Robust Tracking Control for the Yaw Control of Helicopter with Time-varying Uncertainty

Xingang Zhao, Jianda Han
Robotics Laboratory, Shenyang Institute of Automation, CAS, Shenyang, Liaoning, 110016 China
zhaoxingang@sia.cn, jdhan@sia.cn.

Abstract—This paper deals with the problem of robust tracking control with adaptation mechanism. A linear time-invariant system with time-varying ellipsoidal uncertainty is considered. The proposed controller affinely depends on the uncertainties. The application of this approach to the yaw control of a small-scale helicopter mounted on an experiment platform shows the effectiveness.

I. INTRODUCTION

Over the last three decades, considerable attention has been paid to analysis and synthesis of helicopter [1-3]. The increasing interest about this topic can be understood by the fact that helicopter can operate in different flight modes, such as vertical take-off/landing, hovering, longitudinal /lateral flight, pirouette, and bank-to-turn and be invaluable for terrain surveying, surveillance, localization of targets, tracking, map building and others.

Helicopter flight control system design has been dominated by linear control techniques. The last decade has witnessed remarkable progress in small-scale helicopter research including modelling [4], control [5-6], which are based on the accurate and fixed model. However, the complicated dynamics of helicopter lead to both parametric and dynamic uncertainty. Unmeasurable states, sensor and actuator noise, saturation, bandwidth limitations, friction and delays, all of these may perturb the resulting closed-loop system out of the region of stability, so the controller should be designed to robust to those effects.

Recently, a considerable amount of work has been done to design robust controllers for linear system with parameter uncertainty. Since an adequate level of performance is required in practice, recent literatures have focused on quadratic stabilizing control with some performance such as LQR, H_\infty or H_2 [7-9] disturbance attenuation and closed-loop pole location based on LMI or other methods. While a single controller with a fixed gain is considered, the resulting controllers designed by these methods inherently become conservative. On the other hand, adaptive control theories[10] have been long developed as controller design methodologies for system with uncertainties. The typical adaptive control scheme is the parameter adaptive control, in which unknown parameters are estimated explicitly, and control parameters are determined based on these estimates. However, even in the so-called “ideal case”, a stable adaptive controller doesn’t necessarily guarantee good transient response.

It is worthwhile considering incorporate some kind of adaptation mechanism into robust control methods. In this paper, an adaptive robust tracking controller is presented in order to reduce conservatism inherent in a robust control method with a fixed gain and to improve transient behaviour in time-response. Both system matrix A and input matrix B have time-varying parameter uncertainty. It belongs to an ellipsoidal set, which often appears in the results of set member identification in practical systems [11]. The transient behaviour can be improved directly in a real-time fashion, where the gain of controller is tuned on-line based on the information about parameter uncertainties. Since the complicated dynamics of helicopter lead to both parametric and dynamic uncertainty and the parameter uncertainty are time-varying, the proposed method will be applied to the yaw control of helicopter.

The paper is organized as follows. In Section II, we give the yaw dynamic of helicopter and the simplified model. Section III gives the adaptive robust tracking control method. The application of the proposed controller to the yaw control of small-scale helicopter is given in Section IV. Both linear and nonlinear simulations of yaw dynamic model are performed. Finally, Section V gives the conclusion.

II. MODELING YAW DYNAMIC OF HELICOPTER

In this paper a framework of the simulation model for the helicopter-platform (see Fig. 1) is set up using rigid body equations of motion of the helicopter fuselage. In hovering and low-velocity flight, the torque generated by main and force generated by tail rotor are dominant [12]. By simplifying the fuselage and vertical fin damping, the yaw dynamics can be rewritten as:

\[
\begin{align*}
\dot{\phi} &= r \\
I_\phi \dot{r} &= -Q_{mr} + T_r l_r + b_1 r + b_2 \dot{\phi}
\end{align*}
\]  

where $Q_{mr}$ is the torque of main rotor, $T_r$ is the thrust of tail rotor, $l_r$ is the distance between the tail rotor and z-axis, $b_1$ and $b_2$ are damping constants. The expressions of $T_r$ and $Q_{mr}$ has been given in [13]:

\[
T_r = C_t \theta_r + \frac{1}{2} C_2 (C_2 + \sqrt{C_2^2 + 4C_1 \theta_r})
\]  

The 33rd Annual Conference of the IEEE Industrial Electronics Society (IECON)
Nov. 5-8, 2007, Taipei, Taiwan
with
\[
C_1 = \frac{1}{2} \rho a_p b_p c_p \Omega_x^2 (R_y^2 - R_w^2)
\]
\[
C_2 = \frac{1}{2} \rho a_p b_p c_p \sqrt{2/ \rho \pi R_p^2} (R_y^2 - R_w^2)
\]
where \(\rho_p, a_p, b_p, c_p, \Omega_x, \theta_x, \theta_w, \theta_y, \Omega_w, v_t, \Omega_t\) are respectively, density of air, slope of the lift curve, number of the rotor, chord of the blade, speed of the tail rotor, pitch angle, radial distance, induced speed of the tail rotor and area of the tail rotor disc.

\[
Q_{mr} = \frac{c\Omega}{48\pi^2} \left[ 8C_a \Omega \rho \pi R^2 (2C_2 \theta_x + C_0 \sqrt{C_4^2 + 4C_2 \theta_w} \Omega) (R_y^2 - R_w^2) + \right.
\]
\[
+ 4a \Omega \rho \pi R^2 (C_2 \theta_x + C_4 \sqrt{C_4^2 + 4C_2 \theta_w} \Omega^2) (R_y^2 - R_w^2) +
\]
\[
+ 6C_2 C_0 \Omega \rho \pi R^2 (C_2 \theta_x + 4C_2 \theta_w) R_y (R_y^2 - R_w^2) +
\]
\[
+ 6C_2 C_0 \Omega \rho \pi R^2 (C_2 \theta_x + 4C_2 \theta_w) R_y (R_y^2 - R_w^2) +
\]
\[
+ \frac{c}{48\pi^2} \left. \right] 6C_a \rho \pi R^2 (C_4 \theta_x + 4C_2 \theta_w) \Omega (R_y^2 - R_w^2) +
\]
\[
+ 3C_a \rho \pi R^2 (C_4 \theta_x + 4C_2 \theta_w) \Omega (R_y^2 - R_w^2) +
\]
\[
+ 3C_a \rho \pi R^2 (C_4 \theta_x + 4C_2 \theta_w) \Omega (R_y^2 - R_w^2) +
\]
\[
+ \frac{c}{48\pi^2} \left. \right] 6C_a \rho \pi R^2 (C_4 \theta_x + 4C_2 \theta_w) \Omega (R_y^2 - R_w^2)
\]
\[
(3)
\]
where \(C_a, \theta_w\) are respectively, radial and pitch angle of main rotor, \(a, \alpha, r, c, \phi, \psi, \Omega\) are respectively slope of the lift curve, the angle of attack of the blade element, radial distance, chord of the blade, inflow angle, induced speed and rotor speed of the main rotor.

From (1) we can see that there exist couplings between main rotor torque \(Q_{mr}\) and tail rotor thrust \(T_y\). And (2) and (3) further demonstrate that the models are highly nonlinear and too complex to be used for control design. Instead of the dynamics described by (2) and (3), a simplified model is proposed for control design:

By plotting the torque vs pitch angle, we can find that relation between \(Q_{mr}\) and \(\theta_w\) approximated with quadratic polynomial. Instead of the dynamics described by (1), a simplified model is proposed as follows:

\[
\begin{align*}
\dot{\phi} &= r \\
\dot{r} &= k_r r + k_\theta \theta_x + k_\psi \Omega + k_\phi
\end{align*}
\]
\[
\begin{bmatrix}
\phi \\
r
\end{bmatrix} = \begin{bmatrix}
f(x,u) \\
c(x,u)
\end{bmatrix}
\]
\[
(4)
\]
where \(k_r, k_\theta, k_\psi, k_\phi\) are coefficients, and \(\Omega\) is the speed of the main rotor.

The nonlinear dynamics (4) can be represented by a state space description:

\[
\dot{x} = f(x,u)
\]

where \(x = \begin{bmatrix} \phi & r \end{bmatrix}^T, u = \theta_x\).

Furthermore, the model above can be linearized at a trim point \((x_0, u_0)\)

\[
\dot{x} = Ax + Bu
\]

where \(A = \frac{\partial f}{\partial x} \bigg|_{x_0,u_0} = \begin{bmatrix} 0 & 1 \\
1 & k_r \end{bmatrix}\), \(B = \frac{\partial f}{\partial u} \bigg|_{x_0,u_0} = \begin{bmatrix} 0 \\
k_r \end{bmatrix}\) and \(a = 2k_r \theta_x + k_\theta + k_\phi \Omega\).

III. ADAPTIVE ROBUST TRACKING CONTROLLER DESIGN

In this section, firstly, we propose the control method which is general for a linear model with time-varying affine uncertainty. Then the proposed control method will be applied to the yaw dynamics of helicopter.

A. Problem statement and preliminaries

Consider the following linear uncertainty model described by

\[
\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) + B_o(\theta(t))\omega(t)
\]
\[
y(t) = Cx(t)
\]
where \(x(t) \in R^n\) is the state, \(u(t) \in R^m\) is the control input, \(y(t) \in R'\) is the measured output and \(\omega(t) \in R'\) is an exogenous disturbance which belongs to \(L_2[0, \infty)\), respectively.

The system matrices have the following time-varying structure

\[
A(\theta(t)) = A_0 + \sum_{i=1}^{n} \theta_i(t) A_i, \quad B(\theta(t)) = B_0 + \sum_{i=1}^{n} \theta_i(t) B_i
\]
\[
B_o(\theta(t)) = B_o + \sum_{i=1}^{n} \theta_i(t) B_o_i
\]
where \(A_0, A_1, \ldots A_n, B_0, B_1, \ldots B_n, B_o, B_o_1, \ldots B_o_n\) are known constant matrices. The time-varying parameter vector \(\theta(t) \in R^n\) represents unknown parameters which belong to the N-dimensional ellipsoidal set expressed as

\[
\Delta = \{ \theta \in R^n \mid \theta^T(\Sigma^{-1} \theta) \leq 1 \}
\]
\[
\Sigma = \text{diag}(\sigma_1, \ldots, \sigma_n)
\]
where \(\Sigma \in R^{n \times n}\) represents the size of the ellipsoid.

The ellipsoidal set can be obtained by set membership identification method. Set membership identification is one of the identification technique that use a priori assumptions about a parametric model to constrain the solutions to certain sets. In this approach, uncertainty is described by means of an additive noise which is known only to have given bounds. The motivation for this approach is that in many practical cases the
Unknwon but Bounded (UBB) error description is more realistic and less demanding than the statistical description. In Section IV, the Fogel-Huang Algorithm[14] is used for the parameter identification.

Control Objective: design a robust controller such that:
1. The closed-loop system is stable for all \( \theta(t) \in \Delta \) with a guaranteed level of disturbance attenuation.
2. The output \( y(t) \) tracks the reference signal \( r(t) \) with zero steady-state error, that is \( \lim_{t \to \infty} e(t) = 0 \) where \( e(t) = r(t) - y(t) \).
3. Satisfactory transient performance in time-response by adding a controller with adaptation mechanism.

It is well known that integral control can effectively eliminate the steady tracking error. In order to obtain a robust tracking controller with state feedback plus tracking error integral, the following augmented state-space description is introduced:

\[
\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) + \bar{B}_o(\theta(t))\omega(t) \quad (9)
\]

where \( x(t) = \begin{bmatrix} \begin{bmatrix} I \end{bmatrix} e^T(t) \end{bmatrix}^T \begin{bmatrix} e^T(t) \end{bmatrix} ^T \begin{bmatrix} \omega(t) \end{bmatrix} \end{bmatrix} \), \( \omega(t) = [r(T) \ \omega^T(T)] \)

And

\[
\tilde{A}(\theta(t)) = \begin{bmatrix} 0 & -C_1 \\ O & A(\theta(t)) \end{bmatrix}, \quad \tilde{B}(\theta(t)) = \begin{bmatrix} 0 \\ B(\theta(t)) \end{bmatrix}
\]

\[
\tilde{B}_o(\theta(t)) = \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix} B_o(\theta(t)) \]

Choose the controlled output \( \bar{z}(t) \in R^a \), defined by

\[
\bar{z}(t) = C\bar{x}(t) + D^o u(t) + \Gamma \omega(t) \quad (10)
\]

where \( C \) and \( D \) are constant weighting matrices which can be adjusted to achieve satisfactory response.

Then the design problem can be reduced to the following:
Find a robust controller \( u(t) \) such that:
1. The augmented closed-loop system is robust stable for all \( \theta(t) \in \Delta \).
2. Transient performance improves in time-response.

Next, we will propose the robust control method with adaptation mechanism.

First, we introduce a target model with adjustable parameters which is determined so as to ensure quadratic stability of the error system between the state trajectory of the plant and that of the target model. Then a controller for the target model is designed. Consequently, an adaptive robust controller to improve transient behaviour in time-response is established.

**B. Adjustable target model and parameter adjustment law**

In order to obtain on-line information on the parameter uncertainty, we introduce the following target model described by

\[
\dot{\bar{x}}_o(t) = \tilde{A}(\hat{\theta}(t))\bar{x}_o(t) + \tilde{B}(\hat{\theta}(t))\bar{v}(t), \quad \bar{x}_o(0) = \bar{x}_0 \quad (11)
\]

\[
\bar{z}(t) = C\bar{x}_o(t) + D\bar{v}(t) \quad (12)
\]

where \( \hat{\theta}(t) \in R^m \) denotes the adjustable parameter vector, and let the matrices \( \tilde{A}(\hat{\theta}) \) and \( \tilde{B}(\hat{\theta}) \) have the same structure as the system matrices of (9). The input \( \bar{v}(t) \) is determined so as to improve the output \( \bar{z}(t) \) according to the adjustable parameter \( \hat{\theta}(t) \). If we define the error vector as \( e = \bar{x} - \bar{x}_o \), then the error equation between (9) and (11) is written as

\[
\dot{e}(t) = \tilde{A}(\theta)e(t) + \tilde{B}(\theta)(u - \bar{v}) + \tilde{B}_o(\theta)\omega(t)
\]

where \( e(0) = \bar{x}(0) - \bar{x}_o \) is determined to satisfy the following equation:

\[
\bar{z}(t) = \tilde{A}(\theta)e(t) + \tilde{B}(\theta)(u - \bar{v}) + \tilde{B}_o(\theta)\omega(t)
\]

(13)

where \( E(\bar{x}_o) \in R^{\alpha \times \gamma \times \nu} \), \( E(\omega) \in R^{\alpha \times \gamma \times \nu} \) is given by

\[
E(\bar{x}_o) = \begin{bmatrix} [A_{\bar{x}_o}, \ | \ A_{\bar{y}_o} \bar{x}_o] \end{bmatrix}, \quad E(\omega) = [\bar{B}_o \bar{v}, \ | \ \bar{B}_o \bar{v}] \]

By considering the control input

\[
u(t) = v(t) + F(\hat{\theta})(\bar{v}) \quad (14)
\]

where \( F(\hat{\theta}) = F_0 + \sum_{i=1}^{N_1} \theta_{j=1}^{N} F_j, \quad F_0, F_1 \cdots F_{N_1} \) are the error feedback to be designed. Then (13) can be written as

\[
\dot{e}(t) = (\tilde{A}(\hat{\theta}) + \tilde{B}(\hat{\theta})F(\hat{\theta}))e(t) + \tilde{B}(\hat{\theta})(u(t) - \hat{\theta}) + \tilde{B}_o \omega(t)
\]

(15)

In addition, if we define \( \tilde{z} = e - \bar{z} \) then from (10) and (12), we can get

\[
\dot{\tilde{z}} = (C + DF(\hat{\theta}))(\tilde{z}) + \Gamma \omega
\]

(16)

Here, for the error system (15) and (16), we determine the parameter vector \( \hat{\theta}(t) \) and the gain matrix \( F(\hat{\theta}) \) so as to ensure quadratic stability and an \( L_2 \)-gain bound \( \gamma > 0 \) from the exogenous signal \( \omega(t) \) to the output error signal \( \tilde{z} \), i.e.

\[
\int_0^\infty \tilde{z}^T \tilde{z} dt \leq \gamma^2 \int_0^\infty \omega^T \omega dt \quad \text{for all} \quad \theta(t) \in \Delta \quad \text{and} \quad \omega(t) \in L_2(0, \infty)
\]

(17)

for zero-state initial conditions.

**Theorem 1:** The error system (15) and (16) is stable and its \( H_{\infty} \) disturbance attenuation is no more than \( \gamma \) if there exist \( H_0, H_i (i=1 \cdots N) \) and \( P > 0 \) such that

\[
\begin{bmatrix} \tilde{A}(\theta) & \tilde{B}(\theta) \\ \tilde{A}(\theta)M + \tilde{B}(\theta)H(\hat{\theta}) \end{bmatrix} \begin{bmatrix} \tilde{z}(t) \\ \bar{v}(t) \end{bmatrix} \leq 0
\]

(18)

where, \( * \) denotes the symmetric part, \( M = P^{-1}, \quad H(\hat{\theta}) = H_0 + \sum_{i=1}^{N} H_i \hat{\theta}, \quad H_0 = F_0 M, \quad H_i = F_i M, \quad i=1 \cdots N \)

and also if \( \hat{\theta}(t) \) is determined according to the adjustment law
Moreover, parameter setting (18) implies and inequality, from $V_7(\theta) = e^{V_7(Tj_0)}$ where $e^{-A_{\text{max}}T}$ is the derivative supmax $\mathcal{E}(X)$ and matrices are adjusted on-line, the problem can be reduced to check (18) for all $\theta(t), \hat{\theta}(t) \in \Lambda_{w_0}$, where $\Lambda_{w_0}$ is a convex set of positive definite ellipsoids.}

Remark 1: The adjustable parameter $\hat{\theta}(t)$ satisfies $\hat{\theta}(t) \Sigma^2 \hat{\theta}(t) = 0$, which means that $\hat{\theta}(t)$ is adjusted on the boundary surface of the prespecified ellipsoidal set $\Delta$.

Remark 2: In order to transform (11) to a convex problem, a substitute set for the ellipsoidal set $\Delta$ can be used in Theorem 1, that is $\tilde{\Lambda} = \{\theta(t) \in R^n | \theta(i) \leq \sigma_i, i = 1, \ldots, N\}$

Then since $\theta(t), \hat{\theta}(t)$ appear affinely in (11), the problem can be rephrased to check (18) for all $\theta(t), \hat{\theta}(t) \in \tilde{\Lambda}_{w_0}$, where $\tilde{\Lambda}_{w_0}$ is the set of $2^n$ vertices of $\tilde{\Lambda}$.

Remark 3: The error feedback gain $F_0, F_1, \ldots, F_N$ can be chosen to minimize the $L_2$ gain ($H_\infty$ norm) from disturbances $\omega_i(t)$ to state error $\tilde{e}(t)$, that is

$$\min \delta \text{ s.t. (18) and } M > 0$$

where $\delta = \gamma^2$.

C. Adjustable Controller Design for Target Model

Since the parameter $\hat{\theta}(t)$ is available on-line, we will establish $\nu(t)$ in a state feedback form with a parameter-dependent gain $K(\hat{\theta})$

$$\nu(t) = K(\hat{\theta}(t)) \bar{x}(t)$$

where

$$K(\hat{\theta}(t)) = K_{\omega} + \sum_{i=1}^{N} \theta_i(t) K_i$$

Substituting (25) and (26) into (11) results in the closed-loop form

$$\dot{\bar{x}}(t) = (\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta}) K(\hat{\theta})) \bar{x}(t)$$

Choose cost function

$$J = \int_0^\infty (\bar{x}^T \bar{Q} \bar{x} + \nu^T R \nu) dt$$

Theorem 2: The target model (11) with the adjustable controller (25) is stable if there exist a positive definite matrix $S$ and matrices $W_0, W_i, M_i (i = 1, \ldots, N)$ such that

$$W_0 + \sum_{i=1}^{N} W_i M_i \geq 0, M_i \geq 0, i = 1, \ldots, N$$

By pre-and post-multiplying formula (24) by diag$\left(\begin{array}{lll} P^{-1} & I & I \end{array}\right)$, we can get that $\phi(\theta) < 0$ is equivalent to (18).
where
\[ \Xi = \tilde{S}^T + W^T \tilde{B}^T \tilde{A} \tilde{S} + A \tilde{B} W \tilde{A} + \sum_{i=1}^{N} \tilde{A}_i \tilde{M}_i \]
\[ S = P^{-1}, \quad W(\tilde{A}) = W_0 + \sum_{i=1}^{N} \tilde{A}_i \tilde{W}_i, \quad W_0 = K_o S, \quad \tilde{W}_i = K_i S \]
\[ i = 1 \cdots N. \]

Moreover, the upper bound of the cost function (28) with respect to \( \tilde{A}(t) \in \tilde{\Delta} \) is given as
\[ J \leq \tilde{x}_i^T(0) \tilde{P} \tilde{x}_i(0) = \tilde{x}_i^T S^{-1} \tilde{x}_i \quad \text{for all} \quad \tilde{A}(t) \in \tilde{\Delta} \quad (31) \]

**Proof:** Chose the following Lyapunov function
\[ V = \tilde{x}_i^T(0) \tilde{P} \tilde{x}_i(0) \]
Then if follows
\[ J \leq \int_0^T \tilde{x}_i^T \tilde{P} \tilde{x}_i + v^T \tilde{W} \tilde{V} + \frac{d}{dt} \tilde{V} \tilde{x}_i(0) \tilde{P} \tilde{x}_i(0) \]
\[ \leq \int_0^T \tilde{x}_i^T \tilde{P} \tilde{x}_i + \tilde{x}_i^T(0) \tilde{P} \tilde{x}_i(0) \]
where
\[ \tilde{P} = \tilde{M} \tilde{S}^{-1} \tilde{M} \]

The following is the algorithm to optimize the cost function (28) of target model.

**Algorithm:** The cost function \( J \) is minimized if the following optimization problem is solvable
\[ \underset{\tilde{x}_0, \tilde{w}_i}{\text{Min}} \quad \text{Tr}(Z) \quad \text{s.t.} \quad (29), (30) \quad \text{and} \quad \begin{bmatrix} Z & I \\ I & S \end{bmatrix} \geq 0 \]

After designing adjustable controller \( v(t) \) for target model, now the controller \( u(t) \) has the following form
\[ u(t) = K^T \tilde{x}_i(t) + F^T \tilde{A}(t) \tilde{x}_i(t) \]
where \( \tilde{A}(t) \) is determined by (19), \( K_o, K_i, F_0, F_i, i = 1 \cdots N \) can be obtained from Theorem 1 and Theorem 2, respectively.

Now, we can conclude that the total system is guaranteed to be stable because both target model and error system have been stabilized.

**IV. Simulations**

The proposed control algorithm is verified by the simulation model obtained from the helicopter-on-arm platform, shown as Fig. 2. A small-scale electrical helicopter is mounted at the end of a two-DOF arm, while the weight of the helicopter is perfectly balanced at the other side of the arm. First, the parameters of the nonlinear yaw dynamic model are identified and followings are the result:
\[ \phi = r \]
\[ \dot{r} = k_r \dot{r} + k_{\theta_p} \theta_p \]
where \( k_4 = -1.38, k_2 = 63.09, k_3 = 11.65, k_4 = -0.14, k_5 = -3.33 \), \( \Omega = 1200 \). System (32) is linearized and the system matrices are as follows:
\[ A_k = \begin{bmatrix} 0 & 1 \\ -3.33 & -1.38 \end{bmatrix}, \quad A_i = \begin{bmatrix} 0 & 0 \\ -3.33 & 0 \end{bmatrix} \]
\[ B_o = \begin{bmatrix} 0 \\ 72.32 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 72.32 \end{bmatrix} \]
The unknown parameter uncertainty \( \theta \) is assumed to vary within an ellipsoidal set expressed as
\[ \Delta = [\theta] [\theta]^T \Omega \leq 1, \quad \Omega = \text{diag}(0.5, 0.3, 0.2) \]

We choose the controlled output matrices as
\[ C = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = 1 \]
In the following simulations, the initial conditions are:
\[ \phi(0) = 0, \quad r(0) = 0 \]
The tracking command of \( \phi \) is
\[ \phi(t) = -30, \quad 0 \leq t \leq t_{off} \]
and the following disturbance is used:
\[ \phi(t) = 0, \quad t < 0 \]
\[ \phi(t) = 5, \quad 0 \leq t \leq 6 \]
\[ \phi(t) = 0, \quad t > 6 \]

An adaptive robust tracking controller is designed to control yaw model of the helicopter using the proposed approach in Section III. We can get the gains \( F_i, K_i, i = 0, \cdots, 3 \) of robust controller:
\[ F_o = [3.44 -7.63 -2.43] \]
\[ F_1 = [0 \quad 0.20 \quad 0] \]
\[ F_2 = [0 \quad 0 \quad 0] \]
\[ F_3 = [-0.29 \quad 0.64 \quad 0.20] \]
\[ K_0 = [0.95 \quad -1.61 \quad -0.96] \]
\[ K_1 = [0 \quad 0 \quad 0] \]
\[ K_2 = [0 \quad 0 \quad 0] \]
\[ K_3 = [-0.08 \quad 0.14 \quad 0.08] \]
We have also implemented the fixed gain robust control [16] with \( K_4 = [3.70 \quad -8.20 \quad -2.60] \) on yaw dynamic of the helicopter.

From Fig. 3 and Fig. 4, it is easy to see that using the proposed robust controller the closed-loop nonlinear system is stable and has zero tracking error even in presence of disturbance. And also the proposed method improves the transient behaviour of the robust stat-feedback control with fixed gain.

Summarizing these simulations, it is noted that the proposed robust controller design method can improve the system performance in the presence of disturbance and time-varying
V. CONCLUSIONS

In this paper, a new robust tracking controller design method is proposed for linear systems with time-varying parameter uncertainties. A linear time-invariant system with time-varying ellipsoidal uncertainty is considered. The proposed method with adaptation mechanism can reduce conservatism inherent in a robust control with a fixed gain and improve transient performance in time-response. The application of this approach to the yaw control of a small-scale helicopter has demonstrated to provide superior performance comparing with conventional robust tracking controller with fixed gain.

REFERENCES


