1}
$$

$$
= \lim_{N \to \infty} trCov\{\theta_i(N)\} = \lim_{N \to \infty} tr\sigma^2 \left( \phi_i^T (N)\phi_i(N) \right)^{-1}
$$

$$
= \lim_{N \to \infty} tr \frac{1}{N} \sigma^2 \left( \frac{1}{N} \phi_i^T (N)\phi_i(N) \right)^{-1} = 0
$$

Because $\theta = (\theta_1, ..., \theta_n)$, $\hat{\theta}$ is unbiased and consistent estimate for the real parameter vector $\theta$. For cost functions about linear combination of magnitude and phase, the traditional frequency method in Fig 2 is unbiased and consistent estimate for unbiased and consistent initial estimate and has

$$
\lim_{N \to \infty} \begin{bmatrix} \hat{A}(N) \\ \hat{B}(N) \end{bmatrix} = [A \ B],
$$

where $N$ is the number of the selected frequency points. Thus, if we select different frequency points for every iteration $i$ and considering $E\left\{ (1-\alpha)\theta^{(i)} + \alpha \bar{\theta} \right\} = \theta$, $\bar{\theta}$ is the current parameter estimation by minimizing cost function (18).

Then, we have

$$
\lim_{i \to \infty} \begin{bmatrix} A^{(i)} \\ B^{(i)} \end{bmatrix} = [A \ B].
$$

### C. Adaptive set-membership filter based joint estimation
In Section B, we can obtain the nominal parameters for the unmanned helicopter in certain flight mode, however, the parameters are only effective for the flight mode, and will change in full flight envelope, which includes many flight modes. The changing parameters and unknown process disturbance bring model error to nominal controller, and cause tracking errors in real flight.

To eliminate the model error influence, based on the above parameterized model (3-5), model error must be estimated and eliminated online in real flight. This part adopts the joint estimation to get the boundaries of model error, based on adaptive set-member filter (ASMF), and the strategy for the elimination of the estimated model error will be described in the next part.

Joint estimation is a method that estimates the states and model parameters at the same time. During the estimation, the model parameters are considered as the variables of the system to form the extended states. Thus, the parameters are identified online when system states are estimated by filters. With the following operations, the joint estimation can be used for the model error estimation.

Let the extended state $X^a = \left( X \quad f^T \right)^T$ and consider only measurable states, then we can obtain the discrete equation for estimation as

$$
X_{k+1}^a = A_d^a X_k^a + B_d^a U_k + W_k^a
$$

$$
Y_k = C_d^a X_k^a + V_k
$$

where $A_d^a = \begin{pmatrix} A_d & B_f \\ 0_{13 \times 13} & I_{13 \times 13} \end{pmatrix}$, $B_d^a = \begin{pmatrix} B_d \\ 0_{13 \times 4} \end{pmatrix}$, $C_d^a = \begin{pmatrix} C_d & 0_{8 \times 13} \end{pmatrix}$, $W_k^a = \left( W^T \quad h^T \right)^T$, $B_f = I_{13 \times 13}$ and $f$ is a 13×1 vector for model errors. Here, $t$ is the sampling time, $I_{m \times m}$ is the $m \times m$ unit matrix and $0_{m \times n}$ is the $m \times n$ zero matrix. $\{A_d, B_d, C_d\}$ is the discrete expression of system $\{A_0, B_0, C\}$.

Because the model error comes from the linearization, neglected coupling dynamics and disturbance, the process noise vector $W^a$ may not have normal distribution in full flight envelope, and KALMAN type filter, which needs the known noise distribution, does not work well here.

To estimate the model error and states, adaptive set-membership filter is used as it does not need the initial distribution of the noise and only requires it to be bounded. The adaptive set-membership filter was first proposed in, and it can estimate not only the value of the states, but also the corresponding boundary. This boundary provides robustness for the estimator and is used for formulating an optimal strategy for model error compensation, which will be developed later in this paper.
For Equation (25), we can build the adaptive set-membership filter as (26), where $Q^a$ and $R^a$ are the initial elliptical boundary of process noise and measurement noise respectively, $r_m$ is the maximum eigenvalue of $R$, $p_m$ is the maximum eigenvalue of $C^a P_{l|k-1} C^{aT}$, $Tr(X)$ is the trace of matrix $X$, and $\delta_k$ and $\beta_k$ are the adaptive parameters of the filter. Thus, we can also obtain the boundary of the $i$th extended state $\hat{a}_i^a X$ as $\mathbb{E} \left[ \hat{a}_i^a X \right] = \mathbb{E} \left[ \hat{a}_i^a X \right] + \mathbb{E} \left[ \hat{a}_i^a X \right]$, where $p_{ii}$ is the $i$th element in the diagonal of matrix $P$. The details of adaptive set-membership filter and the explanation of the variables and parameters of ASMF, please see reference 14.

D. Optimal strategy for the elimination of the model error

In order to compensate the model error in Eq. (25), the control vector has to match the following equation, which can be directly obtained from Eq. (25):

$$B_d U_k + B_f f_k = B_d U_k$$

(27)

where $U_k$ is the control vector need to be calculated by a nominal controller, designed based on the original model (25) without the model error. This nominal controller, here, can be designed based on the identified model in section B through some controls methods, such as PID control (we use it for flight test in Part V), model reference control, $H_\infty$ control and etc. This section only discusses the compensation for the nominal controller, and the type of the nominal controller is not cared about here.

The control input at sampling time $t$ cannot be solved directly from Eq. (27), because:

1) Eq. (27) is difficult to be implemented because the dimension of $U_k$ is less than that of $f_k$. Thus, only the approximate solution can be obtained with respect to (27);

2) $f_k$ is actually an uncertainty set, a static optimal problem must be considered.

Thus, we introduce the following cost function with quadratic form to solve the above problem 1).

$$U_k^* = \arg \min_{U_k} J_k(U_k)$$

(28)

$$J_k(U_k) = \left( B_d U_k + B_f f_k - B_d U_k^0 \right)^T H \left( B_d U_k + B_f f_k - B_d U_k^0 \right)$$

where $H$ is a weight matrix, which can be selected.

On the other hand, $f_k$ is obtained from the ASMF algorithm introduced in section IV.C, thus its convergence is very important for the validity of the whole controller. Actually, the convergence of ASMF algorithm is also influenced by the control action $U_k$. This is because the stability of the ASMF can be represented by the filter parameter $\delta_k$, while $\delta_k$ in Eq. (26) can be rewritten as follows.

$$\delta_k = 1 - (Y_k - C_d \hat{a}_{k|k-1})^T W_k^{-1} (Y_k - C_d \hat{a}_{k|k-1})$$

(29)

$$= 1 - (Y_{k+1} - C_d (A_d^a \hat{a}_{k|k} + B_d^a U_k))^T W_k^{-1} (Y_{k+1} - C_d (A_d^a \hat{a}_{k|k} + B_d^a U_k))$$

It has been shown the stability of the ASMF can be represented by the filter parameter $\delta_k$, i.e., the ASMF is stable when $\delta_k > 14$.

Firstly, define

$$J_k^f(U_k, Y_{k+1}) = (Y_{k+1} - C_d (A_d^a \hat{a}_{k|k} + B_d^a U_k))^T W_k^{-1} (Y_{k+1} - C_d (A_d^a \hat{a}_{k|k} + B_d^a U_k))$$

(30)

Thus, from Eq. (29), in order to maintain $\delta_k > 0$, the maximum value of $J_k^f(U_k, Y_{k+1})$ with respect to $\hat{a}_{k|k}$ should be less than or equal to 1, i.e.,

$$J_k^{\delta^*}(U_k, Y_{k+1}) = \max_{\hat{a}_{k|k}} J_k^f(U_k, Y_{k+1})$$

(31)
In general, larger $\delta_k$ often means more rapid convergence of ASMF algorithm. That is, we should select a $U_k$ to make $J_k^{\delta^*}(U_k, Y_{k+1})$ small as far as possible, that is,

$$J_k^{\delta^*}(Y_{k+1}) = \min_{U_k} J_k^{\delta^*}(U_k, Y_{k+1})$$

(32)

We introduce the following cost function $J_k(U_k)$ with consideration of both (28) and (31) at the same time:

$$J_k(U_k) = \min_{U_k} J_k(U_k)$$

(33)

where $\alpha = 1 - \delta_k \in R$ are the positive definite weight matrix. To minimize $J_k(U_k)$, considering $J_k(U_k) > 0$, the control can be obtained at $\frac{\partial J_k(U_k)}{\partial U_k} = 0$, i.e.,

$$\frac{\partial J_k(U_k)}{\partial U_k} = 2(MU_k + N)$$

(34)

where

$$M = B_d^THB_d + \alpha B_d^TC_d W_k^{-1}C_d^a B_d^a$$

$$N = B_d^TH(B_f f_k - B_dU_k^0) - \alpha B_d^TC_d W_k^{-1}Y_{k+1}$$

(35)

For the unknown measurement at time $k+1$ in Eq. (30), we consider that the control system is stable, so, $Y_{k+1} \in \Delta(Y_k)$. Here, $\Delta(Y_k)$ is the elliptical domain of $Y_k$. Because $J_k^{\delta^*}(U_k, Y_{k+1})$ in Eq. (31) is positive definite, its maximum value point must be on the boundary, which can be estimated by the ASMF. Thus, we first define array $i_kS$ to include the estimate of the $i$-th element’s two boundary endpoints as

$$S_k^i = \left\{ \hat{y}_{k+1}^i \mid \hat{y}_{k+1}^i + (1)^h \left( \max_{j=1,2,...,13} \left| \frac{C_d Col(j)}{\sqrt{\sum_{j=1}^{13} |Col(j)|}} \right| \right) \right\}$$

(36)

where $y_k^i$ is the $i$-th element in the vector $Y_k$, $\hat{y}_{k+1}^i$ is the corresponding output $Y_{k+1}$’s endpoints estimation. For set $S_k^i$, $i \in \{1,2,...,8\}$ and $h$ is 0 or 1 for every $i$, $|\bullet|_i$ is the absolute value of the $i$-th element in vector $\bullet$, and the function $Col\{j\}$ is defined as follows:

$$Col\{j\} = (j_1 \ldots j_3)^T$$

(37)

Then, we define a set $S_k$ to describe all possible endpoint vector of the $Y_{k+1}$ as

$$S_k = \{ \hat{y}_{k+1}^{EP} \mid (S_k^1 \ldots S_k^{13}) \}$$

(38)

where $\hat{y}_{k+1}^{EP}$ is the possible endpoint (EP) for output $Y_{k+1}$ at next sampling time $k+1$.

Thus, the proposed active modeling based predictive controller can be implemented by using the following steps:

**Step I: Make nominal control**

Based on the current measured state $\hat{X}_{k+1}^a$, use the nominal controller, which is designed based on the nominal model, to obtain the nominal control input $U_k^0$.

**Step II: Model error estimation and elimination**

Based on $U_k^0$, compute the optimal control input $U_k$:

a) Estimate the values and boundaries of state $X_k$ and model error $f_k$, using ASMF in (26);

b) Calculate the corresponding $U_k(\hat{y}_{k+1}^{EP})$ for every $\hat{y}_{k+1}^{EP}$ in set $S_k$ by Eq. (35);
c) For every $U_k(\hat{Y}^{EP}_{k+1})$ in step 1), use Eq. (30) to obtain the maximum of function $J^\delta_k(U_k(\hat{Y}^{EP}_k), \hat{\gamma}^{EP}_{k+1})$, and get the $^*\hat{Y}^{EP}_{k+1}$ to let $^*\hat{\gamma}^{EP}_{k+1} = \arg\max_{\gamma^{EP}_{k+1}}\{J^\delta_k(U_k(\hat{Y}^{EP}_k), \hat{\gamma}^{EP}_{k+1})\}$.

d) The corresponding $U_k(\hat{Y}^{EP}_{k+1})$ is the optimal control $U_k$ at time $t$, i.e. $U_k = U_k(\hat{Y}^{EP}_{k+1})$.

**Step III: Go back to step 1 at the next time instant $k+1$.**

Thus, the system with the strategy of model error elimination for the small-size helicopter dynamics has been designed, and the structure of the active modeling controller is described in Fig. 5.

![Fig. 5 The structure of the active modeling based controller](image)

**V. Experiments and Results**

**A. Parameter Identification through the frequency sweeping inputs**

For each flight, the pilot applied a frequency sweep sequence into one of the four control inputs via the remote control unit. While doing so, the pilot used the other three control inputs to maintain the helicopter close to hovering. The real frequency sweeping inputs of roll is shown in Fig. 6 as an example, x axis is the time and y axis is the percentage of the maximum roll input of the control unit. By frequency sweeping inputs, the pilot inputs some sine-waves of different frequency (1HZ-20HZ) and the dynamics of the system is excited, and the response provides a large frequency bandwidth for identification. According to the different inherent frequency between rotor and fuselage in reference 18, the low frequency component, which is below 3 Hz, is used for fuselage dynamics identification, and the high frequency component, which is above 10 Hz, is used for rotor dynamics identification. The selection of frequency points for identification can be realized by the proposed optimal estimation.

During the experiment, all control inputs and all vehicle state variables were recorded and sampled at 50 Hz ($>4\pi / T_d$, $T_d=1$). The data (of Euler angles, angular body rates, body accelerations and body velocities) was filtered (-3dB at 10HZ) to remove effects of structural vibrations. The length of the data segments collected for the hovering conditions were 60 seconds. We also collected 5 seconds hovering data, which is not in frequency sweep mode, to be used for real time verification.

It should be noted that the actuator dynamics are not considered here, and we apply the enhanced method of optimal estimation in the frequency domain described in Section 3 to identify the parameters in each decomposed part. The results of identified parameters are listed in Table 2.

The ultimate model accuracy verification is done in time domain to compare the response from model with the hovering flight data, which is shown in Figs. 7 and 8. All of the calculations were done in MATLAB. For the initial states $\Delta X_0$, we consider that all of the differential initial state $\Delta X_0$ should be zeros when hovering. Thus, we can obtain the initial state for simulation as

$$\Delta X_0 = -A^{-1}B\Delta U_0$$  \hspace{1cm} (39)
To verify the precision of the proposed semi-decoupled model by numerical evaluation further, the prediction accuracy of the model output is defined by the following root mean square criterion:

\[ V = \sqrt{\frac{1}{TN} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_i(t) - \hat{y}_i(t))^2} \]  

(40)

where \( y_i(t) \) and \( \hat{y}_i(t) \) are the \( i^{th} \) true and estimated output, respectively, and \( N \) is the relative dimensions of the output. The criterion \( V \) represents the average error of the model, and the value for the simplified model is listed in Table 3. The results from the simulations (Fig. 8) above and numerical accuracy (Table 3) show that the structure of the semi-decoupled model and identified parameters are accurate enough to describe the characteristics of a small-size unmanned helicopter in hovering conditions.
Table 2 The values of identified parameters

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>Parameter value</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_X$</td>
<td>-0.2446</td>
<td>$v_Y$</td>
</tr>
<tr>
<td>$a_X$</td>
<td>-4.962</td>
<td>$b_Y$</td>
</tr>
<tr>
<td>$lat_X$</td>
<td>-0.0686</td>
<td>$lat_Y$</td>
</tr>
<tr>
<td>$lon_X$</td>
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<td>$lon_Y$</td>
</tr>
<tr>
<td>$u_M$</td>
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<td>$L_Y$</td>
</tr>
<tr>
<td>$a_M$</td>
<td>46.06</td>
<td>$L_p$</td>
</tr>
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<td>$lat_L$</td>
</tr>
<tr>
<td>$lon_M$</td>
<td>3.394</td>
<td>$lon_L$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.1628</td>
<td>$B_d$</td>
</tr>
<tr>
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<td>$B_{lat}$</td>
</tr>
<tr>
<td>$A_{lon}$</td>
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<td>$B_{lon}$</td>
</tr>
<tr>
<td>$C_{lat}$</td>
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<td>$D_{lat}$</td>
</tr>
<tr>
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<tr>
<td>$\tau_f$</td>
<td>0.5026</td>
<td>$\tau_s$</td>
</tr>
</tbody>
</table>

Table 3 Performance of the simplified model

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Model Part</th>
<th>Lateral accuracy</th>
<th>Longitudinal accuracy</th>
<th>Yawing-Vertical accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>0.0698</td>
<td>0.0434</td>
<td>0.1832</td>
<td></td>
</tr>
</tbody>
</table>

B. Flight experiment for estimation and elimination of the model error

Flight experiments have been conducted to verify the performance of the proposed strategy for estimate and elimination of the model-errors on the ServoHeli-40 platform. Because we had implemented position and velocity tracking using SISO PI feedback loops on the ServoHeli-40, we can obtain the PI parameters based on the identified model, and set the SISO PI feedback loops as the nominal controller. The nominal PI controller’s structure is shown in Fig. 9 (roll controller for example) and the controller parameters are listed in Table 4.
Fig. 8 Comparison between the response predicted by the identified model and the response obtained during flight test in hovering conditions

<table>
<thead>
<tr>
<th>Table 4 Parameters for roll controller</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K_P</strong></td>
</tr>
<tr>
<td>Position Loop</td>
</tr>
<tr>
<td>Attitude Loop</td>
</tr>
</tbody>
</table>

In this experiment, the identified hovering model is selected as nominal model for PID controller parameters calculation. The helicopter flies between two selected points, point A and point B. In the flight, the helicopter turns.
head 180° to point B at point A first, then, the helicopter increases the longitudinal velocity from 0 m/s to 10 m/s. When the helicopter arrives at point B, it decreases the longitudinal velocity from 10 m/s to 0 m/s, and flies back to point A in the same way. To verify the strategy for estimate and elimination of the online model error, during the flight, we only use the PI controller without the strategy of model error elimination before time 150 seconds, and compensate for the model error after that time in order to show the necessity of the strategy for model-error estimation and elimination. The flight path and position tracking errors are shown in Figs. 10-11, and Fig. 12 shows longitudinal velocity in the flight experiment. Figs. 13-14 show the estimated model errors, and Fig. 15 show the compensation for control inputs based on the proposed strategy for elimination of the model errors. The wind in the flight environment is 3-6 m/s from the southeast.

![Fig. 10 Flight path for test](image1.png)

**Fig. 10 Flight path for test**

![Fig. 11 Vertical tracking error in real flight](image2.png)

**Fig. 11 Vertical tracking error in real flight**
Fig. 12 Longitudinal velocity in the flight

Fig. 13 Vertical and yaw model errors in real flight
Fig. 14 Lateral and longitudinal model errors in real flight

Fig. 15 Compensation control inputs for elimination of model errors in real flight
It is clearly seen in Figs. 12-14 that when the helicopter increases the longitudinal velocity and changes flight mode from hovering to cruise, the adaptive set-membership filter estimates the non-zero model errors and the estimated model error boundaries converge into the constant ellipse intersection, which means that the filter is stable. The estimated transient model errors are completely eliminated, in Figs.15, through the compensation for the nominal control input, which is calculated by the proposed online compensation strategy. The effect of the strategy can be clearly seen in Figs. 10-12, the lateral and vertical position errors decrease into 0.5m and the accuracy of longitudinal velocity tracking increases after the compensation, while nominal PID controller cannot get rid of the wind disturbance and changing flight mode that cause position error. It can also be seen from Figs.13-14 that when the helicopter changes flight mode (at 50s and 160s, from hovering to cruise, 120s, from cruise to hovering) and turns (at 25s and 150 s), the model errors increase, which shows the model error happens as we analyzed during the full envelope flight, and the boundaries maintains stable in constant ellipse intersection to avoid making the estimator and controller unstable for the parameters’ sudden change.

VI. Conclusions

With linear regression based selecting of initial estimation of the unknown parameters and proposed converge criterion, this paper first developed an enhanced frequency method for unmanned helicopter identification. The iteration convergence of the proposed identification process was verified. To simplify the small-size helicopter modeling, the hovering dynamics was decoupled into three parts. The results from the simulations above show that the structure of the semi-decoupled model and identified parameters are accurate enough to describe the characteristics of a small-size unmanned helicopter in hovering conditions. The method used in the experiment and data processing are effective, and the simplification of the semi-decoupled model structure for identification is very feasible in practice. The results also show that the modification and application of the method of the optimal estimation of parameters in the frequency domain for the MIMO state-space modeling of a small-size helicopter are effective and have the following advantages:

a) Output measurement noise or process noise that is uncorrelated with the control inputs does not bias the frequency-response estimates. This type of disturbances is automatically separated from data.

b) It is possible to individually specify the frequency range used for fitting each input-output pair, and it is particularly effective for faster rotor dynamics. This is particularly effective in addressing the frequency disparity between dynamic modes, such as rigid body dynamics and faster rotor dynamics.

c) Selection of specific frequency ranges is also effective at separating information that is relevant for the identification of helicopter dynamics from irrelevant information, which is often characterized by their frequency content.

d) The formulation of the cost function in the frequency domain requires fewer data points than in the time domain, resulting in more efficient parameter identification.

Then, based on the adaptive set-membership filter and the established helicopter hovering model, a strategy for transient model error estimation and elimination is developed to maintain the nominal controller stable when flight modes change instantly. Through optimizing the weighted function of the filter indicator and the model error, the proposed strategy minimizes the on-line tracking error and keeps the filter healthy at the same time. Real flight experiments have been conducted to verify the tracking accuracy and stability of the strategy for transient model error estimation and elimination. The whole flight experiments show that the proposed model enhanced identification process and strategy for model error estimation and elimination work together well and are feasible for full envelope flight of small-size unmanned helicopters in practical applications.

Reference


