SEAFLOOR TRANSPONDERS CALIBRATION USING PERPENDICULARS INTERSECTION

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Seafloor transponders can track underwater targets, for instance, the underwater robots, in $x$-$y$ space from their received acoustic signals, if the transponder positions are known exactly. However, it is not always possible to deploy the transponder precisely, so a calibration phase is needed to estimate the position of each transponder before doing any tracking or localization. An acoustic transponder can be calibrated from a towed interrogation transducer. We derive a calibration method called perpendiculars intersection (PI), which is different from traditional methods for the transponders in the deep ocean. Based on the solid and analytic geometry, this method is used to obtain robust and high-resolution acoustic calibration process. The PI method is formulated combing the selection of measurement data, geometry and the motion dynamics of the moving interrogation transducer. In addition, a least square estimate (LSE) is presented using the outlier rejection of Chauvenet criteria, along with a Markov process that actually simulates the motion of the research vessel. The Mont-Carlo simulation shows that the error of the estimated transponder position is much less than the GPS error. The probability statistics method of the confidence interval with the confidence degree 0.99 on the transponder position estimates is also derived to prove that the PI calibration method is exact and robust. The performance of the calibration algorithms is demonstrated on the simulation data. The results show that the position estimates error is about 0.4 m when the size of GPS error is 3 m.

1. Introduction

Acoustic transponders at the bottom of the deep ocean can be used to localize and track targets through acoustic ranging methods (Vaganay, et al. 1996; Matos, et al. 1999; Kussat, et al. 2005; Li, et al. 2006). There has been some previous work in calibrating acoustic transponders using beacons or moving sources (Osler&Beer, 2000; Hiroshi, et al., 2000, 2002). The calibration scenario considered in the paper (Hiroshi, 2000, 2002) employs a method called MIL (Method of Incremental Layers), using the basic nonlinear equations obtained by a variation principle about ray acoustics, to determine the transponder position in the horizontal plane. The multi-path effects are ignored, while deriving these calibration methods. Moreover, it is assumed here that the calibration device can acquire the depth of the transponder at the bottom in the deep ocean. In fact, the depth of the seafloor transponder is not available before the calibration. Besides that, the sources of error, which are associated with position of the interrogation transducer and the two-way travel time from the interrogation transducer to each transponder, are not considered.

Osler&Beer (2000) introduce two algorithms, which are EGS (Exhaustive Grid Search) and FitDS (Objective Function Minimization) to determine the absolute geographic coordinates and depths of multiple acoustic transponders given slant range measurements from known interrogation positions. Both algorithms are based on principle of the intersection of the three spherical shells, the centers of which are interrogation positions. A transponder location survey is performed by circling the transponder on a specifically radial while simultaneously collecting two-way acoustic propagation times and GPS data. The algorithms adopt the depth-averaged speed of sound in water, ignoring the ranging error introduced by acoustic ray curves and velocity variation.

Ji (2006) propose a new calibration method, which is called Perpendiculars Intersection, but it remains a few problems needed to improve. Due to vessel motion, ocean swells and cur-
rents, the ship track is not a straight line. The improved PI calibration method is proposed here, which has been easily utilized to fix on the geographic coordinates in the maritime trial in October 2006. To analyze the effect introduced by the above factors on the transponder position estimates, Monte-Carlo simulation is used with the Markov process of vessel motion and the GPS error.

The organization of the paper is as follows. Section I introduces the principle and experimental procedure of PI calibration. Section II formulates the problem and presents the PI calibration with the least square estimates and outlier rejection. Section III gives computer simulations of typical scenarios. Conclusion is provided in Section IV.

2. PI Calibration Method and Procedure

This calibration method (Fig. 1) applies solid geometry principia to get the coordinate of transponder. Navigating vessel sails along the two tracks with a determinate angle \( \alpha \). It is likely to find the point in each track which is the nearest to transponder and make perpendiculars to the tracks respectively across the point. Then the location of the transponder in the horizontal plane is just the intersection of the two perpendiculars. The detection time-delay of transponder doesn’t affect the calibration accuracy if only the time-delay distribution is as much as regular. Independent of the depth, PI calibration possesses well engineering practicality because of the simple calibration process and acoustic measurement system.

![Fig.1. The sketch map of PI calibration](image)

Experimental procedure of PI calibration with the long positioning system (LBL) of the AUV is much similar to the one described by Osiel & Beer (2000). In the maritime trial of the underwater robot, acoustic transponders, which the underwater robot utilizes to position or navigate it, are fixed to a rope tethered to the seabed. The geographic locations and depths of the pieces of equipment are required for the conduct of various experiments. These are determined by towing an interrogation transducer from the research vessel. An interrogation transducer is mounted on a heavy plumb and towed behind the research vessel at a depth of 15 to 50 m according to the sound depth-speed profile in the trial district. The transducer transmits interrogation pings at 12.5 kHz in CW pulse, typically every 30 s. The position of the interrogation transducer for each ping is determined by recording the GPS position of the vessel, typically with a differential correction, and the horizontal offset of the interrogation transducer from the antenna (the layback). The layback correction assumes that the interrogation transducer is located directly at the stern of the research vessel, and is typically 30 m. The receiver transmits the outgoing pings and processes the received signals to determine the two-way travel time from the interrogation transducer to each transponder. The transponders are configured to reply at discrete frequencies in the band from 12 to 14 kHz; the threshold of the receiver channel is tuned to the discrete frequencies as required.

There are errors associated with position of the interrogation transducer and the two-way travel time from interrogation transducer to each transponder. The geographic position of the GPS antenna on the research vessel will have errors that depend on the mode in which the GPS system is operating. The layback correction includes a fixed component for the distance from the
antenna to the tow point on the research vessel and a variable component for the distance from the tow point to the interrogation transducer. The second component has to be estimated from the length of cable deployed and its slope. During the trial in South China Sea, approximately 60 meters of cable is deployed.

There are errors associated with the two-way travel time measurements, principally due to a delay in the matched filter processing in the receiver. It is a non-trivial function of the bandwidth and signal-to-noise ratio required for detection. This delay has been calibrated for a variety of pulse lengths, transmission power levels, receiver gain settings, and transponder frequencies. For typical settings of the electronic circuit of the transponder, the time-delay is 4.2±1.1 ms, though the uncertainty can increase substantially if inappropriate pulse lengths or receiver gain settings are selected.

3. PI Calibration Method

In this section, we present an improved PI method for the horizontal position of the transponder, given the local measurement of propagation time at an interrogation point of the transducer. The research vessel sails along the calibration track, simultaneously collecting two-way acoustic propagation times and GPS data. The position of the seafloor transponder would be searched via the perpendiculars intersection as shown in Fig.2. Point O represents seafloor transponder and $O$ represents the projection in horizontal plane of bottom transponder. The transducer pings every 30 seconds at the interrogation point ($A_i B_i, i = 1, 2, ..., n$) in the calibration track. The geographic position of the point is supplied by GPS device. The total numbers of interrogation points are $n$ and $m$ in each track respectively, and $l = ceil(n/2)$.

![Fig.2. The calibration tracks are neither straight, nor parallel with coordinate axes.](image)

Line segment $OA$ is vertical to line segment $A_i A_{i+1}$, and Line segment $OB$ is vertical to line segment $B_i B_{i+1}$. Thus line segment $OA$ in horizontal plane is vertical to $A_i A_{i+1}$, and line segment $OB$ in horizontal plane is vertical to $B_i B_{i+1}$. The intersection $O$ of line segments $A_i A_{i+1}$ and $B_i B_{i+1}$ just is the projection of point $O$. We obtain the first horizontal position estimate of the seafloor transponder. Then we can get $l$ position estimates using the same process. The average of position estimates is the optimal result.

3.1 PI Solution

We convert the geographic coordinates of interrogation points into the relative coordinates system. The known interrogation point supplied by GPS and the layback correction is defined as $X_j = [x_j, y_j]^T$, using the following relations:

$$x_j = (L_j - L_0) \cdot l_F \cdot F_0 (1), \quad y_j = (F_j - F_0) \cdot l_F$$

(2)

Where \(L_j\) and \(F_j\) - the longitude and latitude of \(j-th\) interrogation point; \(L_0\) and \(F_0\) - longitude and latitude of the point we have taken as coordinates beginning; \(l_F\) - distance equivalent to latitude
angle unit; \( l_p(1') = 11120 \text{m}; l_p(1) = 1852 \text{m} \). Using \( \chi_{11}^A, \chi_{21}^A, \chi_{11}^B \) and \( \chi_{21}^B \), the slopes of the line \( A_{i-m}A_{i} \) and \( B_{i-m}B_{i} \) are given by

\[
\begin{align*}
    k_a &= \frac{\chi_{11}^A(2) - \chi_{11}^A(1)}{\chi_{21}^A(1) - \chi_{11}^A(1)} y^A - \frac{\chi_{11}^A(2) - \chi_{11}^A(1)}{\chi_{21}^A(1) - \chi_{11}^A(1)} x^A, \\
    k_b &= \frac{\chi_{11}^B(2) - \chi_{21}^B(2)}{\chi_{21}^B(1) - \chi_{11}^B(1)} y^B - \frac{\chi_{11}^B(2) - \chi_{21}^B(1)}{\chi_{21}^B(1) - \chi_{11}^B(1)} x^B.
\end{align*}
\]

Where \( m = 1, 2, \ldots, I \). In order to obtain the equations of the line \( O'A \) and \( O'B \), the coordinates of points \( A \) and \( B \) must be determined. Let \( c_n \) denote the average speed of sound in the sea, \( \tau^A, \tau^B \) and \( \tau^B \) represent the two way travel time of sound at points \( A_{i-m}, A_i, B_{i-m} \) and \( B_i \). If a two-way travel time has a great deviation to its neighbor ones, it should be eliminated. The distance between the interrogation transducer and the bottom transponder equals \( c_n \tau \). Then the length of line segment \( A_{i-m}A_i \) and \( B_{i-m}B_i \) are calculated by

\[
\begin{align*}
    A_{i-m}A_i &= O_i \cdot \cos \angle A_{i-m}O_i A_i = O_i \cdot \frac{(\chi_{11}^A)^2 + (\chi_{21}^A)^2 - (\chi_{11}^A)^2}{2 \chi_{11}^A}, \\
    B_{i-m}B_i &= O_i \cdot \cos \angle B_{i-m}O_i B_i = O_i \cdot \frac{(\chi_{11}^B)^2 + (\chi_{21}^B)^2 - (\chi_{11}^B)^2}{2 \chi_{11}^B}.
\end{align*}
\]

The selection of the pair of points, such as \( A_{i-1}A_{i+1} \) and \( B_{i-1}B_{i+1} \), shown in the Fig.2, which are used to find the foot of the perpendiculars is considered for compensating the random disturbance because of the motion dynamics of the research vessel. For the \( \triangle A_{i-1}A_i O \) and coordinates of point \( A_{i-m} \) are known, the coordinates of \( A_i \), so as well \( B_i \) because of the same principle, is given by

\[
\begin{align*}
    x_i^A &= x_i^A + \frac{\chi_{11}^A}{\chi_{21}^A} \cos \beta_a, \\
    y_i^A &= y_i^A + \frac{\chi_{11}^A}{\chi_{21}^A} \sin \beta_a, \\
    x_i^B &= x_i^B + \frac{\chi_{11}^B}{\chi_{21}^B} \cos \beta_b, \\
    y_i^B &= y_i^B + \frac{\chi_{11}^B}{\chi_{21}^B} \sin \beta_b.
\end{align*}
\]

Where \( \beta_a = \arctan(k_a), \beta_b = \arctan(k_b) \).

Vertical to lines \( A_{i-1}A_i \) and \( B_{i-1}B_i \), lines \( O'A_i \) and \( O'B_i \) can be determined from equations (3)\&(4)\&(6)\&(8). The point \( O \) is just the one of intersection of lines \( O'A_i \) and \( O'B_i \). Thus the horizontal coordinates of the seafloor transponder are gained by the following expression

\[
\begin{align*}
    x_i^O &= \frac{y_i^O - y_i^O + (k_a y_i^O - k_a y_i^O)}{1/k_a - 1/k_a}, \\
    y_i^O &= \frac{-1}{k_a} (x_i^O - x_i^O) + y_i^O.
\end{align*}
\]

Where \((x_i^O, y_i^O)\) denotes the coordinates of the transponder respect to lines \( O'A_i \) and \( O'B_i \). Due to various measurement errors, it is necessary to utilize the Least Square Method to estimate the actual value. The observation equation is

\[
\chi_i^O = \hat{X} + \varepsilon_i.
\]

Where matrix \( \chi_i^O = [x_i^O, y_i^O]^T \) is the observation vector, and \( \varepsilon_i \) is an additive noise. The LSM estimate \( \hat{X} \) would be obtained by minimizing the expression

\[
\min \sum (\chi_i^O - \hat{X}^T) (\chi_i^O - \hat{X}^T) = \min [\chi_i^O - \hat{X}^T + \varepsilon_i^O - \hat{Y}^O]^T.
\]

Thus
\[
\frac{\partial}{\partial x} \sum_{i=1}^{n} (x'_i - \bar{x})^2 = 0, \quad \text{And} \quad \frac{\partial}{\partial y} \sum_{i=1}^{n} (y'_i - \bar{y})^2 = 0
\]  
(12)

Through the calculation about the equation (12), we obtain the following results

\[
x = \frac{1}{l} \sum_{i=1}^{l} x'_i = \bar{x}, \quad \text{And} \quad \bar{y} = \frac{1}{l} \sum_{i=1}^{l} y'_i = \bar{y}
\]  
(13)

The equation (13) means that the position vector estimates can be obtained via the average of each component of the total coordinates. The Chauvenet criteria are used to eliminate the outliers of the position solutions of the equation (9). Define \( \phi(\omega) \) as the normal distribution function, a position solution \( x'_m \) or \( y'_m \) is considered false if

\[
| x'_m - \bar{x} | > \omega \sigma_x, \quad \text{Or} \quad | y'_m - \bar{y} | > \omega \sigma_y
\]  
(14)

Where \( \omega_x \) is a parameter satisfying this equation \( \phi(\omega_x) = 1 - \frac{1}{4n} \). The statistical variables \( \sigma_x, \sigma_y \) are formulated as \( \sigma_x = \left[ \frac{1}{n-1} \sum (x'_m - \bar{x})^2 \right]^{1/2}, \sigma_y = \left[ \frac{1}{n-1} \sum (y'_m - \bar{y})^2 \right]^{1/2} \). \( \omega_x, \sigma_x, \sigma_y \) are chosen to prevent spurious position solutions from being considered in the estimation of the transponder position. Then we transform the horizontal coordinates into geographic coordinates using the inverse transform formulas (1) and (2).

There is another error source that the transceiver moves a little distance between the transmission and reception of sound. If the research vessel sails very slowly, the ratio of the speed of the transceiver to the one of sound is much small so that it is rational to assume that the motion of transceiver is negligible during the two way time of sound.

The sensitivity of the slope of the line determined by the two chosen points is obviously affected by the relative position of the two points in the reference frame. Let \( k = \frac{y_2 - y_1}{x_2 - x_1} \), where \((x_1, y_1)\) and \((x_2, y_2)\) represent the two interrogation points in the calibration track. The relation between the slope and the relative position is given by

\[
\frac{\partial k}{\partial (x_1 - x_2)} = -k \cdot \frac{1}{x_1 - x_2}, \quad \frac{\partial k}{\partial (y_2 - y_1)} = k \cdot \frac{1}{y_2 - y_1}
\]  
(15)

If \(|x_1 - x_2| > 600 \) and \(|y_2 - y_1| > 600 \), then it can be argued that GPS error with the size of 30m has a very small impact on the sensitivity of the slope. This leads to an intuitive result: when the distance between the two interrogation points is large and the research vessel sails in the calibration track with some extent of a yaw with respect to \( x \)-axis, the perturbation of the slope \( k \) error is tiny, without much sensitive to GPS error.

3.2 Confidence Interval Estimation of the Position Error

Assume that the position measurements of the transducer at each time are supplied by the GPS device and the layback correction. It is important to determine the exactness of the position estimates error. We define \( \xi \) as a position error in the 2D plane.

\[
\xi = \sqrt{\xi_x^2 + \xi_y^2} = \sqrt{(\hat{x} - x)^2 + (\hat{y} - y)^2}
\]  
(16)

Where \((x, y)\) is the actual position of the bottom transponder, \( \hat{x} = (\hat{x} - x) \) and \( \hat{y} = (\hat{y} - y) \) represent the deviations of coordinate components of the position estimates respectively. Assume that the random variables \( \xi_1, \xi_2, \ldots, \xi_n \) are independent of each other, and obey the same probability distribution: the \( \mu = E(\xi), \sigma^2 = D(\xi) > 0 (i = 1, 2, \ldots, n) \). According to the central limit theorem under the condition of a large sample, we use the maximum likelihood (ML) method and unbiased estimator to estimate both the mean value \( \hat{\mu} \) and standard deviation \( \hat{\sigma} \). The unbiased estimation variables are expressed as follows
\begin{equation}
\hat{\mu} = \bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i, \ \hat{\sigma} = S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{X})^2}
\end{equation}

Where the statistical variables \( \bar{X} \) and \( S \) represents the mean and standard deviation of the sample.

We can approximate the random variable \( Y = \frac{\bar{X} - \mu}{S/\sqrt{n}} \) with a normal probability density

\begin{equation}
\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim N(0,1)
\end{equation}

with the large sample. Given the confidence degree \( \alpha (0 < \alpha < 1) \), the confidence interval of the position error can be expressed as

\begin{equation}
(\bar{X} \pm S/\sqrt{n} \cdot z\alpha/2)
\end{equation}

where the confidence degree \( \alpha \) satisfies the equation \( P\left(\left|\frac{\bar{X} - \mu}{S/\sqrt{n}}\right| < z\alpha/2\right) = 1 - \alpha \).

4. Simulations

The GPS errors are modeled as IID Gaussian, \( \nu_x \sim N(0, \sigma_x^2 I) \). In many practical scenarios, the size of the GPS errors is generally on the order of 3~30m with different kind of GPS device. The motion dynamics of the research vessel in the calibration track can be regarded as Markov process. We model the perturbation of the yaw with one step transition probability at j point in the track as zero mean IID Gaussian, \( \omega_y \sim N(0, \sigma_y^2 I) \). According to the steering experience of the research vessel, the yaw deviation is about from 3° to 10° at the sea situation inferior to level 4. Then we can observe the effects of both the GPS error and the motion of the vessel with the non-straight-line track on the final calibrated position estimate. Calibration is repeated 1000 times with three different standard deviations of GPS error (3m, 15m and 30m), using two intersectant tracks, as shown in Figure 3. The y-axis represents the \( \xi \) and \( \xi_y \) respectively defined in the (14). PI estimates using acoustic data are shown with solid line marked with dots.

![Fig.3. The simulation results of the PI calibration](image)

The distance between the two chosen points in the calibration track is greater than 900 m. Fig.3 shows the results of a Mont-Carlo run for the calibration algorithms in the two tracks (Fig.4). It demonstrates that the effects of the GPS errors on the PI solution are obvious in the scope of the errors. However, in general, the authors observed that the PI algorithm is not sensi-
tive to GPS errors because of the error dots trending to the center. In Fig.3 (a), (b) and (c), the slope of the estimated position errors is much less than the GPS error, including the cases with tiny error, minor error and a substantive amount of error.

Using computer simulations, our objectives are to 1) show the effects of the GPS errors on the estimation performance of the various sizes of errors, and 2) demonstrate the PI calibration algorithms and measurement principle. The simulation examples use two calibration vessel tracks with a constant speed of 2 knots. The layback correction is assumed accurate. One track starts at (0, 1500) with an initial yaw 15°, and the other starts at (1500, 0) with an initial yaw 75°. Assume that the yaw deviation \( \sigma_y = 8° \). The transponders are all located with a depth of 3000m at (3000, 3000). The interrogation transducer collects the two ways time of sound every 30s. The total time for the calibration in each track is 100.5 minutes, and the GPS error standard deviation is \( \sigma_x = 3 \) m, 5m and 30m.

![Fig.4. The sail tracks in PI calibration](image)

The transponder is situated at (3000,3000) In horizontal plane. Calibration vessel with Markov process completes two tracks beside the transponder for calibration, corresponding to a 201 minutes run. Using the results of the above simulation, table 1 compares the performance of PI algorithms with the statistical variables defined in the expression (14) which are the mean, the standard deviation of the sample and

<table>
<thead>
<tr>
<th>( \sigma_x ) (m)-</th>
<th>( \bar{x} ) (m)-</th>
<th>( S ) (m)-</th>
<th>Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS Error</td>
<td>Mean</td>
<td>Standard Deviation</td>
<td>Interval (m)</td>
</tr>
<tr>
<td>3</td>
<td>0.4037</td>
<td>0.2255</td>
<td>0.3802–0.42</td>
</tr>
<tr>
<td>15</td>
<td>1.9334</td>
<td>1.0823</td>
<td>1.8205–2.04</td>
</tr>
<tr>
<td>30</td>
<td>3.9789</td>
<td>2.2559</td>
<td>3.7434–4.21</td>
</tr>
</tbody>
</table>

The confidence intervals with the confidence degree \( a \) is 0.99. We find that the mean of the position errors is only the about 13 % of the GPS error with the approximation of the data of the first column dividing the data of the second one in the table. Moreover, the lengths of the confidence interval for the simulation are very short. Therefore it indicates that the position estimates using the PI algorithm has a favorable exactness. Especially, the position estimates error is only about 0.40m when the 3 m error of GPS device is used in the calibration.
5. Conclusions

A PI calibration method for calibrating the seafloor transponder, which is different from the traditional methods in the deep ocean, was demonstrated given a calibration target that carries a GPS system. The PI solution was demonstrated and compared with the different extents of the GPS errors. Through the performance of the simulation, the PI calibration algorithm can be used as a flexible and suitable for field data processing because it can incorporate the response and motion compensation directly into the calibration. In addition to owing a good exactness, the proposed method possesses an engineering practicality because of the simple operation and acoustic measurement system.

References