Passive Creeping of a Snake-like Robot

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Abstract—The control of a snake-like robot is a challenging problem because of the complex dynamics. In this paper, we present a novel method, called passive creeping, to control the serpentine locomotion of the snake-like robot. The kernel of this method is composed of the following two concepts: 1) dynamic shift brings the tendency toward the serpentine locomotion; and 2) energy links the environments and the dynamics with the control law based on passivity. The head module leads the movement, and the body modules push the robot forward. The movement is a dynamic process from an unordered state to an ordered state, and the maximal Lyapunov exponent explicates the orbital stability of the movement in the phase space. Especially, the snake-like robot can adapt to the environments with different friction coefficients according to the dynamic state not the environment information. The validity and adaptability of the method is studied through simulations.

I. INTRODUCTION

A snake with hundreds of joints can perform various locomotion gaits, e.g. serpentine locomotion, according to different environments ([1]), so the snake has great locomotion capability and environment adaptability. A snake-like robot has these merits potentially by imitating the snake. However, these characteristics, such as the redundant degrees of freedom (DOFs), the interaction between the robot and the environment, and the nonexistence of a fixed base, make the dynamics of the robot very complicated.

Thus, one finds that it is difficult to construct an effective control method for the snake-like robot. According to the classification in [2], the existing control modes of the serpentine locomotion can be mainly divided into three categories, namely sine based, model based, and central pattern generator (CPG) based. First, the sine based method controls the relative angles as sine functions to approximate the serpenoïd curve [1], [3]. Because the method has the merits potentially by imitating the snake, it can influence the locomotion by feeding back the state to the control law.

The advantages of passive creeping are as follows: 1) The concept of passivity relating to energy has an explicit physical meaning. 2) The snake-like robot can adapt to variable environments without sensors measuring the environments. 3) The dynamic state (i.e., energy) of the snake-like robot can influence the locomotion by feeding back the state to the control law.

II. MODEL OF A SNAKE-LIKE ROBOT

Before presenting the concept of passive creeping, we recall the kinematic and dynamic model first. The details can be found in [7].

A. Kinematic Model

The snake-like robot is a 2-D articulated multi-link system as shown in Fig. 1. A module of the snake-like robot is denoted by $U_i$ ($i = 3, \ldots, n$). $U_1$ and $U_2$ are assigned to the virtual modules which are not referred to in the paper. See [7] for details, and the length, mass, and inertia tensor of the $i$th module are $l_i$, $m_i$, and $I_i^b$ respectively. The configuration of the robot in coordinates can be written as

$$x = [x^1, x^2, x^3, x^4, \ldots, x^n]^T$$

where $[x^1, x^2, x^3]^T$ is the position and orientation of the robot in the inertial coordinate frame S, and $[x^4, \ldots, x^n]^T$ represents the relative joint angles of the robot. Note that $x^i$ is short for $x^i(t)$, where $t$ is time.

The position and orientation of the $i$th ($i = 3, \ldots, n$) module in the global coordinates is described by the product-of-exponentials formula (POE) as follows:

$$g_{s,i} = e^{\xi_1 x^1} e^{\xi_2 x^2} \cdots e^{\xi_i x^i} g_{s,1}(0) = \begin{bmatrix} R_{s,i} & b_{s,i} \ 0 & 1 \end{bmatrix}$$

The position and orientation of the robot in the inertial coordinate frame S, and $[x^4, \ldots, x^n]^T$ represents the relative joint angles of the robot. Note that $x^i$ is short for $x^i(t)$, where $t$ is time.
for \( i = 3, \ldots, n \), where 1) \( g_{s,i}(0) \) is the position and orientation at the initial moment; 2) \( R_{s,i} \) is the rotation matrix, and \( b_{s,i} \) is the position vector; and 3) \( \xi_j \) is a twist. The body velocity of the \( j \)-th module relative to the global coordinates is denoted by \( V_{s,i} \), and can be expressed as

\[
V_{s,i}^b = \begin{bmatrix} \omega_{s,i}^b \vspace{1mm} \\ \nu_{s,i}^b \end{bmatrix} = J_{s,i}^b \dot{x}
\]  

(2)

where \( \omega_{s,i}^b \) and \( \nu_{s,i}^b \) are the angular and linear velocity component respectively, and \( J_{s,i}^b(x) \) is a \( 6 \times n \) body manipulator Jacobian matrix. In addition, the real velocity can be decomposed into the tangent and the normal velocity along the robot body as \( v_{s,i} = [v_{s,i}^t, v_{s,i}^n] \).

### B. Dynamic Model

The dynamic equation of the snake-like robot in closed form can be written as follows:

\[
\sum_{j=1}^{n} M_{kj} \ddot{x}^j + \sum_{i=1}^{n} \sum_{j=1}^{n} \Gamma_{ijk} \dot{x}^i \dot{x}^j - f Y_k = \tau_k
\]

(3)

for \( k = 1, \ldots, n \), where

1) \( M_{kj} \) is the element of the metric matrix \( M = \sum_{i=1}^{n} (J_{s,i}^b(x))^T M_i^b J_{s,i}^b(x) \) wherein \( M_i^b = \begin{bmatrix} I_i^b & 0 \\ 0 & m_i \end{bmatrix} \) is a generalized inertia matrix with \( I \) denoting a \( 3 \times 3 \) identity matrix (Specially, \( M_1^b = M_2^b = 0_{6\times 6} \));
2) \( \Gamma_{ijk} \) is the Christoffel symbol of the first kind;
3) \( \tau_k \) (\( \tau_1, \tau_2, \tau_3 = 0 \)) is the torque applying at the \( k \)-th joint;
4) \( f Y_k \) is the general force of friction in the direction of \( x^k \). Additionally, Coulomb friction is used to describe the interaction between the robot and the environment as

\[
f_i = [-\mu_{i}^t m_i g \cdot \text{sgn}(v_{s,i}^t), -\mu_{i}^n m_i g \cdot \text{sgn}(v_{s,i}^n)]
\]

(4)

for \( i = 3, \ldots, n - 1 \), where \( \mu_{i}^t \) and \( \mu_{i}^n \) are the friction coefficients in the tangent and the normal direction respectively, and \( \text{sgn}(\cdot) \) is the sign function.

Being similar to manipulator dynamics, the first, the second, and the third term in (4) are the acceleration-related inertia, the Coriolis and centrifugal, and the friction term, respectively; and the right-hand side is the driving torque.

Let \( C_{jk} = \sum_{i=1}^{n} \Gamma_{ijk} \dot{x}^i \). Equation (3) can be rewritten as

\[
M \ddot{x} + C \dot{x} - f Y = \tau
\]

(5)

where \( f Y = [f_{Y_1}, \ldots, f_{Y_n}]^T \), and \( \tau = [\tau_1, \ldots, \tau_n]^T \). We multiply (5) by \( \dot{x} \), and obtain that

\[
\dot{x}^T M \ddot{x} + \dot{x}^T C \dot{x} - \dot{x}^T f Y = \dot{x}^T \tau.
\]

(6)

In addition, \( M \) is symmetric and \( M - 2C \) is skew-symmetric, so the change rate of the kinetic energy

\[
\dot{E} = \frac{d}{dt} \left( \frac{1}{2} \dot{x}^T M \dot{x} \right) = \dot{x}^T M \ddot{x} + \dot{x}^T C \dot{x}.
\]

(7)

According to the principle of virtual power, the third term of (6) can be computed as

\[
P_d = -\dot{x}^T f Y = \sum_{i=3}^{n-1} \left( \mu_i^t m_i g \left| v_{s,i}^t \right| + \mu_i^n m_i g \left| v_{s,i}^n \right| \right).
\]

(8)

Thus, the first two terms in (6) describe the change rate of the kinetic energy; the third term represents the frictional dissipation; and the right-hand side is the input power. In fact, (6) satisfies the work-kinetic energy theorem as

\[
\dot{E} = \dot{x}^T \tau - P_d.
\]

(9)

The system (9) with \( E \) lower bounded and \( P_d \geq 0 \) is said to be passive. Furthermore, \( \int_0^t P_d > 0 \) for all \( t > 0 \) in creeping, so the system is dissipative as well [8].

### III. CONCEPT OF PASSIVE CREEPING

The idea of passive creeping is inspired by the passivity-based control of the bipedal locomotion [9]. The key of the passivity-based control is energy. In the research of the snake-like robot, the energy relates with the following two aspects of the serpentine locomotion.

First, the energy relates to the environment. Because the interaction between the robot and the environment brings energy dissipation as shown in Fig. 2, and (9) describes the relationship mathematically. The efficiency of the snake-like robot can be deduced from (9), so a control objective is

\[
\max_{\eta} \eta = \frac{\dot{E}}{\dot{E} + P_d}.
\]

(10)
Thus, the rotational and the translational kinetic energy of the phase order is denoted as

\[
E_i = \frac{1}{2} (V_{s,i}^b)^T M_i b^b \omega_{s,i} + \frac{1}{2} m_i (v_{s,i}^b)^T v_{s,i}^b.
\]  

(11)

Thus, the rotational and the translational kinetic energy of the whole robot can be defined as

\[
E_R = \frac{1}{2} \sum_{i=3}^{n} (\omega_{s,i}^b)^T I_i b_i^b \omega_{s,i} + \frac{1}{2} \sum_{i=3}^{n} m_i (v_{s,i}^b)^T v_{s,i}^b, \quad E_T = \frac{1}{2} \sum_{i=3}^{n} m_i (v_{s,i}^b)^T v_{s,i}^b.
\]  

(12)

Because \( E_T \) is directly correlated with the forward velocity of the robot, the proportion of the translational kinetic energy to the whole kinetic energy \( \eta_2 \) is an important quantity in a stable state. Therefore, another control objective is

\[
\max \eta_2 = \frac{E_T}{E_R + E_T}.
\]  

(13)

The proportion \( \eta_2 \) which represents the energy assignment of the serpentine locomotion in a stable state is more proper to describe the efficiency of the locomotion. Contrarily, \( \eta_1 \) describes the instantaneous efficiency of the robot system generally not to the serpentine locomotion specially.

Accordingly, the energy is a pivotal role in the serpentine locomotion. Naturally, the passivity-based control and a reference energy level are introduced to the control method of the snake-like robot in order to carry out above notion.

Being different from the bipedal locomotion, the serpentine locomotion relates to the assignment of the energy (or coordination) between the numerous joints. The reasons are 1) the snake-like robot has much more actuated joints than the bipedal robot, and 2) the serpentine locomotion is the resulting effect of the movements in all the joints. Dynamic shift which is a \textit{a priori} knowledge of the snake motion is proposed to deal with this problem. Biologically, the head-to-tail undulatory wave propels a snake onwards. In the sine-based method, the undulatory wave is considered as the phase shift between the joint angles. Therefore, the shift control ([10]) is just a translation operation of the joint angles as

\[
x^{n-i}(t) = T_\Delta \{x^{n-i+1}\} (t)
\]  

(14)

for \( i = 1, \ldots, n-4 \), where \( T_\Delta \) is a translation operator with a time difference \( \Delta \), and \( T_\Delta \{x^{n-i+1}\} (t) = x^{n-i+1}(t-\Delta) \).

However, the amplitudes in the different parts of the wave can be nonhomogeneous, e.g., the amplitude near the head of the snake and that in the middle of the body are different actually. From the head-to-tail undulatory wave, we can only know the order of the phases of the joint angles. Abstractly, the phase order is denoted as

\[
x^n \succ x^{n-1} \succ \ldots \succ x^5 \succ x^4
\]  

(15)

where \( \succ \) is the order relation of the phases. According to the above fact, we propose dynamic shift as follows:

\[
\tau_{n-i} = A_{n-i} \left(x^{n-i+1}(t - \Delta') - x^{n-i}(t)\right)
\]  

(16)

for \( i = 1, \ldots, n-4 \), where \( A_{n-i} \) is a proportional coefficient controlling the amplitude of the torque; \( \Delta' \) is a time delay, and let \( \Delta' = 0 \) for convenience in this paper. The current torque is influenced by the difference between the anterior joint angle and the current, so the current joint angle must lag behind the anterior. Consequently, the angles satisfy the phase order. In addition, the snake-like robot are directly controlled by the torques rather than by the planned angles. Therefore, we call this phase control method as dynamic shift, which brings the tendency toward the basic serpentine locomotion. Seemingly, dynamic shift is similar to the time-delayed feedback control [11]. However, the time-delayed feedback control affects on one state vector guaranteeing the target orbit to be an inherent unstable periodic orbit of a system, while dynamic shift (16) involves two joints in tandem impelling the latter joint to follow the former joint. In fact, dynamic shift expects to realize the synchronization of the joint movements in a stable state when creeping.

Combining the above two aspects, we present the sketch for the concept of passive creeping in Fig. 3. Dynamic shift produces the serpentine locomotion tendency; the environment affects the serpentine locomotion through the energy dissipation; and the robot dynamics influence the motion through the energy transformation.

IV. CONTROL METHOD OF PASSIVE CREEPING

According to the concept of passive creeping in Section III, we propose a detail control method of the snake-like robot as shown in Fig. 4. The mathematic equations of the passive creeping control are as follows.

□ Control torque of head joint:

\[
\tau_n = a I_n \left| K_n \left( \int_0^t (E_{\text{ref}} - E) dt \right) + E_{\text{ref}} - E \right| \cdot (\ddot{x}_n^a + k_d (\dot{x}_d^a - \dot{x}^a) + k_p (x_d^a - x^n) + \delta_r)
\]  

(17)

□ Control torque of body joint:

\[
\tau_{n-i} = K_{n-i} \int_0^t (E_{\text{ref}} - E) dt \left(x^{n-i+1} - x^{n-i}\right)
\]  

(18)
for $i = 1, \ldots, n - 4$, where
1) $a$, $k_d$, and $K_p$ are three coefficients;
2) $I_n = I_n^0 + m_n l_n^2/4$ is the moment of inertia of the head module about the head joint axis;
3) $K = [K_n, K_{n-1}, \ldots, K_4]^T$ is a vector of the integrating amplification factors;
4) $E_{ref}$ and $E$ are the reference mechanical energy and the real mechanical energy of the snake-like robot, respectively;
5) $x_n^d$ is a reference angle of the head joint;
6) $\delta_\tau$ is a turning parameter. If $\delta_\tau = 0$, the snake-like robot moves straightly. Otherwise, the robot turns right or left.

The torque of the head joint $\tau_n$ is used to implement the top-level control, such as leading and turning, in the head-to-tail serpentine locomotion. The difference between the reference angle and the real one causes the persistent excitation that drives the head joint swaying in a certain way. The torque $\tau_n$ is partially based on the computed torque control, but the difference from the standard one is that the head module rotates around an unfixed axis. Additionally, $\int_0^t (E_{ref} - E) dt + E_{ref} - E$, which is a PI control law for the head joint, has three functions as follows: 1) compensating the steady energy error because of the integral part, 2) quickening the startup due to the proportional term, and 3) adapting the amplitude of the head joint swaying to the uncertain environment.

Correspondingly, the torques of the body joints $\tau_{n-i}$’s are used to realize the serpentine locomotion (i.e., wriggling and pushing). The difference between the anterior joint angle and the current one produces the movement tendency. The energy error integral $\int_0^t (E_{ref} - E) dt$ has the following two functions: 1) adjusting the amplitude of the body joint based on the energy control, and 2) adapting the body wriggling to the uncertain environment. In fact, the snake-like robot was hardly controlled by direct torque inputs without a reference curve (e.g., serpentine curve), because no one knew how many torques were proper to the locomotion in various instances. However, the energy error integral can adaptively adjust the torque amplitudes to resolve the problem.

In short, the head module leads the movement, and the body modules realize the serpentine locomotion and push the robot forward. In order to distinguish the two different tasks, the passive wheel is not installed on the head module as shown in Fig. 1. According to the above control law, passive creeping can adapt the environment only through obtaining the environment information, not as some existing methods through measuring the environment information (e.g., [6]).

V. SIMULATION STUDY OF PASSIVE CREEPING

In this section, we address the following two fundamental problems with computer simulations as: 1) the stability of passive creeping based on the maximum Lyapunov exponent, 2) the adaptability of passive creeping with the change of the environment parameters (e.g., friction coefficients). The simulations are programmed based on Open Dynamics Engine (ODE). The basic fixed parameters of the simulations are presented in Table I. In addition, the variable parameters are the environment parameters (namely, $\mu_c$ and $\mu_l$) and the control parameters (namely, $E_{ref}$ and $\delta_\tau$). For convenience, a triple $(E_{ref}, \mu_c, \mu_l)$ is used to denote the variable parameters when the turning parameter $\delta_\tau$ is zero.

### A. Stability

By comparing the configurations, torques and angles in the initial stage with those in the final stage as shown in Fig. 5, the movement of passive creeping is a dynamic process from an unordered state to an ordered state, and realizes synchronization finally. The energy can reach the reference value with fluctuation, but cannot converge at it as shown in Fig. 6. This is because: 1) the energy feedback makes the kinetic energy near the reference one; however, 2) the head swing which is a persistent exciting effect leads the energy fluctuation; and 3) the control law cannot forecast the complex relationship between the swing and the fluctuation. Therefore, the energy fluctuation have not been restrained from the serpentine locomotion. In fact, the slight fluctuation of the energy is intricate but endurable in the movement.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of real units</td>
<td>$n' = n - 2$</td>
<td>10</td>
</tr>
<tr>
<td>Length of the i th unit</td>
<td>$l_i$</td>
<td>0.08 m</td>
</tr>
<tr>
<td>Mass of the i th unit</td>
<td>$m_i$</td>
<td>0.50 kg</td>
</tr>
<tr>
<td>Inertia of the i th unit</td>
<td>$I_i$</td>
<td>0.00027 kg·m²</td>
</tr>
<tr>
<td>Time step of simulation</td>
<td>$T$</td>
<td>0.01 s</td>
</tr>
<tr>
<td>Reference angle</td>
<td>$x_n^d$</td>
<td>$0.5 \sin(2t + \pi/2)$ rad</td>
</tr>
<tr>
<td>$a$</td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td>$k_d$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>$K_n$</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$K_{n-1}$</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td></td>
</tr>
<tr>
<td>$K_4$</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

Let each real unit be of the same length, mass, and inertia respectively.
The serpentine locomotion can be stabilized gradually under passive creeping as shown in Fig. 5, notwithstanding the energy fluctuation. We describe the stability of the snake-like robot by using the maximal Lyapunov exponent \( \lambda \). In fact, the Lyapunov exponent of a dynamical system is a quantity that characterizes the exponential rate of divergence or convergence of infinitesimally close trajectories in a phase space [12]. Because the joint movements which represent the locomotion mode are interested, the position and orientation of the snake-like robot in the inertial coordinates are not included in the phase space. The state of the movement in the phase space can be written as

\[
z = [x^4, \ldots, x^n, \dot{x}^4, \ldots, \dot{x}^n]
\]

Quantitatively, \( \lambda \) is determined by

\[
d(t) \simeq \delta(0) e^{\lambda t}
\]

where \( \delta(t) = \| z_1(t) - z_2(t) \|_\infty \) is the distance some time \( t \) ahead between the two trajectories emerging from two close points \( z_1(0) \) and \( z_2(0) \). According to (19), we estimate \( \lambda \) as shown in Fig. 7. At the initial stage (A), the system is divergent, because the energy is injected by the motors under the control of passive creeping. After startup (B), the energy is also injected, while the friction between the snake-like robot and the environment leads the energy dissipation, wholly, the energy of the system is increased. However, the serpentine locomotion is gradually established and synchronized under the persistent exciting effect of passive creeping at this stage, so \( \lambda < 0 \) and the phase volume decreases. At the final stage (C), the injected and the dissipated energy are in a dynamic balance, so \( \lambda \approx 0 \) and a limit cycle is generated in the phase space. Finally, the locomotion of passive creeping with certain parameters is locally stable. We point out that the phase volume cannot decrease infinitely, because of the energy fluctuation and the numerical error.

### B. Adaptation

Observing the energy error integral and the energy proportion \( \eta_2 \) in Fig. 8, we find that the integral which determines the torque amplitudes adjusts with different conditions and the proportion seems changeless under some conditions. This inspires us to research the adaptation of passive creeping with the variable parameters (0.5, 0.50, 0.010). In a logarithmic presentation of the distance \( \delta(t) \) over \( t \), the slope of the fitted regression line corresponds to \( \lambda \). From nine distance sequences, the estimations for \( \lambda \)'s in the time intervals B and C are at \(-0.059 \pm 0.009\) and \(0.000 \pm 0.005\) respectively.

![Fig. 6. Kinetic energy of the snake-like robot. The energy can reach the reference value but fluctuates about the value.](image)

![Fig. 7. Estimation of the maximal Lyapunov exponent \( \lambda \) of passive creeping with the variable parameters (0.5, 0.50, 0.010). In a logarithmic presentation of the distance \( \delta(t) \) over \( t \), the slope of the fitted regression line corresponds to \( \lambda \). From nine distance sequences, the estimations for \( \lambda \)'s in the time intervals B and C are at \(-0.059 \pm 0.009\) and \(0.000 \pm 0.005\) respectively.](image)
When $E_{ref}$ is small and $\mu^t$ nears zero, the amplitude increases rapidly and the proportion decreases fast as shown in Figs. 9 and 11. The robot confusedly swings here, because $E_{ref}$ can be reached without the harmonious movement.

VI. CONCLUSIONS AND FUTURE WORKS

In this research, we have presented the concept of passive creeping by combining the property of the serpentine locomotion and the notion of passivity. The control law which is composed of dynamic shift and energy-based feedback has then been designed. The snake-like robot can realize the stable serpentine locomotion finally, and the local orbital stability of passive creeping has been explained by the maximal Lyapunov exponent. The especial advantage of passive creeping is that the robot can adapt to the environments with different friction coefficients according to the dynamic state of itself. The turning locomotion has not been presented for economy of space. In addition, we point out this method has not been experimented on a physical robot.

Finally, this paper leads to a number of interesting topics to be studied in future as follows: 1) analyzing the non-stationary process and the synchronization of passive creeping, 2) estimating the basin of attraction for the control law, 3) applying the method to a flexible snake-like robot.

REFERENCES